ACCESSIBILITY AND RESILIENCE
IN COMPLEX NETWORKS

(Preliminary Version)

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ABSTRACT

The recent enormous interdisciplinary interest in network concepts, analysis, and modelling – arising from the study of complex interconnected dynamic systems – underlines the simplicity of laws describing these phenomena. Networks often show common behaviour, based on their topological characteristics, and this connectivity behaviour is mainly modelled by exponential/power form distributions.

In other words, the topological properties of a network can give useful insights into: how the network is structured; which are the most 'important' nodes/agents; and how network topology can influence the patterns emerging from the conventional spatial economic laws (such as equilibrium theory, spatial interaction theory, etc.). This topology structure is expressed by very simple laws, and in most cases these laws can be interpreted in a spatial economic framework.

In this framework, it is still an open research issue which specific and novel contributions network analysis can offer to spatial economic analysis, and – vice versa – whether the solidity of spatial economic laws needs to be reconhanded in the light of recent advances in complexity and network theory. Hence, a dual analysis is necessary, in order to explore potential connections between these two approaches (spatial economics and network analysis).

In this context, the accessibility indicator – although expressed in a very simple form – can play a strategic role, since it may detect the most connected network centres (hubs), as well as trace their development and decline.

This paper is devoted, first, to a methodological analysis, by exploring the interrelationships between theories and models in spatial economics and network analysis. This interdisciplinary synthesis calls for further reflections on network complexity and the simplicity of the associated models and indicators – such as accessibility.

A further issue worth to be explored is the connection between accessibility and resilience. The question is whether the accessibility indicator can be related to the identification of resilience/fragility in connectivity network structures.

Current policy strategies which focus on resilience show the relevance of this issue and the need for continuing research on the links between complex transport networks, accessibility and resilience, mostly by exploring this relationship at different scale levels and its impact on the whole network.
1 BACKGROUND: COMPLEX NETWORKS

In the recent years, there has been great interest in the interdisciplinary study of complex networks\(^1\) with special reference to the relevance of the connectivity structures. Surprisingly, scientists from many different disciplines (spatial economics, sociology, physics, biology, etc.) have simultaneously faced – although from different perspectives and with different aims – the issue of the emerging interaction processes (known as emergence’ phenomenon) as the result of complex (evolving) networks of connections between the different ‘units’ involved.

The idea that underlies this approach is the belief that the topology (or architecture) of these interactions is an essential part of many processes, and it cannot be ignored without losing a crucial ingredient of the phenomena concerned (Vega-Redondo, 2007). The topology issue implies a focus on the network configuration and its properties (such as connectivity, centrality, clustering etc.), in order to analyse the related impact on the behavioural dynamics of the network itself. Over the years, this topological perspective has shed new light on the study and modelling of complex networks, by producing a great amount of models/maps/software. It is now the moment to reflect on the scientific discoveries in this modelling field, by focussing on the objectives, problems and future directions.

If, on the one hand, it is clear that all the scientific efforts have as their ultimate objective the acquisition of ‘knowledge’ (‘scientia’ in Latin), on the other hand, it also essential to build up an ‘interpretative’ phase, oriented to meditate on the existing theories and models, in the light of the possible empirical analyses and forecasts, and related policy issues. In other words, we need to explore the following methodological questions:

- Are there ‘new’ theories, or are we using – in several disciplines connected to spatial-economic science – the same ‘universal’ principles?
- How relevant is the network interaction/connectivity through space?
- How do different network topology/typology structures affect the evolutionary (complex) behaviour of actors, also in the light of the network sustainability/resilience issues?

The above ‘network embedding’ implications for complexity theory raise the issue of the search for a ‘hidden’ order/simplicity (Reggiani and Nijkamp, 2009). In this context, a dual analysis (Spatial Economic Science vs. Network Science) seems essential. This will be briefly discussed in Section 2.

Linked to the relevance of different network topologies is the issue of the different utility functions to access these networks, viz. the role of accessibility. In particular we will show how accessibility can be the ‘simplest’ model which – by embedding connectivity – can link network science and spatial-economic science, and thus capture the homogeneity/heterogeneity of the spatial system concerned (Section 3).

