

A Dynamic CGE Analysis of Exhaustible Resources: The Case of an Oil Exporting Developing Country

By

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Abstract: An extensive literature concerned with optimal depletion of an exhaustible resource, with only a few exceptions, ignores the economy-wide and sectoral distribution effects of resource depletion. This paper presents a dynamic computable general equilibrium model to link the underlying natural resource base to economic performance. The model consists of an intra-temporal price endogenous model of a market economy, embedded in an inter-temporal optimal growth and development model. It is an optimization model that determines the optimal development path of the economy, hence, the inter-temporal depletion problem subject to workings of a multi-sector market economy. This general equilibrium approach captures the economy-wide and sectoral distribution effects of resource depletion. The model, benchmarked to Iranian data, is used to examine the issues related to optimal extraction of an exhaustible resource, optimal savings in the economy, and the allocation of investment funds.

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1 Introduction

An extensive literature concerned with optimal depletion of an exhaustible resource, with only a few exceptions, ignores the economy-wide and sectoral distribution effects of resource depletion. Typically, capital accumulation and consumption are discussed within the limited framework of the one-sector neoclassical growth models (Aarrestad 1978). These models do not consider the role of prices in influencing production and consumption decisions of firms and households, and undermine the significance of inter-sectoral interaction on the optimal depletion profile. The treatment of the optimal depletion of an exhaustible resource independently from the rest of the economy is only justified when perfect capital markets prevail.¹ Clearly, in the case of oil exporting developing countries where well functioning capital markets do not exist, the rate of resource depletion is closely related to activities in the rest of the economy. In any realistic circumstance, therefore, the intensity of interaction among various sectors and markets across the economy has significant bearing on the depletion program, as does the level of domestic and international prices. Private and public consumption and savings decisions as well as the investment allocation mechanism of a country directly affect its level of resource extraction. In these instances a general equilibrium approach that fully captures the economy-wide effects of resource depletion is the appropriate tool.

This paper presents a dynamic computable general equilibrium model to link a country's underlying natural resource base to its economic performance. The model consists of an intra-temporal price endogenous multisectoral model of a market economy,

embedded in an inter-temporal optimal growth and development model. This general equilibrium approach captures the economy-wide and sectoral distribution effects of resource depletion. The model is benchmarked to Iranian data and is used to examine the issues related to optimal extraction of an exhaustible resource, optimal savings in the economy, and the allocation of investment funds. Our interest is with general equilibrium effects of oil extraction and investment policies within a window of time during which the oil reserves abound and the oil sector plays a crucial role in the economy. Hence, the issues related to full depletion of oil and transition to a non-oil era are not considered in this study.²

Devarajan (1988) reviews the CGE applications to natural resources and taxation issues in developing countries and identifies three categories of models:

- 1- "Energy Management Models" that generally focus on energy-economy interactions³;
- 2- "Dutch Disease Models" that study the effects of an export boom on the rest of the economy; and
- 3- "Optimal Depletion Models" that take into account the exhaustibility of the resource and establish optimal extraction of the resource in a multisectoral context.

Devarajan (1988) sketches out the formal structure of the last two classes of models and presents some results from the application of these models. The model to be proposed here belongs to the optimal depletion category of computable general equilibrium models. It is an optimization model that determines the optimal development path of the economy, hence, an inter-temporal depletion problem subject to the workings of a multi-sector market economy. Such a formulation establishes general equilibrium linkages

between the depletion profile of the resource and the rest of the economy working through both factor and product markets. The main focus is on the optimal rate of exhaustible resource depletion.

Section 2 presents the overall theoretical structure of the model by discussing the nature of the economic institutions or "actors" in the economy and the ways in which they interact. The following section presents the equations of the dynamic model and discusses in detail the objective function and the two important intertemporal linkages in this model: depletion of the exhaustible resource oil, and optimal savings and investment allocation. A full description of the equations of the static sub-model are in Section 4. In section 5 implementation of the model and some simulation results are described. A summary and conclusions are presented in the last section.

2 Structure of the Model

The four major actors in the economy are producers, households, government, and rest of the world. Figure 1 depicts an economy-wide circular flow of income and provides an overall picture of links among actors in the economy. Note that the model ignores the monetary side of the economy; the capital market or the financial sector acts only as a "savings pool", where all savings in the economy are collected and are channeled to real investment expenditure.⁴ The following sections provide a detailed discussion of the main institutions of the economy and conclude with an overview of how the major dynamic choices of government regarding resource depletion and investment are addressed in the model.

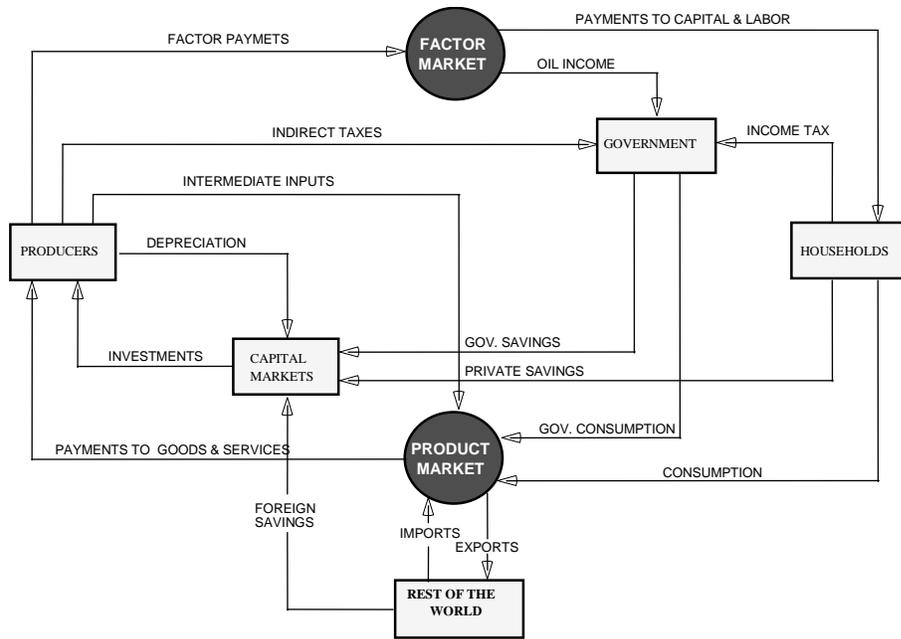


Figure 1: Major actors in the economy and circular flow of income

2-1 Producers

Producers are industries or sectors of production of the economy. Each sector is assumed to behave as a single representative firm producing a single homogeneous good. There are four sectors in the economy, of which one extracts the non-renewable resource of oil. This sector is called the "oil sector" and the remaining sectors -- agriculture, industry and service -- will sometimes be referred to as "non-oil sectors". The outputs of producers may be consumed domestically, used as material inputs (intermediate inputs) in the production of other goods, or exported.

There are three primary factors in the economy: man-made capital or "capital" for short, a natural capital or "resource", and labor. Households own capital in non-oil sectors and also their labor; government owns both physical and natural capital in the oil sector. All sectors employ capital, labor, and intermediate inputs in their production processes. It is assumed that intermediate inputs are demanded in fixed proportions to the level of gross output while the production technology for the primary factors is described by a neoclassical constant returns to scale production function. The oil sector

also is assumed to have a fixed coefficient demand for intermediate inputs and employs physical capital along with labor to extract the exhaustible resource oil.

All non-oil sectors are assumed to maximize short run profits. Given wage rates and rentals on capital, they decide on the input factor levels that maximize their profits. The oil sector is also assumed to behave as a short run profit maximizer in its labor hiring but the capital requirements of the sector are determined by the government (issues related to the oil sector will be discussed later in more detail). Aggregation of factor demands across sectors determines the total demand for primary and intermediate inputs. Supplies of goods and services, given the availability of factors, are determined by the production technology of the firms.

As shown in Figure 1, producers make payments for their primary inputs to the owners of factors. They also pay other production sectors for using their products as intermediate inputs. Other outlays of the producers include depreciation expenditures, which go to the total savings pool, and indirect taxes, which are collected by the government. Producers receive payments from the households, the government, and the rest of the world when they purchase goods and services in the product market. Inflow of funds from the savings pool augments the production capacity of the firms for future production.

2-2 Households

There is a single representative household in the economy that owns the capital in the non-oil sectors, and labor. This household, as illustrated in Figure 1, supplies factor services and receives payments made for them.

The household provides a fixed amount of labor, assumed to be an aggregation of various skill categories, and receives factor payments for labor. Competitive short run profit-maximizing behavior assures that the nominal wage rate equals the value of the marginal product of labor. The household is also owner of the man-made capital in the

non-oil sectors and receives payments made to this capital. There exist potential factor market distortions in the economy, so wage rates and capital returns may vary across sectors.

The household can either save or consume its income. The consumption of the household, however, follows a fixed pattern, that is, the household spends a fixed portion of its income on the goods of each sector. In other words, the sectoral private consumption shares are constant. This specification is a simplified version of a linear expenditure system and implies unitary income and price elasticities of demand. These assumptions may be too restrictive for the long term, where the share of total consumption expended on certain goods might change in the course of development. However, we retain this simple demand structure to avoid excessive complexity.