\(^1\) Networks (literally: operations via nets) may be interpreted as an ordered connectivity structure for dynamic spatial communication and transportation which is characterized by the existence of main nodes (vertices) which act as receivers or senders (push and pull centres), and which are connected by means of corridors and edges (links) (Nijkamp and Reggiani, 1998). For the definition of complexity, see Section 2.2.
Starting from these considerations, the next section will explore the relationships between accessibility and an interesting concept linked to the robustness of the network: i.e. the resilience (Section 4).

Finally, the concluding section will offer some reflections and suggestions for future research (Section 5).

2 COMPLEX NETWORKS AND SIMPLE LAWS

2.1 Prologue

As previously outlined, much emphasis is currently being given to the complexity of the spatial dynamic interrelationships – and related feedback network effects – between transport, land use, and socio-economic systems with, inter alia, a view to their impact on sustainability. Such a ‘network perspective’ is considered essential for understanding the relationships between (physical and virtual) flows and related socio-economic activities (Reggiani, 2009).

The next section will explore the main questions raised in Section 1. In particular, we will briefly summarize the main findings in both spatial economics and network analysis, with reference to the main models that aim to identify the (evolutionary) network patterns, in the light of their capability to capture the essential driving forces of the complex phenomena concerned.

2.2 Complexity vs. Simplicity in Spatial Economics

Defining complexity is fraught with many difficulties. Horgan, in 1995, provided a list of more than 30 known definitions of complexity2 (Reggiani, 2004). We will refer here to two interlinked meanings. Simon in 1962 refers to complexity as the large number of parts that interact in a non-simple way. Casti in 1979 conceives of complexity as the mapping of a system’s non-intuitive behaviour, particularly the evolutionary patterns of connections among interacting components of a system whose long-run behaviour is hard to predict. In particular, Casti (1979) offers the following classification:

a. *Static complexity*. refers to the network configuration (e.g. the type of hierarchical structure, the connectivity pattern, the variety of components and the strength of interactions), where the components are put together in an interrelated and intricate way.

b. *Dynamic complexity*. concerns the dynamic network behaviour governed by nonlinearities in the interacting components (e.g. computational complexity and evolutionary complexity). The latter can be measured by means of nonlinear evolutionary models, such as chaos models, which are able to identify the dynamic (random) network patterns.

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2 For a review of the various notions of complexity, see, among others, Donaghy (2009) and Seel and Waters (2009).
Modelling network complexity can, in principle, be pursued by following two different approaches (Reggiani 2004): i) the deductive approach: complexity emerges from the (static or dynamic) equations which model the network concerned; and ii) the inductive approach: complexity emerges from data analyses by means of appropriate approaches.

With reference to the above notion of static complexity, we can observe that the most widely used transport models, which aim to map out the static complexity, by planning and predicting transport network behaviour at different scales of analysis, have their roots in spatial economics.

Spatial economics seeks to identify the factors governing the distribution/location of economic activity over space. The fundamental models in spatial economics can be summarized as follows:

a) The rank-size rule/Zipf’s law (Zipf, 1949);

b) Gravity Models (Isard, 1954);

c) Spatial Interaction Models (Wilson, 1967; 1970);

d) Discrete Choice Models (McFadden, 1974).

These four types of models are static models, with very simple formulations. Their common characteristic is that space is measured in the form of cost/utility function (at the aggregate or disaggregate level). It is well known that we can observe an analytical compatibility between the Spatial Interaction Model (SIM) and all the other models (Reggiani, 2004; 2009). In particular, we can note that:

- SIM is linked to the rank-size rule and gravity (Newton) model;
- SIM is linked to statistical information principles and entropy maximization (Wilson, 1967);
- SIM is linked to logit model and thus to micro-economic theory (stochastic utility maximization) (Anas, 1983).

By considering a dynamic setting, we can expect to find the same ‘constancy’ of the ‘universal’ law of the spatial interaction form in an evolutionary complex perspective. It can be shown that SIM represents the steady state of network evolution. Vice versa, the dynamic version of the logit form (compatible with SIM) leads, under particular assumptions, to the well-known (Verhulst) logistic function (for a demonstration, see Nijkamp and Reggiani, 1992).

In other words, the above simple ‘laws’ a)-d), and in particular SIMs, appear to be the conceptual and operational instruments able to ‘decode’ the complexity of the space-time phenomena concerned (Reggiani, 2009). Thus, the search for a ‘hidden’ order/simplicity seems to have governed the scientific arena in spatial economic science in the past few decades. On the other hand, network analysis, which aims to study network representations of physical, biological, and social phenomena leading to predictive models of these phenomena\(^3\), again underlines these ‘simplicity laws’.