2-3 The Rest of the World (ROW)

The rest of the world is linked to the model through exports, and imports. The model constructed in this study uses an intermediate specification of foreign trade that has become standard practice in nearly all developing country CGE models, namely Armington (1969). This approach treats domestically produced goods and imported goods as imperfect substitutes. In other words, consumers can choose between imports and domestic goods that are not identical. The price of domestic products can deviate from that of the imported products to the extent that the users do not find them substitutable. Analogously, imperfect transformability is assumed on the export side. This specification allows divergence between domestic price of exports and their world prices.

Crude oil is by far the largest component of foreign trade in oil exporting countries, hence, changes in the world prices of oil have significant implications for the domestic economy. The model retains the small open economy assumption, indicating

that the economy is a price taker in the international markets. That is, the world prices of both exports and imports are exogenous to the model.

Capital flows in the form of investing abroad or foreign borrowing are other important links between an open economy and the rest of the world. As labor abundant oil exporting countries do not consider investing in foreign assets as a viable investment alternative,⁵ the model assumes that foreign savings is a fixed proportion of the GDP. The assumption of fixed foreign savings, in addition to ruling out investing abroad, implies that the country cannot borrow and must rely on its domestic resources to expand its production capacity. Clearly this has strong implications for the depletion of oil and will be discussed later.

2-4 The Government

The government plays a central role in the economy. The aim of the present study is to describe the optimal actions of the government. Notwithstanding its pivotal role, the government does not work within the environment of a command economy. It strives to achieve societal objectives within the more realistic environment of a mixed economy in which market also plays an important role. Thus, the government is an optimizing agent that faces the institutional constraints posed by the workings of a market economy, where producers and households independently pursue profit or utility maximization.

The government is assumed to be a benevolent selfless entity that is motivated solely by social welfare.⁶ The optimizing producers and households through the price mechanism establish a one-period equilibrium, or more precisely, a sequence of one period equilibria. The government, on the other hand, with information on current and future prices determines the long-run dynamic behavior of the economy by maximizing an inter-temporal social welfare function subject to the total availability of the exhaustible resource, adjustment costs in the accumulation of capital, and constraints

implied by the set of competitive within-period equilibria. The government with perfect foresight determines the private savings rate and the rate of investment in the oil sector to maximize social welfare. This social welfare is represented in the objective function of the model as the present value of a representative household's utility of consumption, plus the present value of the terminal capital stock and the resource remaining in the ground.

The government's behavior is constrained by yearly balanced budgets. That is, the government revenues are either saved or consumed; hence, there is no possibility of government budget deficit or surplus. The government's total expenditures include purchases of goods and services from producing sectors on a fixed share basis. The net savings of the government is the residual of its revenues less its expenditures.⁷ The government expenditures are viewed as administrative input required for running the overall economy but are not valued in the objective function. However this does not mean that the government services have no effect on social welfare. On the contrary, the government by producing services such as health and education improves and increases factor productivity, hence, significantly affects the production.

As shown in Figure 1, the government earns its revenues through direct and indirect taxes, tariffs, and revenues from the oil sector. Tax and tariff rates are assumed to be exogenous and fixed over time -- we could vary these rates but we chose to focus on the oil sector as the major constraint of development policy. Oil revenues are the total revenues of the oil sector (value added plus the user charge or the rent for the resource) less the wage bill.

2-5 Choices of Government

The core of government's decisions, and the focus of this study, are the fundamental dynamic choices of oil economies, namely: optimal rate of depletion of oil,

optimal level of investment and investment allocation. The instruments that the government uses to address these issues are further discussed below.

Oil Depletion

The government as the owner of both physical and natural capital in the oil sector receives returns from these factors. Oil revenues are the major source of government revenues and significantly affect activities in the rest of the economy. Given domestic prices, world prices of both imports and exports, and international trade elasticities, the government, as the owner of the oil resource, at the intra-temporal level manages the oil sector as a short-run profit maximizing firm. At the inter-temporal level, however, the government determines the magnitude of the physical capital in the oil sector, hence, the rate of resource extraction.

The oil sector's labor demand is determined by the assumption of competitive and short run profit maximization behavior of the sector in the labor market. This assumption is plausible for the following reasons. First, the wage rate in the economy is determined by the market mechanism and the government is a price taker in the labor market. Second, the share of labor in oil production, compared to the contribution of physical capital, is very small; thus, the effect of any alternative assumption about labor is inconsequential.⁸ Finally, the assumption of profit maximization implies an efficient management of the sector in its day to day activities, which is consistent with the long run goal of the government. It is possible to drop the assumption of the competitive and profit maximizing behavior in the labor market and let both factors of production -- capital and labor -- be determined by the government. This would only have a minor effect on the production level and the interpretation might be more difficult.

Savings and Investment Allocation

The government influences household saving decisions through its tax policies and other instruments, which are not explicitly modeled. In other words, economy-wide

savings is determined by the government's choice of the rate of private savings as it optimizes a social welfare function. Total savings includes government savings. The base model assumes that government consumption is a fixed portion of its revenues and government savings is found as a residual. This assumption is consistent with the experience of developing countries generally and oil economies in particular. Of course, in reality the public sector does not shrink when oil revenues decline, but this is not a concern here. Other formulations of government consumption-savings decision are possible. The level of government savings, for example, can be linked to GDP or oil revenues.

A variation of the base model, which in its objective function values government consumption along with private consumption, is used to find optimal government savings. By controlling the economy's savings level, the government indirectly determines the optimal level of all the activities in the economy, including the level of oil extraction, subject to workings of a competitive economy.

Once the savings level is determined the next question is how investment funds are allocated among sectors. The government concerned with long run social welfare decides the investment share of the oil sector. The remainder of the investment fund is distributed among non-oil sectors. This residual investment is allocated such that the more productive and profitable sectors of the economy receive a larger share. An alternative to the present formulation is one in which the government determines the investment shares for all sectors. This formulation would imply a much larger role for the government in the economy. Clearly, the greater command of the government would result in a different optimal path for the economy, including a different oil depletion path. Another approach to investment allocation is to introduce perfect foresight for individual firms. Each firm would make its investment decisions to maximize its net present worth. This approach to dynamic behavior of producers is more recent and less widely adopted in multisectoral models.⁹

3 The Dynamics of the Model

3-1 The Objective Function

In our model, we maximize the welfare of the representative household, which includes the present value of the utility of consumption over time and the present value of end-of-planning-horizon capital stock and oil reserves:

$$\text{MAX } J = \int U(C_t) * e^{\Delta t} dt + \left[\overline{PK} * \sum_i K_{i,T} + \overline{PR} * RSRV(T) \right] * e^{-\Delta T}$$

Here, (C_t) represents Cobb-Douglas aggregation of consumption of $CD_{i,t}$ of goods from sector i in time period t with fixed consumption shares ch_i :

$$C_t = CD_{1t}^{ch_1} * CD_{2t}^{ch_2} \dots CD_{nt}^{ch_n} \quad \text{where} \quad \sum_{i=1}^n ch_i = 1$$

and \overline{PK} is the price of terminal capital stock; \overline{PR} is the price of resource at terminal period; and Δ is the social discount rate. The utility function is concave, reflecting the diminishing marginal utility of consumption. In other words, as the society gets richer the value of an additional unit of consumption declines. The general form of the utility function is $U(c) = \frac{1}{1-\Phi} c^{1-\Phi}$ with $\Phi \neq 1$, where a higher constant elasticity of marginal utility (Φ) implies a higher degree of consumption smoothing over time. The positive social discount rate (Δ) implies that when faced with the choice between a unit of consumption today or the same unit tomorrow, the society chooses the first option.

The statement of our problem, with the objective function written in a discrete form, is summarized as:

(1) Objective function

$$\text{MAX } J = \sum_{t=0}^T \frac{1}{(1+\Delta)^t} * \frac{1}{(1-\Phi)} * \left[\prod_i (CD_{i,t})^{ch_i} \right]^{1-\Phi} + \frac{1}{(1+\Delta)^T} * \left[\overline{PK} * \sum_i K_{i,T} + \overline{PR} * RSRV(T) \right]$$

Subject to: equations 2-47, to be described in the following sections.

3-2 Optimal Depletion of Oil

The major focus of this study is characterizing the extraction path for an exhaustible resource in a multisectoral framework. The optimal path is identified for a given planning period during which the economy enjoys substantial oil reserves. Our interest is with the economy-wide effects of oil extraction; namely: the optimal intertemporal pattern of extraction constrained by workings of a market economy, the optimal intertemporal pattern of accumulating physical capital, and the allocation of investment funds. This model is designed to focus on a period that oil reserves abound and the oil sector plays a crucial role in the economy. There are, however, a number of important questions that can only be addressed using a more general model which is capable of explicitly dealing with the full exhaustion of oil reserves and transition to a non-oil era. Such a model would place the depletion problem in a broader context and can be used to address the issues related to the transition period.