2.3 Complexity vs. Simplicity in Network Analysis

In the network literature, two main types of network structures have been observed, modelled, and discussed. On the one hand, the existence of homogeneous centres/nodes, where all vertices are

\(^3\) [http://en.wikipedia.org/wiki/Network_science](http://en.wikipedia.org/wiki/Network_science)
equivalent and any pair of vertices are connected with the same probability, identifies what is called the Random Network (RN) (Erdős and Rényi, 1959). In this case, the statistical distribution of the links between the centres/nodes decays is expressed by a Poisson distribution. In other words, an RN predicts that the probability $P(k)$ that a node $k$ in the network is connected to $k$ other nodes according to a Poisson distribution:

$$p(k) \propto e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!} \cdot \tag{1}$$

RNs are also called exponential, because the probability that a node is connected to $k$ other sites decays exponentially for large $k$.

On the other hand, the presence of agglomeration economies/high attracting centres (hubs) identifies a network as Scale-Free (SF), because of its intrinsic characteristic of exhibiting a power-law distribution $P(k)$ in its connectivity structure (Barabási and Albert, 1999; 2000):

$$P(k) \sim k^{-\alpha} \cdot \tag{2}$$

In particular, in the power law (2), a value of the exponent $\alpha$ between 2 and 3 implies a ‘hierarchy of hubs’; a value of $\alpha = 2$ suggests the existence of a hub-and-spoke network, centralized on a major super-connected node; and a value of $\alpha$ greater than 3 again implies an RN (Barabási and Oltvai, 2004).

Surprisingly, if we carry out a dual analysis, i.e. spatial economic analysis vs. network analysis, we find two main similarities, as follows (Reggiani, 2009):

- the statistical distribution of the nodes (with $k$ links) in network analysis revisits the rank-size/Zipf’s rule;
- the homogeneity and heterogeneity of the topological structures (SF vs RNs) in network analysis fits the homogeneity and heterogeneity of the economic centres in spatial economic analysis (which is embedded in the different impedance functions of SIMs).

In summary, the various models a)-d) applied to interaction/movement and the statistical distribution of the nodes seem to constitute two hands of the same coin (see also Section 3.1). We can infer then that the same ‘universal principles’ – although with different interpretations – are used in both spatial economics and network analysis.

### 2.4 Final Remarks

In the previous sub-sections we have outlined how the methodological instruments used for the analysis of the economic variables in spatial networks are the same as those used for identifying the underlying topological (connectivity) structures.

A further interesting aspect of network analysis (see, e.g., Albert et al., 2000) deals with the vulnerability issue associated with the type of network connectivity; this fragility problem can be then inferred also for the corresponding economic centres. In particular, the SF networks (networks with hubs) are highly resistant to random failures, since a substantial number of links can fail
without affecting the performance of the network as a whole; however, the SF networks are very vulnerable to shocks and attacks in the major hubs. In this latter case, the SF network would break down earlier than the random network.

Given the aforementioned ‘correspondence’ between network analysis and spatial economic analysis, understanding the underlying spatial network topology of urban/regional systems provides useful insights into the evolution of socio-economic patterns, particularly in the light of vulnerability/resilience matters.

In this context, in Section 3 below we show how the accessibility concept and its measure can be a ‘very simple’ benchmark instrument, which links spatial interaction and network analysis, not only from the methodological and empirical viewpoint, but also from the policy viewpoint. This latter feature will be particularly examined in the framework of resilience analysis (Section 4).

3 THE ROLE OF ACCESSIBILITY IN COMPLEX NETWORKS

3.1 Accessibility and Network Structures

Accessibility plays an important role in the ‘network’ framework. On the one hand, it relates to all the nodes in the network. On the other hand, it can explore both the slow network dynamics – characteristic of the network supply hand (infrastructure development) – and fast network dynamics – typical of the demand hand (mobility/communication increase). Consequently, accessibility may be used for investigating the (un)even distribution of economic activities in the nodes, or the (dis)equilibrium in the development of different regional performances (Martin and Reggiani, 2007; Reggiani, 2008).

The concept of ‘accessibility’ came to the fore in the 1950s, after the legitimation of gravity theory by Isard at the first Regional Science Meeting in Detroit in 1954. The ‘founding fathers’ of the accessibility concepts were, first, Hansen (1959), who defined accessibility as the potential of opportunity for interaction (p.73), and, second, Weibull (1980), who considered accessibility as a property of configuration of opportunities for spatial interaction (for a review on the accessibility concept, see Reggiani, 2008).