The oil sector differs significantly from other sectors in that it uses a resource that is nonrenewable and is owned by the government. There are a number of issues that are exclusive to the oil sector. Therefore, we describe our theoretical treatment of the oil sector, and the ways in which this model departs from conventional models of exhaustible resources.

An exhaustible resource is different from other goods and resources in that it is limited in quantity and cannot be reproduced. By consuming a unit of resource today we

forego a value that might have been realized in the future. In the case of an exhaustible resource, the normal efficiency condition of a competitive economy, where price is equated with marginal cost of production, does not hold. Instead, the price exceeds the marginal extraction cost by an unobservable amount called economic rent.¹⁰ Theories of optimal depletion of exhaustible resources attempt to describe the behavior of this rent over time. Optimal depletion, as the term is commonly used in the literature, means the pattern of depletion that maximizes the present value of the resource's net benefits stream.

In a classic work, Hotelling (1931) demonstrated that in a competitive economy the price of an exhaustible resource, net of its marginal cost of extraction, must grow at a rate equal to the rate of interest. For the monopolist, he showed, the net marginal revenue, not the net price, will grow at the interest rate.¹¹ Hotelling derived these results assuming constant marginal extraction costs, although he was aware of the effects of cumulative production (Devarajan and Fisher 1982). This assumption implies that the extraction costs are independent of the remaining stock of resource or that the resource is of a constant quality. But as originally suggested by Ricardo (1817), deposits of exhaustible resources, just like land, occur in varying grades and the higher qualities (i.e., the lower-cost) are exploited first.¹² Therefore, it is more realistic to assume that the cost of extraction is negatively related to the stock of resource, that is, it gets more costly to extract as the stock dwindles.¹³

The specification of the oil sector in our model assumes that the extraction cost is a function of stock size and rises as the stock is depleted. We further assume a known extractable quantity of reserves, that no exploration activity takes place, and that there is

no uncertainty.¹⁴ The present model differs significantly from the conventional single sector partial equilibrium models of exhaustible resource extraction in that it characterizes a depletion profile within the constraints imposed by the workings of a market economy. The level of activities in the non-oil sectors, domestic and world prices, and the behavior of various autonomous economic actors have direct bearing on the extraction plan.

For the purpose of exposition let us use a continuous time version of the optimal control framework to cast the optimization problem of the oil sector in the same way that is typically done in single sector partial equilibrium models. This exposition helps to highlight some of the notable similarities and differences that exists between the conventional partial equilibrium models of exhaustible resources and the formulation of the resource depletion in this model (a multisectoral general equilibrium framework).

The optimization problem of the oil sector in a partial equilibrium model of exhaustible resource can be viewed as selecting an extraction program $\{XD_t, t=1, 2, \dots\}$ to maximize the present value of a stream of benefits. In symbol the problem is:

$$\begin{aligned} & \text{MAX}_{\{XD_t\}} \int_0^T \{P * XD_t - c(XD_t, S_t)\} * e^{-rt} dt \\ & \text{s.t.} \\ & \int_0^T XD_t dt \leq S(0) \end{aligned}$$

where $c(\cdot)$ is the extraction cost function and is negatively related to stock size, that is

$\frac{\partial c}{\partial S} < 0$. If we write the original quantity of oil as $S(0) = S_0$ and the stock left at the

last period as $S(T) = S_T$, the constraint can be rewritten as:

$$\dot{S}_t = -XD_t$$

This constraint (equation of motion), similarly, in our general equilibrium framework captures the dynamic updating of the oil reserves; it enters in discrete form, as shown below, into the computer program that solves the model:

(2) *Oil reserve updating*

$$S_{t+1} = S_t - XD_{oil,t}$$

Note that the optimization problem, described in the previous section, in our general equilibrium model is much broader than intertemporal profit maximization of a single sector. Also note that in our model the economy in addition to resource availability constraint faces numerous other constraints; notably, it incorporates imperfections in capital markets.

The Hamiltonian for the above problem is:

$$H = P * XD_t - c(XD_t, S_t) - \omega * XD_t$$

where in the language of optimal control theory ω is the costate variable (the shadow price of oil), XD is the control variable, and S is the state variable. The first two terms in the Hamiltonian measure net current benefit and the last term is the future losses due to not having the resource. The Hamiltonian is maximized along an extraction program such that at the margin the net benefit from extracting a unit is equal to the loss of that unit from the stock of resource.

The necessary conditions are:

$$\begin{aligned} \text{(a)} \quad \frac{\partial H}{\partial XD_t} &= 0 & \Rightarrow P - \frac{\partial c}{\partial XD_t} - \omega_t &= 0 \\ \text{(b)} \quad \frac{\partial H}{\partial S} &= -\dot{\omega} + r * \omega_t & \Rightarrow r * \omega_t &= \dot{\omega} - \frac{\partial c}{\partial S_t} \\ \text{(c)} \quad \frac{\partial H}{\partial \omega} &= \dot{S}_t & \Rightarrow -XD_t &= \dot{S} \end{aligned}$$

The first condition of optimal depletion says that, along the optimal path, the price of resource P is equal to the marginal cost of extraction $\frac{\partial c}{\partial XD}$ plus the shadow price of resource (rent) ω . The condition (b) is a description of the behavior of the rent over time. With the assumption that the extraction costs are negatively related to the remaining stock this condition implies that the rate of change in the rent, $\frac{\dot{\omega}}{\omega}$, is less than the interest rate r .¹⁵ The third condition is just a restatement of the constraint.

The explicit functional form of the cost function, $c(XD, S)$, depends on the technology and can be derived by solving the following minimization problem:¹⁶

$$c(XD_t, S_t) = \min W * F$$

s. t.

$$XD = f(L, K; S) = A(S) * L^\alpha * K^{1-\alpha}$$

where W is the vector of factor prices and F is the vector of factors. The explicit functional form of $f(L, K; S)$ in our model is the production function for the oil sector that will be discussed in detail later.

It must be recognized, however, that there are fundamental differences between the assumptions about the economy in the control problem of an exhaustible resource and the CGE model presented in this study. First, the capital is sector specific and perfectly immobile in the CGE model, but it is perfectly mobile in the control model. Second, the CGE model assumes a diminishing marginal efficiency of investment and incorporates costs of adjustment for capital stock. This is incompatible with the factor market assumptions in the control model, which assumes that capital services are perfectly malleable, i.e., any amount of capital can be rented at the given market interest rate.

Third, an important feature of the CGE model is that it solves for market clearing equilibrium prices, but in the control model the price is exogenous.

3-3 Savings and Investment Allocation

One important feature of the present model is its explicit treatment of the dynamic inter-period market equilibrium. The government chooses the private marginal propensity to save (MPS) and the rate of investment in the oil sector ($ISHR_{oil}$) so as to maximize the social welfare function as represented in equation (1). The non-oil sectors receive the remainder of investment funds based on their relative profitability in past and current periods. This specification of investment allocation assumes that non-oil sectors have myopic expectations (Dervis et al. 1982). Specifically, each non-oil sector's share of investment funds, $ISHR_{in}$, is equal to its share in aggregate capital income, SP_{in} , adjusted upward if the sector's profit rate is higher than the average profit rate and adjusted downward otherwise:

(3) *Investment shares in non-oil sectors*

$$ISHR_{in,t+1} = SP_{in,t} + \Omega * SP_{in,t} * \left[\frac{RP_{in,t} - AVGRP}{AVGRP} \right]$$

where RP_{in} is the sectoral profit rate, $AVGRP$ is the average profit rate for the economy as a whole, and Ω is an investment mobility parameter, a measure of the responsiveness of capital markets to sectoral profit rates.¹⁷ The following three equations show how profit shares, SP_{in} , profit rate, RP_{in} , and average profit rate, $AVGRP$, are determined. Note that the profit rate, RP_{in} includes R_{in} , rate of return on capital as well as capital gains (d_i is the sectoral depreciation rate).

(4) *Share in overall profits*

$$SP_{in} = R_{in} * K_{in} / \sum_{jn} R_{jn} * K_{jn}$$

(5) *Determination of profit rates*

$$RP_{in,t+1} = R_{in,t+1} + [PK_{in,t+1} - (1 + d_{in}) * PK_{in,t}] / PK_{in,t}$$

(6) *economy wide profit rate*

$$AVGRP = \left[\sum_{in} RP_{in} * K_{in} \right] / \sum_{in} K_{in}$$

The investment funds in each sector augment the sector's capital stock but at a decreasing rate as shown below:

(7) *Dynamic capital equation*

$$K_{i,t+1} = K_{i,t} * (1 - d_i) + \theta_i * K_{i,t} \left[1 - \left[1 + \frac{DK_{i,t}}{2 * \theta_i * K_{i,t}} \right]^{-2} \right]$$

where θ is the investment cost adjustment coefficient. This specification embodies an absorptive capacity constraint, i.e. the marginal efficiency of sectoral investment declines if investment grows too rapidly.¹⁸ As the rate of investment, $\frac{DK}{K}$, rises, the return to additional DK declines. Technically, with such an absorptive capacity constraint, the rate of increase in capital stock, K , would be smaller than the rate of increase in investment as a percentage of capital stock, DK/K .