It should be noted that Litman (2008) highlights the following definitions:

I. ‘accessibility’ as a general concept used to describe the degree to which a product, device, service, or environment is accessible by as many people as possible;

II. ‘access’ refers to connections to adjacent properties.

Along these lines, accessibility can be viewed – in general – as the "ability to access" and the possible benefit to some system or entity. In the specific field of regional economics/geography, accessibility refers to the relative ease of reaching a particular location or area. In this context, accessibility emerges in its analytical form, as the potential of opportunities, as follows:

\[ A_i = \sum_j D_j f(\beta, c_{ij}), \]  

(3)
where $A_i$ defines the accessibility of node/zone $i$; $D_j$ is a measure (or weight) of opportunities/activities in $j$; and $f(\beta, c_{ij})$ is the impedance function from $i$ to $j$, with $\beta$ its cost-sensitivity parameter. Equation (3) has always governed the scientific literature, being the core of the various theoretical formulations (see Annex A). Additionally, it is well known that Eq. (1) also emerges as the inverse of the calibration factor of the (single or double) constrained spatial interaction models (Wilson, 1970; see again Table A1 in Annex A). In addition, Eq. (1) can be interpreted in the random utility framework as the benefit (expected utility) provided to an individual in a spatial choice situation (Ben-Akiva and Lerman, 1979; Geurs and van Wee, 2004; Leonardi, 1978; Miller 1998; Reggiani et al. 2011a; Willigers et al., 2007).

Although very simple, formulation (3) is very interesting since it encapsulates the network connectivity structure – by means of the cost matrix $c_{ij}$ – as well as the (aggregate) behavioural responses to socio-economic activities in $j$ – by means of the cost-sensitivity parameter $\beta$ embedded in the decay functions $f(\beta, c_{ij})$.

Consequently, accessibility plays a fundamental role as an instrumental tool able to link spatial-economics and network analysis. On the one hand, accessibility can be interpreted in a utility framework, thus offering an economic insight into empirical regularities emerging from (3). In fact accessibility $A_i$ is explained by the sum of the discounted economic activities (workplaces) $D_j$, by means of a generalized non-linear discounted factor $f(\beta, c_{ij})$. Hence, accessibility appears to be extremely useful for forecast analyses and related policy strategies.

On the other hand, the different forms of the impedance function $f(\beta, c_{ij})$ can reveal the topology of the connectivity structure concerning the network concerned.

In particular, different types of impedance functions might be used, depending on the topology of the opportunities (destinations) under analysis. For example, already in 1967 Wilson advised the use of a negative exponential function$^4$ for homogeneous/isotropous areas. On the other hand, in the presence of large agglomeration/metropolitan areas, impedance functions of the power type seem more appropriate, as also indicated by Richardson (1969), and by Fotheringham and O’Kelly (1989). In fact, the power-decay function has been found to be suitable for long-distance interactions, since the power decay shows a higher tail than the exponential. This means that users are able to overcome long distances/high costs to reach high attraction centres/poles (see, among others, Reggiani et al., 2011b).

It is worth noting that Fotheringham and O’Kelly (1989, pp. 11-13) discuss four issues concerning the employment of the negative power decay function vs. the negative exponential decay function. One issue is extremely significant: it emphasizes the ‘scale-free’ characteristic of the power decay (scale-independent parameter estimates) vs. the scale-dependency of the exponential-decay function. In other words, we can remark – from a spatial-economic perspective – the relevance of the negative power vs. the negative exponential function, which was also outlined by Barabási and his group – from a network connectivity perspective (Section 2.3).

Obviously, other decay functions, which express the connectivity topology in SIM and in the related accessibility, might be used as well. For example, five decay functions have been adopted in the accessibility analysis of the German commuting network of the 439 districts for 2003 and 2007 (Reggiani et al., 2011b). The five functions are:

$^4$ See also the discussion by Olsson (1980, Chapter 17, “On the Mythology of Negative Exponential”), as well as Taylor (1971).
(a) the exponential-decay function:
\[ f(c_{ij}) = e^{-\beta c_{ij}}, \]  
(4)

(b) the exponential-normal decay function:
\[ f(c_{ij}) = e^{-\beta c_{ij}^2}, \]  
(5)

(c) the exponential-square-root decay function:
\[ f(c_{ij}) = e^{-\beta \sqrt{c_{ij}}}, \]  
(6)

(d) the log-normal decay function:
\[ f(c_{ij}) = e^{-\beta (\log c_{ij})^2}, \]  
(7)

(e) the power decay function:
\[ f(c_{ij}) = c_{ij}^{-\beta}, \]  
(8)

By considering the accessibility indicator (3), where the decay \( f(\beta, c_{ij}) \) assumes the five different forms (4)-(8), accessibility in Germany embedding a power decay of type (8) shows – in both the considered years (2003 vs. 2007) – a district ranking compatible with the district hierarchy emerging from the network connectivity index (connectivity degree according to the incoming links), as well as from some network centrality indices (betweenness and co-betweenness). In other words, the most connected German districts – according to network analysis – appear to be also the more accessible from the spatial economic viewpoint, but only if we consider the district order emerging from the accessibility indicator which embeds a negative power decay. In fact, this is not the case for the other types of accessibility rankings embedding the other decay forms (exponential, exponential-normal decay, exponential-square-root, and log-normal) (Reggiani et al., 2011b).

In summary, accessibility seems to match the different topological network structures (RN, SF, intermediate), according to the different utility/deterrence functions employed.

### 3.2 Accessibility as the Driving Force in Scale-Free Networks

The results discussed above shed new light on the accessibility formulation (3). From the methodological viewpoint, accessibility can play a fundamental role as a benchmark for exploring universal principles (e.g. SIMs and network connectivity patterns). From the empirical viewpoint, accessibility can be used for testing the development of the economic activities in the network nodes towards an SF or an RN or an intermediate connectivity distribution. For example, the aforementioned results in Germany allow us to infer the commuting network tendency towards a SF network, where a privileged number of nodes, i.e. the ‘city-network’, composed of the main hubs/attractors/districts, dominates the remaining nodes. In this latter context, accessibility can be conhandled as a driving force in the (efficient) organization of SF networks. Good examples are the High-Speed train networks or the air-transport networks. In both these cases accessibility to the main hubs is fundamental for the realization of such directed networks. The same happens for other types of SF network (e.g. worldwide web, the Internet, etc.). If the (virtual and physical)
accessibility to a hub is weak, most probably this centre will not survive in its hub-function. This is because accessibility expressed as in (3), thanks to its ‘compound’ economic value, is more than proximity/access: it is the benefit of reaching the hub.

We can also consider the counterpart. On the one hand, an SF network (as in the case of the Internet, high speed train, air transport system, etc.) is the ‘simple way’ of organizing complex systems, thanks to its hubs, which need to be accessible for their efficiency. On the other hand, these hubs spread out knowledge and information diffusion, again by means of accessibility. From this angle, accessibility makes the role of mobility and interaction patterns in knowledge production operational (Karlsson et al., 2006).

We can then summarize the above discussion as follows: a) conceptually: accessibility can be interpreted as the ‘driving force’ in the development and increase of SF (hub) networks, as well as in the diffusion of knowledge/socio-economic performances; b) methodologically: accessibility can be viewed as the ‘weight’ in the preferential attachment towards a hub, by impacting a link/node creation/destruction, and thus the change of the network structure.

From the policy viewpoint, accessibility seems to be an operational instrument, able to steer the network – in the presence of complex systems – towards a more ordered/concentrated system, with different hierarchical economic levels.

In view of this, given the well-known ‘fragility’ characteristics of SF networks in the presence of attacks/shocks on its hubs (Section 2.4), it is worth exploring the relationships between accessibility and an interesting indicator of the network robustness/stability: i.e. resilience. This will be dealt with in the next section.

4 ACCESSIBILITY AND RESILIENCE

4.1 Resilience and Complex Networks

Resilience is a ‘popular’ term used nowadays in association with several threatening events which show critical and catastrophic phases (terrorism attacks to transport and digital systems, financial crises, epidemic diffusions, natural disasters like earthquakes, tsunami, fire, etc.).

“The etymology of the word ‘resilience’ is the Latin verb ‘resilio’, meaning to rebound” (Rose, 2009, p.1). In general, resilience refers to the “capacity of a system to retain its organizational structure following perturbation of some state variable from a given value” (Perrings, 1994, p. 30).