4 The Static Model

The static portion of the model is a multisectoral general equilibrium model of a Walrasian competitive economy. Apart from the peculiar effects of dynamics of the oil sector, the static model shares many of the features of the family of CGE models

constructed for developing countries by Dervis, de Melo, and Robinson (1982)¹⁹ -- such as imperfect substitution in trade and imperfections in factor markets.

An overall schematic view of the major components of the model is depicted in Figure 2. The figure includes factors, products rates, and prices as well as the various functional forms that link the parts together.

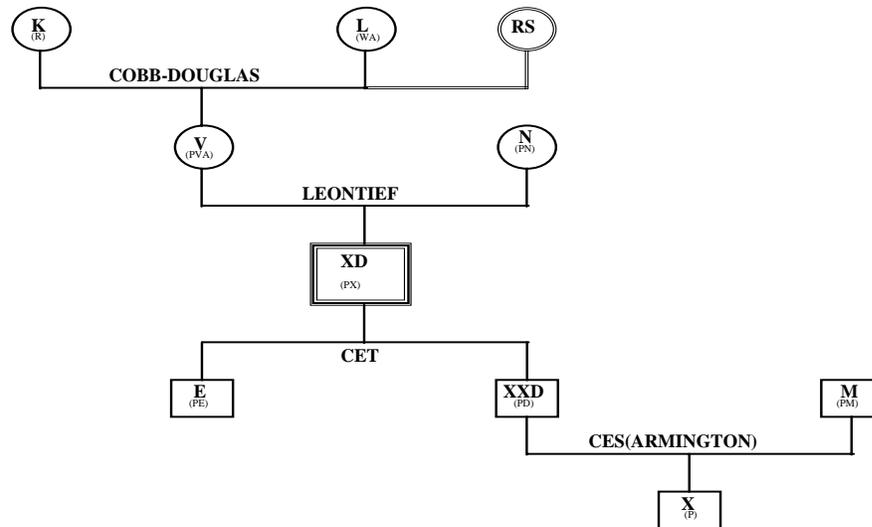


Figure 2. Factors, Prices, and Products in the CGE

Factors & Products:

K: man-made capital
 L: labor
 RS: natural capital (resource)
 V: value added
 N: intermediate inputs
 XD: domestic output
 E: exports
 XXD: domestic sales of domestic goods
 M: imports
 X: composite good

Rates & Prices

R: rate of return on capital
 WA: wage rate
 ω : shadow price of resource
 PV: value added price
 PN: price of intermediates (incl. tax)
 PX: average sales (output) price
 PE: domestic price of exports
 PD: domestic prices
 PM: domestic price of imports
 P: price of composite good

4-1 Production and Factor Markets

The gross output of non-oil sectors is related to inputs according to a Cobb-Douglas production function in the following general form:

(8) Production function for non-oil sectors

$$XD_{in} = ad_{in} * L_{in}^{\alpha_{in}} * K_{in}^{1-\alpha_{in}}$$

where the index "in" refers to non-oil sectors. Parameters ad_{in} and α_{in} are constants and reflect the production technology. In addition to labor and capital, intermediate inputs are also required to produce each sector's output. This amounts to a two level production where at one level capital and labor produce the real value added which in the next level combines with intermediate inputs according to fixed input-output coefficients to produce output (see Figure 2).

With labor and physical capital as the primary inputs, the production technology is a constant-returns-to-scale technology. In this specification of technology the number of firms in the sector does not matter and the whole sector can be seen as a single large firm that takes output and input prices as given.

The production specification for the oil sector is different. The oil produced over the years is ultimately going to be limited by total recoverable reserves. Oil is an exhaustible resource and its cost of production depends crucially on the stock of reserves. The smaller the remaining stock the larger is the cost of extracting a unit. The production function in the oil sector also has a Cobb-Douglas functional form with constant-returns-to-scale and capital and labor as inputs:

(9) Production function for oil sector

$$XD_{oil} = A(S) * L_{oil}^{\alpha_{oil}} * K_{oil}^{1-\alpha_{oil}}$$

where XD , L , and K are output, labor input and capital stock respectively; constant parameter α is the labor share in output. The scale factor $A(S)$ depends on S , the total stock of resource remaining in the ground at each period. Therefore, $A(S)$, decreases over

time as the stock of oil is depleted, reflecting the increase in marginal cost of extraction as seen in the cost function. Specifically, we assume:

$$A(S) = S^{\Sigma} * Z$$

where Z is a positive constant parameter reflecting the technology and Σ is the stock elasticity of resource output.

There are some limitations to the use of Cobb-Douglas production function for the oil sector that must be mentioned. Under this functional form for any strictly positive stock of resource and physical capital, and any strictly positive wage rate and oil price, there exists a profitable, strictly positive extraction level. In other word, with a Cobb-Douglas function it is not profitable to leave any oil in the ground; abandonment of oil extraction is not possible. The reason is that the marginal product of labor rises toward infinity as labor approaches zero (see the necessary conditions for equation 11). Since we are sure that there exists a positive amount of physical capital in the sector (in form of oil rigs), therefore, as long as there is a positive amount of resource in the ground it is profitable to continue to extract. Not being able to abandon the oil production poses no problem in this model since we are looking at a window of time where we always have positive oil reserves and expect oil production to be profitable. Impossibility of abandonment would be a problem in a context where it is optimal to leave positive reserves in the ground as extraction costs become too high.

The amount of capital in each sector, K , is assumed to be fixed within each period. This implies that current investments will add to capacity only in future periods. Capital is a composite good assumed to consist of fixed proportions of different investment goods. These proportions are summarized in the capital composition matrix,

where an element b_{ij} is the amount of capital good originating from sector "i" that will be used to make up one unit of real capital in sector "j". The parameters "ad" and "z" reflect technological progress in each sector and are constant within a period. A Leontief input-output technology is assumed for intermediate inputs which implies such inputs are demanded in fixed proportion to the level of output.

Competitive profit-maximizing behavior in all sectors implies that in each sector the value of the marginal product of each factor must equal its price. Thus, total factor payments in each sector are equal to the total value added by that sector. The (physical) marginal product of labor for each sector is simply the derivative of its production function (equations 8 and 9) with respect to labor. Before we can find the (money) values of these marginal products we need to define net price or value added price. The value added price, PV , is the price that producers use to make their output level and factor demand decisions and is defined as the value of output at producer's price minus the cost of the composite intermediate input. Sectoral value added price is given by:

(10) Definition of value added prices

$$PV_i = PX_i(1 - tn_i) - \sum_{j=1}^n P_j * a_{ji}$$

where: PV_i : value added price for sector i; PX_i : domestic price of sector i's output; tn_i : indirect tax rate; P_j : price of composite good; a_{ji} : input-output coefficients.

Profits are then the difference between revenues (output at value-added prices, which excludes the cost of intermediate inputs) and capital and labor costs. Thus, the profit maximization conditions that wages equal the value of the marginal product of labor for both oil and non-oil sectors can be written as:

(11) Labor demand function

$$WA * wd_i * L_i = XD_i * PV_i * \alpha_i$$

where WA is the economy-wide average wage rate of labor and wd is a wage distortion parameter that measures the extent to which sectoral wage rate, WAS , deviates from the average, WA . Note that this formulation permits labor market distortions, which are measured by parameter wd , $wd_i = WAS_i / WA$, and which is normally fixed over time.

The return to capital in each sector is found as the residual of value added net of payments made to labor. The sectoral capital demands are determined by the following equation:

(12) Capital demand function

$$R_i * K_i = XD_{in} * PV_{in} - WA * wd_i * L_i$$

where R is the rate of return on capital.

4-2 Income Generation and Product Markets

The demand side of the economy consists of four basic blocks: consumption, government, investment, and intermediate demand.

1- Consumption Demand

A single representative household in the economy owns the capital in the non-oil sectors as well as the total supply of labor in the economy, and receives payments made to these factors. Thus household income is total value added less the sum of depreciation expenditures, $DEPR$, and the total payments made to physical and natural capital in the oil sector, $OILREV$:

(13) Household income

$$Y = \sum_i PV_i * XD_i - DEPR - OILREV$$

The household saves a portion of its disposable income (total income less direct taxes, *DIRTAX*) and spends the remainder. Household saving is given below in which *MPS* is the household's marginal propensity to save and is determined through optimizing a social welfare function, as discussed in Section 3-2.

(14) *Household savings*

$$HHSAV = MPS * (Y - DIRTAX)$$

The single household is assumed to have a fixed structure of consumption where it purchases products of various sectors by a fixed expenditure share. This demand specification is a variation of Stone's linear expenditure system and is derived from a Cobb Douglas utility function to be discussed later. The fixed consumption shares imply unitary income and price elasticities:

(15) *Household consumption behavior*

$$CD_i = [ch_i * ((1 - MPS) * Y - DIRTAX)] / P_i$$

where CD_i is total consumption demand for output of sector i ; and ch_i is fixed consumption share.

2- Government Demand

The sources of government revenue include direct and indirect taxes, tariff, and the revenues from the oil sector, *OILREV*. The government revenue *GR* is specified by the following budget equations:

(16) *Government revenue*

$$GR = DIRTAX + IND TAX + TARIFF + OILREV$$

(17) *Direct taxes*

$$DIRTAX = td * Y$$

(18) Indirect taxes on domestic production

$$INDTAX = \sum_i tn_i * PD_i * XD_i$$

(19) Tariff revenues

$$TARIFF = \sum tm_i * M_i * \overline{PWM}_i * ER$$

(20) Oil revenues

$$OILREV = XD_{oil} * PV_{oil} - WA * wd_{oil} * L_{oil} - DEPRO$$

where td and tn_i are direct and indirect tax rates, ER is the exchange rate between US dollars and the Iranian Rials, tm_i is the sectoral tariff rate, and $DEPRO$ is the depreciation expenditure in the oil sector.

Government, analogous to households, is assumed to have a fixed expenditure structure such that it purchases goods and services in fixed proportions, cg_i :

(21) Government expenditure pattern

$$GD_i = cg_i * GR/P_i$$

where GD_i is the government's demand for the output of sector i . Government savings, $GSAV$, is found as a residual;

(22) Government savings

$$GSAV = GR - \sum_i P_i * GD_i$$

3- Investment Demand

We assume that the level of investment demand is determined by the level of total savings available to the economy. Total savings includes private and government savings, depreciation, and foreign savings;

(23) Total savings

$$SAVINGS = HNSAV + GSAV + DEPR + FSAV * ER$$

Foreign savings, $FSAV$, is given by:

(24) *Foreign savings*

$$FSAV * ER = \Psi * \sum_i PV_i * XD_i$$

where Ψ is the share of capital account in GDP. The sum of depreciation expenditures contributes to total investment in the next period;

(25) *Total depreciation expenses*

$$DEPR_{t+1} = \sum_i d_i * PK_{i,t} * K_{i,t+1}$$

where d_i is the given rate of depreciation in sector i , PK_i is the price of a unit of capital employed in sector i defined as:

(26) *Definition of capital goods prices*

$$PK_i = \sum_j P_j * b_{ji}$$

and b_{ij} is an element of the capital coefficient matrix and represents the amount of capital good originating from sector i that will be used to make up one unit of real capital used in sector j .

The inventory investment in each sector, IV_j , is assumed to be a fixed proportion, riv , of the sector's output (in the base run sectoral inventory investments for all periods are assumed to be constant and equal to their base year value in real terms). Sectoral productive investments are determined assuming that investable funds available to sector i is a given proportion, $ISHR_i$, of total productive investment which is total savings less total inventory investment, $TOTIV$.

(27) *Sectoral inventory investment*

$$IV_i = riv_i * XD_i$$

(28) Total inventory investment

$$TOTIV = \sum IV_i * P_i$$

(29) Investment by sector of destination (oil sector)

$$DK'_{oil} = (ISHR_{oil} * (SAVINGS - TOTIV)) / PK_{oil}$$

(30) Investment by sector of destination (non-oil sectors)

$$DK_{in} = (ISHR_{in} * (SAVINGS - TOTIV - DK_{oil} * PK_{oil})) / PK_{in}$$

In equations (29) and (30) DK_i is the volume of investment by sector of destination and $ISHR_i$ is the sector share of investment. The investment share for the oil sector $ISHR'_{oil}$ is optimally determined, as explained in Section 3, and the non-oil investment proportions are in a way measures of profitability of each sector and their determination was also explained in Section 3. Notice that DK_i is investment "to" sector i but we are interested in finding investment demand "from" sector i . This is referred to as "investment by the sector of origin", ID_i , and it is determined using the capital composition matrix, b_{ij} ;

(31) Investment by sector of origin

$$ID_i = \sum_j b_{ij} * DK_j$$

4- Intermediate demand

As a result of the fixed coefficients assumption, intermediate demand is derived as follows:

(32) Intermediate demand

$$INT_i = \sum_j a_{ji} * XD_j$$

4-3 Foreign Trade

Products of sectors are either internationally traded or nontraded. Traded sectors are those that have either imports or exports or both. We start with the discussion of imports but before doing that a word on notation is in order. In the following equations the index "it" identifies traded sectors, while the index "itn" refers to non-traded sectors. The union of subsets "it" and "itn" is "i" the set of all sectors. The index "in", as before, identifies non-oil sectors.

Imports

Imports are assumed to be imperfect substitutes for domestically produced goods. Following Armington's formulation we define a composite commodity, X, to be a CES aggregation of the imported goods, M, and the domestically produced goods, XXD (the relationships between X, XD, XXD, M, and E are shown schematically in Figure 2). The aggregation function is:

(33) *Composite good aggregation for traded sectors*

$$X_{it} = ac_{it} * \left[\delta_{it} * M_{it}^{-\rho_{it}} + (1 - \delta_{it}) * XXD_{it}^{-\rho_{it}} \right]^{-\frac{1}{\rho_{it}}}$$

where ac_{it} is a shift parameter; δ_{it} , is the share of imported good in the composite commodity; and ρ_{it} , the function's exponent parameter is related to the trade substitution elasticity σ by the expression: $\sigma_{it}=1/1+\rho_{it}$. The trade elasticity of substitution, σ , is a measure of the ease with which domestic product and imports can be substituted for each other. If no substitution is possible ($\sigma=0$), then composite good aggregation takes place with fixed proportions and relative price changes cannot directly affect the demand for imports. If, on the other hand, domestic product and imports are perfect substitutes ($\sigma=\infty$) the price ratio is the same for all ratios of imports to domestic products. So the

greater the substitution elasticity the easier it is to substitute the two goods. We use values of the elasticity of substitution greater than zero and less than infinity so that a finite variation in the ratio of price results in a finite variation in M/XXD ratio. Clearly, for sectors such as agriculture σ is large, whereas for capital goods it is quite low.

The CES formulation implies that consumers will choose a mix of domestic goods, XXD , and imported goods, M , on the basis of their relative prices. Consumers are assumed to minimize the cost of obtaining a "unit of utility":

(34) *Value of domestic sales*

$$P_{it} * X_{it} = PD_{it} * XXD_{it} + PM_{it} * M_{it}$$

subject to (33). The solution to this problem yields the ratio:

(35) *FOC for composite good*

$$\frac{M_{it}}{XXD_{it}} = \left[\frac{PD_{it}}{PM_{it}} \right]^{\sigma_{it}} * \left[\frac{\delta_{it}}{1 - \delta_{it}} \right]^{\sigma_{it}}$$

where P is the price of the composite good X , PD and PM are the prices, in domestic currency, of domestic and imported goods respectively. With this specification PD is determined endogenously and is no longer equal to PM , which is fixed exogenously and is linked to the world price PWM by:

(36) *Definition of domestic import price*

$$PM_{it} = \overline{PWM}_{it} * ER * (1 + tm_{it})$$

For sectors with no imports the composite good is equal to domestic sales of domestically produced goods XXD :

(37) *Composite good aggregation for sectors with no imports*

$$X_{in} = XXD_{in}$$

Exports

Similarly, on the export side we allow the domestic prices to diverge from the world price by utilizing product differentiation concepts. Specifically, a Constant Elasticity of Transformation (CET) function allocates domestic output, XD , between domestic use, XXD , and exports, E^{20} :

(38) *CET function*

$$XD_{it} = at_{it} \left[\gamma_{it} * E_{it}^{\phi_{it}} + (1 - \gamma_{it}) * XXD_{it}^{\phi_{it}} \right]^{\frac{1}{\phi_{it}}}$$

where at_{it} is a shift parameter; γ_{it} is the share of exports in domestic output; and the exponent θ_{it} is related to ϕ the elasticity of transformation by the expression $\phi=1/\phi-1$.

Producers can either export or sell in the domestic market. Their problem is to maximize revenue from a given level of output subject to the CET transformation function.

(39) *Value of domestic output*

$$PX_{it} * XD_{it} = PD_{it} * XXD_{it} + PE_{it} * E_{it}$$

The first-order condition represents export supply and is a function of the relative export price to domestic price, the elasticity of transformation between the two uses and the share parameters in the CET function.

(40) *Export supply for traded sectors*

$$E_{it} = XXD_{it} \left[\frac{PE_{it} * (1 - \gamma_{it})}{PD_{it} * \gamma_{it}} \right]^{\frac{1}{\phi_{it} - 1}}$$

Note that implicit assumption in this specification is that there is always a positive amount of export for any positive world price of export. In other words, each traded sector always exports at least some of its output, thus a complete discontinuation of

exports is not possible. Therefore, if one wanted to incorporate the possibility of full depletion of oil reserves, hence zero oil exports, one must drop CET formulation in favor of a more suitable specification.

For sectors with no exports domestic supply XD is equal to domestic sales XXD :

(41) *Domestic sales for non-traded sectors*

$$XD_{it} = XXD_{it}$$

The world market price of exports PWE_{it} is linked to domestic price PD_{it} by te_{it} the fixed export duty and ER , the foreign exchange rate.