Two basic definitions of resilience are: a first concept, engineering resilience, due to Pimm (1984), “takes the resilience of a system to be a measure of the speed of its return to equilibrium” (Perrings, 1998, p. 505). A second concept, ecological resilience, “refers to the perturbation that can be absorbed before the system is displaced from one state to another. This definition, due to Holling (1973; 1986; 1992), does not depend on whether a system is at or near some equilibrium. It assumes that ecological systems are characterized by multiple locally stable equilibria, and the measure of a system’s resilience in any local stability domain is the extent of the shocks it can absorb before being displaced into some other local stability domain” (again Perrings, 1998, p. 505). Reviews of the resilience concepts are numerous (see, among others, Gibson et al., 2000; Gunderson, 2000; Peterson et al., 1998; Reggiani, 2010).
In the past few decades the resilience concept has also been studied and applied to transport networks and to spatial economic infrastructures in general (see, e.g., Cox et al., 2010; Murray and Grubesic, 2007; Matisziw et al., 2009; Murray et al., 2008; Nagurney and Quian, 2009; Reggiani et al., 2002; Schintler et al., 2007). In this context it is clear that fragile communities are more likely to be susceptible to disasters, attacks, disruptions, and, moreover, are likely to experience subsequent weakness and failure after the attacks (Allenby and Fink, 2005). This is particularly true for SF networks, which seem vulnerable in their hubs.

However, operational measures of resilience are not easy to deal with (Reggiani, 2011). An interesting approach in this respect is offered by Cox et al. (2011) who suggest – as a resilience indicator – the percentage avoidance of maximum economic disruption that a given shock could bring about. Cox et al. (2010) take – as an empirical application – the real-world case of the 2007 London subway and bus bombing. In this context, i.e. the possibility to shift to alternative modes in the presence of shocks, the improvement of (physical and virtual) accessibility certainly plays a significant role. The relationship between accessibility and resilience will be proposed in the next section.

### 4.2 Accessibility and Resilience

An interesting issue to be explored is the relationship between accessibility and resilience. In the previous sections we have highlighted that accessibility can be considered a driving force for the implementation and success of SF networks. Being aware that an SF network is vulnerable in its hubs, the question is: What is the impact of accessibility on the resilience of SF networks? In other words, can accessibility decrease or increase the resilience of the SF network?

The relationship between complex networks and resilience has been interestingly investigated in the last decade (e.g. Callaway et al., 2000; Cohen et al., 2001). Discussions have focussed on the clear identification of an SF network, in the light of resilience strategies (for a review, see Schintler et al., 2007). “From the methodological viewpoint several research issues are still open, such as:

- How do (unsustainable) sub-sets of network structures affect the resilience of the whole network?
- How do changes in the network connectivity structures lead to changes in the resilience?
- How does resilience affect the user’s behaviour and hence policy strategies?” (Reggiani, 2011, p. 8).

In this context, to the best of our knowledge there is still no research on the relationship between accessibility and resilience. We can return here to the ‘double’ effect of accessibility, previously outlined in Section 3, i.e. the possibility of spreading knowledge diffusion via the hubs.

In other words, given that the SF structures – even though productive – are fragile and critical in their hubs, an envisaged solution might be the dual-use technologies or other kinds of dual systems, i.e. the possibility of shifting to substitution possibilities, once that the network has been destroyed in one or more modalities. For instance, Allenby and Fink (2005) advise policies that encourage strong tele-working capabilities in local firms. Rose et al. (2007) suggest improving substitution possibilities for key inputs or broadening the supply chain, in the presence of terrorist...
attacks or environmental disasters. All these ‘dual’ solutions might be more efficient – and less costly – in the presence of a viable, accessible network.

In summary, nowadays accessibility is playing a fundamental role, not only in the reinforcement of SF-polycentric networks but also in the enhancement of the related resilience. We can conceive the relationship ‘accessibility-resilience’ – in the framework of SF networks – by interpreting in a new way the Environmental Kuznets Curve (EKC; Kuznets, 1955) which has been proposed and empirically demonstrated by numerous studies (de Bruyin and Heintz, 1999). Figure 1 shows an inverted U curve hypothesis, where the intensity of vulnerability (which can be considered as a particular indicator of environmental externalities in SF networks) decreases after a particular level of accessibility (which can be considered as a particular indicator of GDP in SF networks) has been reached. This particular level (maximum value $a^*$) coincides with the realization of the SF network.