(42) *Definition of domestic export prices*

$$\overline{PWE}_{it} * ER = PD_{it} * (1 + te_{it})$$

Notice that the underlying assumption here is that all export demand is for domestically produced goods rather than for the composite commodity. Put differently, exports are netted out of domestically produced commodities, XD , before the remainder, XXD , plus imports, M , produce the composite domestically traded good, X .

4-4 Market Equilibrium

We have established thus far the dependence of the different components of demand and supply on commodity and factor prices. The equilibrium condition in the product market is given by equation (43). The supply side consists of a composite good, X , which is an aggregation of imports and the portion of domestically produced good that is not exported, XXD . The demand side includes: demand for private consumption (CD), demand for public consumption (GD), investment (ID), inventory demand (IV), and finally demand for intermediate inputs (INT).

(43) *Product market equilibrium*

$$X_i = CD_i + GD_i + ID_i + IV_i + INT_i$$

Total labor supply grows at a constant rate, Γ ; it is also assumed that the labor market clears. These conditions are shown in the following two equations:

(44) *Labor supply updating*

$$LS_{t+1} = LS_t * (1 + \Gamma)$$

(45) *Labor market equilibrium*

$$LS = \sum_i L_i$$

Finally the current account balance defines foreign savings as the difference between the values of imports and exports, or:

(46) *Current account balance*

$$\sum \overline{P} \overline{W} M_i * M_i = \sum \overline{P} \overline{W} E_i * E_i + FSAV$$

Walras' law states that the sum of the nominal values of excess demands of all product and factor markets must equal zero. However, in this model, the system of equations for intra-temporal equilibrium are not independent and thus not sufficient to determine the unknowns. Since all demand and supply functions in the model are homogenous of degree zero in all prices and the wage rate we can specify an additional constraint. This constraint defines the numeraire price index and will not affect any real magnitude in the system.

(47) *Definition of market price index*

$$\overline{P} = \sum_i P_i * \lambda_i$$

where P is price index and λ s are weights in the price index.

5- The Model Implementation and Simulation Results

The model was applied to a data set of the Iranian economy, including a social accounting matrix. The model was implemented in GAMS and used to conduct a number of simulation experiments. The model accurately reproduced the base year data for the Iranian economy and projected the optimal path of the economy for a 20 year period.

The next section contains a general overview of the base run simulation and run results related to the optimal depletion of the exhaustible resource and the optimal savings in the economy.

5-1 The Base Run Results

The numerical solution of the base model accurately reproduces the benchmark 1984 data of the Iranian economy. It also projects the values of all endogenous variables over the planning period. The model was solved for 12 periods where each period is 2 years.²¹ Thus, not counting the last 2 periods,²² our “planning” period covers 20 years from 1984 to 2004. The base run assumes constant world prices and that base-year policies are maintained. The base run is used as a reference against which all subsequent comparative dynamic experiments are compared.

The base model abstracts from an infinite horizon formulation assuming a finite horizon and introducing a salvage value for terminal capital stock and the reserves of oil remaining in the ground. There is no hard rule as to what constitutes an appropriate choice for the salvage value except that the dynamic path of the economy implied by the model should be "reasonable" and not at odds with historical experience. In the base run the time path of the private savings rate served as the main indicator of the reasonability of the salvage value.²³ The private savings rate for the Iranian economy was 21.7 percent in 1984 (the base year of the model) which is low compared to average savings rate in other developing countries. Considering the desirability of achieving higher savings, we

chose our salvage values such that the saving rate smoothly increased in immediate years following the base year and stayed within an acceptable range throughout the planning period.

5-2. Optimal Depletion Profile

This section analyzes the optimal time path of resource extraction. It discusses the optimal depletion profile in terms of the objective function, model constraints, and government instruments. The discussion compares some of the features of our general equilibrium model to those of a traditional partial equilibrium model of exhaustible resources. The sensitivity of the extraction path to model parameters is also discussed.

Figure 3 shows the optimal time path of oil extraction. This path represents the optimal extraction path subject to the constraints of a market economy. The underlying assumption of partial equilibrium models of resource extraction is that activities in the resource sector have no effect on the rest of the economy, and that other parts of the economy do not feedback to the resource market. Our general equilibrium approach, however, recognizes the full range of interactions between the resource sector and the rest of the economy.

The monotonically rising extraction level has an annual average growth rate of 7%. This growth rate initially rises from 5.6% to 6.8%, stays near 7% for a number of periods, and slightly rises again toward the end. During the 19 year period starting with 1955, upon restoration of oil production to its pre-oil-nationalization level, and ending with 1974, the peak production year, oil production in Iran grew on average at a rate of 16% annually. The rate of oil depletion suggested by the model for the 20 years of planning period is significantly lower than the pre-revolution rate but higher than the 4.6% growth rate envisaged in the post-revolution development plan for 1989-1993.

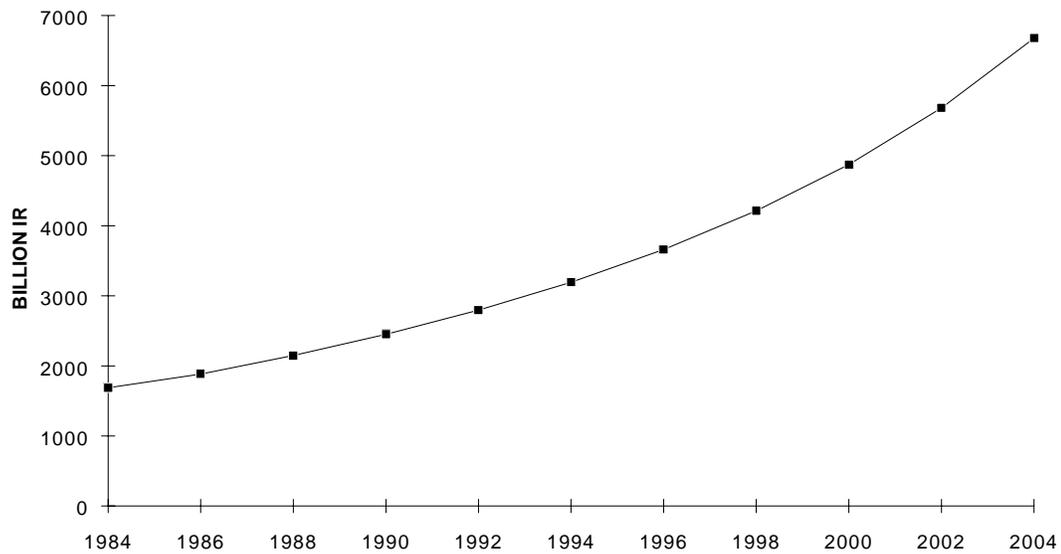


Figure 3. Oil Extraction Profile

The base run extraction program depletes 31% of total reserves by the end of planning period. The depletion program indicates that oil extraction per capita is growing at an annual average rate of 5.5% and oil revenue per capita is growing at a slightly lower rate (5.4%).

Sensitivity of Depletion Profile

The optimal depletion profile is determined by the particular specification of the terminal conditions, the structure and constraints of the model, and the choice of government instruments. There are two features of the terminal conditions that can significantly affect the depletion path. First, the model does not impose complete exhaustion by the end of the planning period.²⁴ A common feature of the early studies of exhaustible resources, as in the analysis of Hotelling, is that the mine or well is completely exhausted, even when the cost of extraction depends on cumulative output. Levhari and Liviatan (1977) in an important paper extended the Hotelling's fundamental findings to cover more general cases. They argue that the assumption that the output of the mine or well is zero at terminal point is unnecessarily restrictive, and show that terminal output may well be positive. Furthermore, these authors demonstrate that under

more realistic assumption of incomplete exhaustion and with increasing extraction costs it is possible to reach a point where extraction is not economical. Hence, production may stop before complete physical exhaustion occurs. Generally, theoretical research on exhaustible resource has shown that when the effects of cumulative extraction are added to a Hotelling model one should not expect standard results; see, for example, Schulze (1975), Pindyck (1978) and Fisher (1981). More importantly, one must bear in mind that when an optimal depletion model is embedded in a general equilibrium model the standard Hotelling model does not apply; see Aarrestad (1978).

Our empirical results clearly substantiate another claim of Levhari and Liviatan, that it is possible to have monotonically rising extraction path. The intuition behind this is that with the cost of extraction rising as reserves fall, there is an incentive to postpone the production for later periods. Because reducing production during the early stages of extraction induces a benefit in the form of lower extraction costs for every period into the future. This benefit, however, declines over time and the end result is a monotonically rising depletion path.

The second important feature of terminal conditions is that the salvage value directly determines the outcome of the model. The scrap term in the objective function consists of salvage values of physical capital stock and unexploited reserves of oil in the terminal period. The values are exogenous to the model and are chosen such that reasonable paths for the economy emerge. The depletion path depicted in Figure 3 is one among various paths that could be generated. An indication of the robustness of this depletion path is that changing the salvage value only changes the end point of the depletion path and not its shape. Experiments with the model also show that the valuation of man-made and natural capital relative to one another has more significant effects on the depletion profile than a change in the salvage value.

A distinguishing feature of the general equilibrium model in this study is that the oil sector is not isolated from the rest of the economy. The oil sector, like any other

sector in the economy, faces prices for factors and output both in domestic and foreign markets and is subject to limitations of investable funds and absorptive capacity constraints. The model characterizes the depletion path subject to constraints and structure of a market economy. Some constraints or parameters of the model have a notable effect on the depletion path and need to be discussed.

The present general equilibrium model assumes that the cost of extraction is negatively related to the stock size, i.e., the extraction costs increase as deeper and thinner layers of the resource are extracted. The depletion effect, therefore, is captured on the cost side. Hence, the notion of economic rent captured in the model is *Ricardian rent*, which is quite different from the Hotelling rent.²⁵ Ricardian rent is defined as the market price of the resource net of production cost, where non-resource factors of production (physical capital and labor) are valued at market rates. As Hotelling rent is associated with exhaustibility, Ricardian rent is associated with resources that occur in a varying quality. An interpretation of the assumption of increasing extraction costs is that the resource occurs in layers of different quality and superior quality, i.e., lower costs deposits, are extracted first.

The assumption of fixed foreign savings rules out investing in foreign assets and borrowing to expand domestic production capacity. This assumption, along with diminishing efficiency of investment, has direct bearing on oil production by reducing capital utilization in the sector. These constraints are realistic and clearly distinguish the general equilibrium approach from the control theory framework of conventional partial equilibrium models of exhaustible resources. In control models, production factors are perfectly mobile and can be instantaneously employed at market prices in unlimited quantities.

The government follows a development strategy that emphasizes investment in the domestic economy without borrowing or investing abroad. The government determines how much to invest in the oil sector. The expansion of production in the oil

sector, however, is subject to diminishing efficiency of investment, which reflects various bottlenecks such as shortages of technical knowledge or skilled labor.

The terminal conditions play a decisive role in the outcome of any finite-horizon optimization model. The optimization literature, however, provides no clear suggestion other than that the resulting time paths must be reasonable. Experiment 1 below describes the effects of terminal conditions on the optimal depletion path. The other parameters and model assumptions that have a significant effect on the depletion path are the discount rate and the assumption of constant world prices. Discounting has important implications for inter-generational equity and directly affects the optimal depletion path. Experiment 2 examines the sensitivity of the model outcome to variation in the discount rate. The assumption of constant world prices has clear implications for the depletion path. For example, with expectations of a higher future price the government would adjust its production policy to reap future benefits. Our assumption of a constant real world price of oil, however, is not far removed from experience.

Experiment 1. Effects of Resource Valuation on Optimal Depletion

The terminal conditions in this model consist of a valuation of total physical capital stock and a valuation of unexploited resource left in the ground. Choosing various salvage values for capital and resource at the terminal period generates alternative paths for the economy. Figure 4 shows the depletion paths when the salvage values of both types of capital are increased 5% or decreased 5% and 50%.

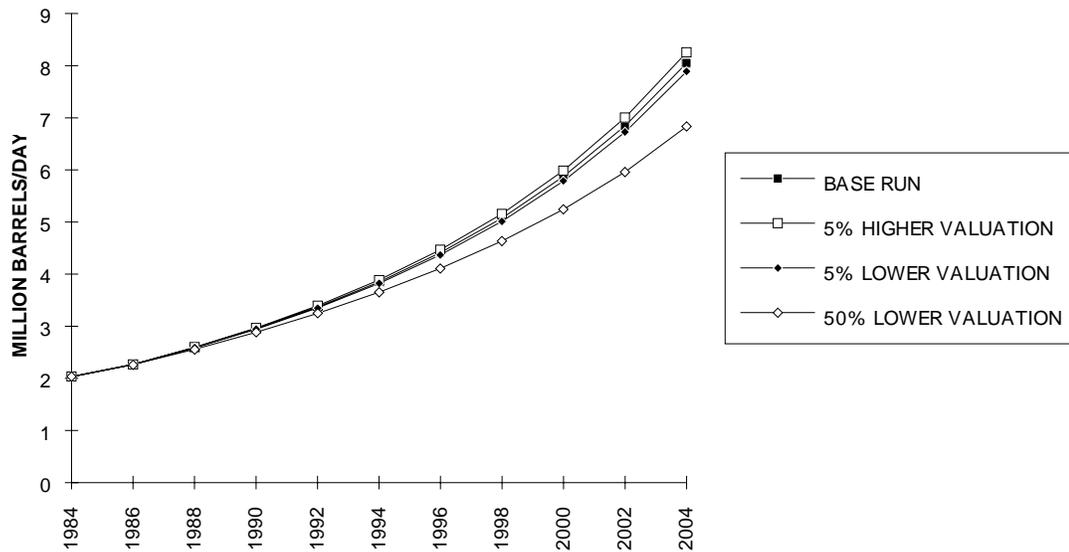


Figure 4. Effects of Various Valuation of Terminal Capital Stock and Resource on Depletion Path

In addition to effects of changes in the salvage value of physical and natural capital, the relative valuation of the two types of capital also has a significant impact on the depletion path. In this part of the experiment we keep the salvage value of physical capital constant (at its base run value) and vary the salvage value of the resource. We ask what depletion path would result if the salvage value of the resource were twice (four times) higher than the physical capital and vice versa. As Figure 5 shows, the relative valuation of these two types of capital has important impact on the depletion path. For example, the experiment shows that if the salvage value of the resource were four times greater than physical capital, we would basically have the same level of extraction as that of the base year. On the other hand if the salvage value of the resource is only one fourth that of capital, we would tend to deplete the resource rapidly. This result is significant in that it reveals that the rate at which a country decides to deplete its exhaustible resource greatly depends on its relative valuation of man-made and natural capital. Intuitively, this says that if a resource-based economy perceives the physical capital to be more valuable than the resource, everything else being constant, it would tend to deplete its

resource at a faster rate. The general tendency in oil economies to convert non-renewable resource to reproducible capital as fast as possible reflects the perception that they can do better with man-made capital than natural resource.

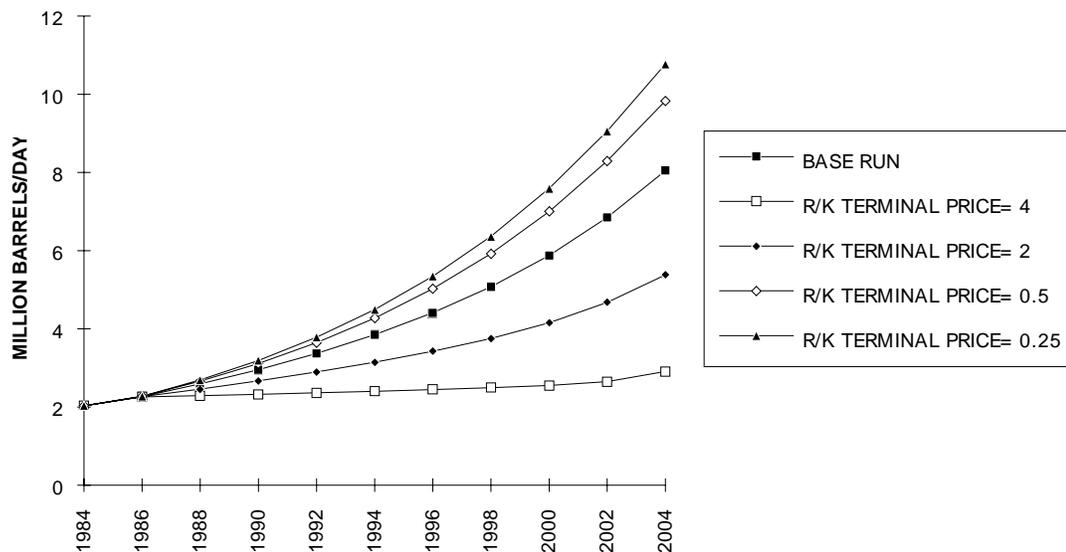


Figure 5. Effects of Change in the Relative Terminal Period Valuation of Capital and Resource on Depletion Path

Experiment 2. Effects of Discount Rate

Variation in the discount rate has profound implications for inter-generational equity. The ethical and welfare issues raised by discounting have been debated by economists for a long time with no clear resolution.²⁶ There seems to be some agreement, however, that in general the social discount rate is below the private discount rate. In particular, when a natural resource is owned publicly, the government is expected to manage with a greater consideration for inter-generational equity. The partial equilibrium models of exhaustible resources have demonstrated that the higher the discount rate, the greater is the tendency to consume by the current generation. To study

the effect of the discount rate on the depletion profile, we conducted a simulation experiment doubling the discount rate.

Figure 6 shows the extraction level for the base case, with a discount rate of 5%, as well as for the case with a discount rate of 8%. Similar to results from partial equilibrium models, generally, the depletion takes place faster under the higher discount rate. A higher discount rate encourages more current consumption and reflects a lower valuation for the resource left in the ground for the future, therefore, the depletion takes place at a faster rate.