![Figure 1: The Resilience-Accessibility Curve on the basis of the EKC](image)

In other words, the hypothesis in Figure 1 implies that, at the beginning of the phase moving towards the SF network, vulnerability and accessibility are at their minimum values (thus high resilience). During the transition phase towards the development of an SF network, vulnerability grows but more slowly than the rate of accessibility – and thus resilience decreases – till a maximum value $a^*$ is reached, which coincides with the realization of the hubs, which present – as previously emphasized – minimum resilience. Above this level, although the SF network persists, accessibility becomes the instrument for policy strategies that aim to reduce the network vulnerability, i.e. to enhance its resilience.

Not only in transport systems but also in socio-economic systems a community with a high prevalence of small, tight, isolated social networks is fragile, with slow information (Wallace and Wallace, 2008). By increasing their linkages, shocks and contagious processes can be prevented.
Wallace and Wallace (2008) suggest a series of policy actions in this respect, based on the reinforcement of weak ties: for example, to encourage economic integration and to provide adequate municipal and private services, especially in poor communities, etc. It is clear that accessibility is also fundamental for the development of these socio-economic networks.

5 CONCLUSIONS

This paper has aimed to review and interpret the concept of accessibility from two interlinked perspectives: spatial economics and network analysis. From the methodological viewpoint, despite its simple formulation and computation, accessibility appears to play a fundamental role: on the one hand – given its economic-utility framework – it can provide insights into the homogeneity and/or disparities/agglomeration economies of the centres; and, on the other hand – given its ‘hidden’ connectivity – it can provide insights into the underlying topological structure and related potential resilience. In the particular context of an SF network, accessibility seems to be the driving force in SF networks, making the SF system viable, in terms not only of its efficiency, but also of the enhancement of its resilience.

The above considerations might constitute the basis for continuing the research on further directions, by investigating:

- the different levels of accessibility – for different socio-economic groups and for different spatial scale levels – in a network (transport, communication, capital network, etc.);
- the different levels of accessibility for different topological networks;
- how to create or maintain the ‘optimal’ accessibility, since too much accessibility might create negative externalities (congestion, etc.).

Meta-analysis on accessibility and transferability of results could therefore constitute the final challenge in a future research agenda devoted to exploring simple models which map out the evolution of complex networks.

References


Taylor, P. J. (1971) Distance Transformation and Distance Decay Functions, *Geographical Analysis* III, pp. 221-238.


ANNEX A

In this Annex we show how accessibility expressed by the potential of opportunity is the main focus in the principal measures of accessibility.

Table A1. Principal measures of accessibility

<table>
<thead>
<tr>
<th>Conventional Name</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential of Opportunity</td>
<td>$\text{Acc}<em>i = \sum_j D_j f(c</em>{ij})$</td>
</tr>
<tr>
<td>Physical Measure</td>
<td>$\text{Acc}<em>i = \sum_j f(c</em>{ij})W_j$</td>
</tr>
<tr>
<td>Expected Utility</td>
<td>$\text{Acc}<em>i = \ln\sum_j D_j f(c</em>{ij})$</td>
</tr>
<tr>
<td>Inverse Function of Competition</td>
<td>$\text{Acc}_i = \frac{1}{A_i}$</td>
</tr>
<tr>
<td>Joint Accessibility</td>
<td>$\text{Acc}_i = \sum_j \text{Acc}<em>j D_j f(c</em>{ij})$ where $\text{Acc}<em>j = \sum_k D_k f(c</em>{jk})$</td>
</tr>
<tr>
<td>Dynamic Accessibility</td>
<td>$\text{Acc}_i(t) = \frac{1}{A_i^*(t)}$</td>
</tr>
</tbody>
</table>

$D_j$ is a measure of opportunities/activities in $j$
$D_k$ is a measure of opportunities/activities in $k$
$W_j$ is a weight calibration factor related to location $j$
$f(c_{ij}), f(c_{jk})$ are the impedance factors, where $c_{ij}$ and $c_{jk}$ are the costs, respectively, from $i$ to $j$ and $j$ to $k$
$\beta$ and $\gamma$ are the cost-sensitivity parameters to be calibrated
$A_i$ is the calibration (balancing) factor in a spatial interaction model
$A_i^*(t)$ is the calibration factor in a dynamic spatial interaction model
$t$ represents the time dimension
(Adapted from Reggiani, 1998.)