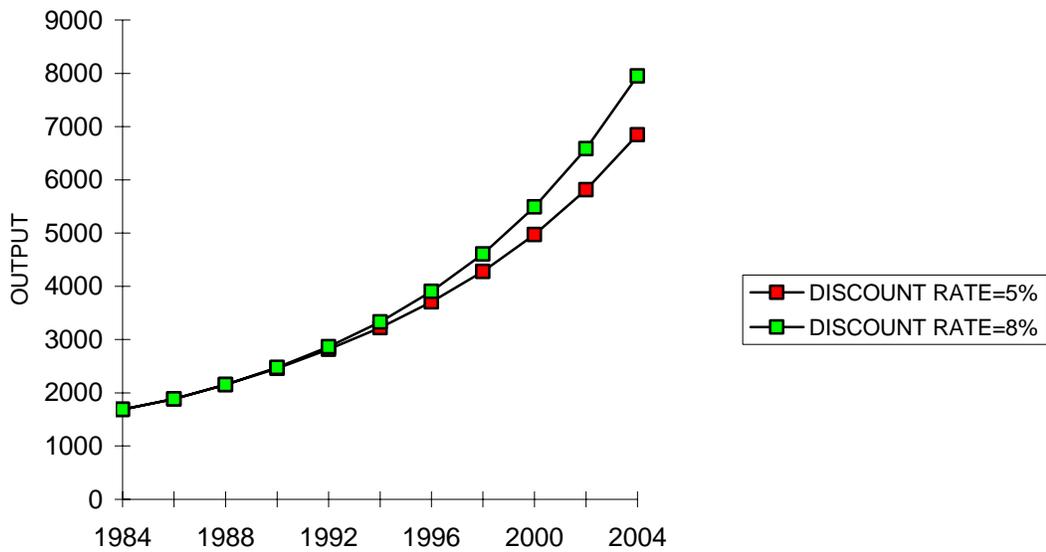


Figure 6. Oil Production Under Two Different Discount Rates

6- Conclusions

This paper presented a dynamic CGE model designed to investigate fundamental questions that oil exporting developing countries must address. Combining elements from exhaustible resources and computable general equilibrium literatures we

constructed a dynamic multisectoral optimization model of the Iranian economy. The model addressed the questions of optimal depletion, optimal savings, and investment allocation in a general equilibrium framework. A general equilibrium approach, in this instance, is superior to a conventional partial equilibrium approach because it captures economy-wide effects. In particular, our computable general equilibrium model takes into account:

- constraints implied by a market economy;
- imperfect substitution in foreign trade;
- imperfections in capital markets such as diminishing marginal efficiency of investment, heterogeneous sector specific capital, and time lags in investment gestation; and
- inter-sectoral interactions.

The results of simulation experiments show that:

- 1- The relative valuation of resource and physical capital has significant effect on resource depletion. A country that imputes a higher value to physical capital would extract its resource more rapidly. This valuation is subjective and may be based on a perception that, for example, the country can derive more benefits from physical capital than a theoretically equivalent quantity of natural capital.
- 2- Similar to results from partial equilibrium models of exhaustible resource, the model shows that higher discount rates encourage more rapid depletion.

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Endnotes

¹ See Devarajan (1988) for a brief discussion and references.

² We use the word '*depletion*' interchangeably with the word '*extraction*' and not as a reference to complete exhaustion of a resource.

³ Among CGE models applied to energy issues in developed countries include Jorgensen (1982), Jorgensen and Wilcoxon (1990), Bergman 1988 and 1990.

⁴ There are a few CGE models of developing countries that include financial side of the economy by including assets and asset markets. For a discussion of this group of "financial" CGE models see Robinson (1991); for examples of these models see Rosenweig and Taylor (1990), Bourguignon, Branson, and de Melo (1991), and Lewis (1992).

⁵ The focus of the model is on labor abundant oil exporting countries which have large enough and diversified domestic economy not to consider investing abroad. The issues regarding capital flows, however, can easily be incorporated in the model.

⁶ A strand of neoclassical political economy concerned with development economics holds that the state intervention in the economy, as in imposing physical quantitative controls over imports or licensing investment in capacity creation and expansion, often results in rent seeking activities where resources are diverted from production to socially wasteful activities (see for example Kruger, 1974 and Bhagwati J. 1982). Here we are not concerned with these issues and retain the view that has been dominant in development economics.

⁷ The common practice in CGE literature is that the level of government expenditure is either exogenously given, as in Jorgenson and Yun (1986), and Zonnoor (1983), or endogenously determined by the balanced budget conditions, as in Benjamin et al. (1989), and Lewis (1992).

⁸ Because of its insignificant role, quite often, labor is left out of oil sector's production function; see for example the Egypt model of Martin et al. (1986).

⁹ Pereira and Shoven (1988) suggest one reason for slow adoption of production-side dynamics is the scarcity of accepted theories regarding the dynamic behavior of firms.

¹⁰ In resource economics literature economic rent (or simply rent) appears under a number of different names including: shadow price of resource, user cost, royalty, opportunity cost (of using a unit of resource today), net price and marginal profit. In essence all these names refer to the difference between price and marginal cost of producing an exhaustible resource. For a concise note on the historical background to rent on exhaustible resources see Hartwick(1989), appendix I pp 129-137.

¹¹ The modern treatment of the economics of exhaustible resources is due to the pioneering works of Gray(1914) and Hotelling(1931). For a brief account of the contributions of Hotelling to this field of economics see Devarajan, Fisher (1981). For comprehensive surveys of the literature see Peterson and Fisher (1977), Fisher (1981), and Hartwick (1989).

¹² Deverajan(1981) identifies two distinct perspectives on natural resources dating as far back as Malthus(1826) and Ricardo(1817). The Malthusian view holds that the resource is in constant quality hence the extraction costs are constant (independent of the stock) as it is assumed in Hotelling's model. The Ricardians, on the other hand, underline the fact that resources occur in varying grades thus higher quality ores get depleted first and lower qualities impose higher extraction costs. Both perspectives of course agree on the existence of a limit on the availability of resources.

¹³ A number of studies that have rigorously extended Hotelling's results to the case of optimal depletion of deposits of varying qualities include: Herfindahl(1967), Heal(1976), Solow and Frederic Y. Wan(1976), and Hartwick(1978).

¹⁴ Fisher(1981) provides an informal discussion of various kinds of uncertainty and summarizes the effects of uncertainty and exploration on the time paths of the output and price of an exhaustible resource. Pindyck(1978) presents a model of optimal exploration and production of a resource; Deverajan(1981), Deverajan and Fisher(1982) present a model of uncertain exploration and show that unobservable resource rent is linked to marginal exploration cost which is observable.

¹⁵ A full discussion of the derivation and interpretation of these conditions can be found in Schluz (1974), Pindyck (1978), or Fisher (1981).

¹⁶ See Varian(1984) pp 28-9.

¹⁷ For a full explanation and limitations of this approach to modeling the investment allocation see Dervis, et al (1982). For an intertemporal forward looking investment behavior specification see Deverajan, and Go (1998).

¹⁸ This is a simplified form of the absorptive capacity function used in Kendrick (1990).

¹⁹ For a later exposition and refinement of this class of CGE models see Devarajan, Lewis and Robinson (1991).

²⁰ The idea of CET specification is due to Powell and Gruen (1968). The idea of product differentiation between domestic output and exports is very common in CGE models of developing countries.

²¹ This means that the economy is stationary within the duration of 2 years and dynamic equations are updated from one period to the next and not annually.

²² The last periods in numerical optimization models are often not counted because of abrupt change in some endogenous variables at the terminal period. For example, in our model marginal propensity to save falls to its lower bound at terminal period since there is no need to save anymore.

²³ There is no objective way to determine whether man-made capital or natural capital contributes more to the welfare at a given point in time. In our base run we assume that contribution to the welfare of one barrel of oil is equivalent to that of one thousand units of physical capital. The market value of a barrel of oil at the base year is equivalent to the market value of more than two thousand units of physical capital. The shadow prices of capital and

resource at the base year, however, depending on the initial assumptions about the terminal conditions, indicate much lower valuation for a barrel of oil than that implied by the market valuation.

²⁴ Conventional partial equilibrium models of exhaustible resources assume complete exhaustion by terminal period. See Levhari and Liviatan (1977) for a full discussion of the implications of assuming complete and incomplete exhaustion.

²⁵ A classic result of the theory of exhaustible resources, due to Hotelling, is that along an optimal path the present value of the net price of a resource is constant across periods. In other words, the owner of the resource is indifferent between extracting today and earning a market interest rate on the proceeds or leaving the resource in the ground and selling the following period. This result may be equivalently stated as the famous "r percent rule" which says that the current valued marginal profit of extracting a resource increases over time at rate of interest. This equilibrium rise in the unit price of an exhaustible resource is also known as the *Hotelling rent*. In the general equilibrium context the domestic price of oil is determined by world prices, trade elasticities, and equilibrium conditions.

²⁶ For a brief account of debate on whether or not to discount and on the appropriate level see Fisher (1981) PP 68-74.