Formula Apportionment, Tax Competition, and the Provision of Local Goods

Santiago M. Pinto*

January 14, 2005

Abstract

The paper develops an analytical framework where regional governments strategically determine the structure of the corporate profit tax system and profits are regionally allocated using an apportionment formula. Two important results emerge in a symmetric Nash equilibrium: (i) investment decisions are distorted, i.e., regional governments will not allow complete deduction of capital costs from taxable corporate profits; and (ii) there is underprovision of the good provided by the regional government, consistent with the literature on property tax competition. The paper also shows that the degree of underprovision may be less severe when the formula employs sales shares to apportion corporate profits. The model allows us to presume that the recent shift by most states in the U.S. towards a formula apportionment that gives a higher weight to the sales proportion may constitute a welfare improvement for all regions, compared to the original formula that weighs all factors equally.

1 Introduction.

If a corporation has business activities established in multiple jurisdictions, regions, or countries,[1] then the local authority can levy a tax on income generated in that location. However, measuring income earned within each region presents a difficult conceptual problem. For instance, the current system of corporate taxation in the European Union requires firms to maintain different accounts for its activities in each country where it operates. The U.S. and Canada, on the other hand, have adopted a system of formula apportionment (FA) to allocate income across states, which relies on observable factors. FA, as used in the U.S., asserts that the proportion of a multi-regional firm's income earned in a given state is a weighted average of the proportion of

*Department of Economics, West Virginia University, 412 Business and Economics Building, P.O. Box 6025, Morgantown, WV 26506-6025. Office Phone #: (304) 293-7871. E-mail: smpinto@mail.wvu.edu. I gratefully acknowledge financial support from the Regional Research Institute, WVU. I would also like to thank Yousam Choi for his efficient assistance.

[1] These three terms will be used indistinctively along the paper.

the firm's total sales, property, and payroll in that state. Thus, the firm's activities in a specific region is approximated by the share of these factors in the region, so the firm is not required to keep different accounts. Specifically, the tax due by a firm to state i is

$$T^i = t^i \left[m^{iK} \frac{K^i}{K} + m^{iW} \frac{W^i}{W} + m^{iF} \frac{F^i}{F} \right] \Omega^i \tag{1}$$

where K^i, W^i, and F^i are property, payroll, and sales in state i, respectively; K, W, and F are total domestic property, payroll, and sales of the firm respectively; m^{ij} is the weight given to factor $j = K, W, F$, in the apportionment formula in state i; Ω^i represents the firm's taxable profits as defined by state i's tax law; and t^i is state i's tax rate.[2] Table 1 shows the weights m^{ij} chosen by different states in the U.S. As it can be observed, there does not exist a single principle followed by all states.

However, this method of apportionment is not, by all means, neutral in terms of its economic effects. If states adopt the same apportionment formula, exactly 100 percent of a corporation's income will be apportioned across states. Non-uniformity, however, can result in more or less than 100 percent of a corporation's income being subject to state income tax. In an effort to encourage tax uniformity across jurisdictions, the Multi-state Tax Compact in 1967 established that the three factors considered in the apportionment formula are to be weighted equally ($m^{iK} = m^{iW} = m^{iS}$ for all regions i). In spite of this, most states have recently deviated from the uniform apportionment formula and moved towards a greater weight on the sales portion of the corporate income tax, as it is clear from Table 1. It has been claimed that by manipulating the formula in this way, officials can offer tax breaks that help the economic development of the region. However, if more states pass such legislation, other states will be compelled to do the same, initiating a "race to the bottom", in which all states end up imposing the same (lower) tax liability.

The apportionment of a firm's total profits across states creates complicated incentive effects. On one hand, firms operating in different regions will react to different formulas by changing the allocation of property, sales and workers across regions. On the other hand, given that the tax policy chosen by different regional governments affects residents of other states, some kind of strategic interaction can be expected. The present paper develops a model that incorporates these effects and analyzes how different formulas affect the provision of regional goods. In our analytical framework, each government can endogenously choose both the corporate tax rate and the corporate tax base. The latter is basically determined by the proportion

[2] Regional governments may define tax bases differently. The present paper considers one way in which tax bases may differ across regions: the proportion of capital costs that can be deducted from the corporate taxable income.

of capital costs that can be deducted from taxable income. In a closed economy, with no cross-border ownership and, consequently, no tax competition, the regional government chooses not to distort investment, i.e., it will allow full deduction of capital costs from taxable income. However, the paper concludes that if a firm operates in many regions at the same time and regional governments interact strategically, investment decisions end up being distorted, i.e., it is optimal to allow only imperfect deduction of capital costs. The paper also shows that there is underprovision of the good provided by the regional authority,[3] and even more interestingly, it demonstrates that the degree of underprovision is more severe when the apportionment formula employs capital shares (or a linear combination of capital and sales shares) compared to one that exclusively uses sales shares. Hence, the shift towards a FA with only sales shares, as evidenced in the U.S., would increase the country's economic welfare according to our model.

Even though there seems to be a great interest in the strategic determination of apportionment formulas among regional governments, only recently this issue has been formally examined. The earlier papers by McClure (1980) and McClure (1981) and Mieszkowski and Zodrow (1985) first elucidated that formulary apportionment mostly transforms the state corporate income tax into three separate taxes on the factors in the apportionment formula. Gordon and Wilson (1986) examine the response of firms to a system of formula apportionment, restricting attention to cases in which all states use the same system, with different corporate tax rates. Most of their analysis is concerned with the component of the tax tied to the allocation of property. Anand and Sansing (2000) provide an explanation for why states choose different apportionment formulas, even though aggregate social welfare is maximized when states use the same formulas. They demonstrate analytically that "importing" states have incentives to increase sales factor weights, while "exporting" states have incentives to reduce the weights on productive factors. This literature is mostly concerned with the allocation of corporate income across states within a given country. However, the conclusions also apply to the relationship between multinational enterprises and national tax systems, in particular, the allocation of foreign direct investment across countries.

The organization of the paper is as follows. Section 2 analyzes the optimal corporate tax rates and tax bases chosen by regional governments in a closed economy: each region has a local firm and a FA system is not required. In this section we assume that the ownership of the firm is entirely in the hands of the local residents. In Section 3, we consider the case of a

[3] Given the complicated incentive effects generated by this tax system, the model analyzes the outcome of a symmetric Nash equilibrium. This approach has been traditionally followed by the literature on property tax competition. See for example, Brueckner and Saavedra (2001).

multi-regional firm with cross-border ownership, and study the incentive effects of a FA system. In Section 4 we derive the optimal corporate tax policy under different FA systems. Finally, Section 5 concludes.

2 The Bencchmark Case: No Cross-Border Ownership

Before analyzing the effects of a FA system and to understand its implications better, we derive the optimal corporate tax policy in a very simple case.[4] The model considers an economy with two identical regions, A and B.[5] An homogeneous consumption good x^i, which serves as the numéraire, is locally produced by a firm operating exclusively in region i. There is a representative immobile consumer-investor in each jurisdiction. The ownership of the firm is entirely in the hands of the local consumer. Capital is the only variable factor, it is in perfectly elastic supply, and can be rented at an exogenous rate r. Output is determined by the production function $f^i = f(k^i)$, which is identical across regions, and satisfies $f_k > 0$, $f_{kk} < 0$.[6] Gross profits in each region i are given by $\Pi^i = f(k^i) - rk^i$.

Consumers derive utility from the consumption of x and from a local good publicly provided g^i. Preferences are represented by a strictly quasi-concave utility function $u^i = u(x^i, g^i)$, where $u^i_j > 0$ represents the marginal utility of good $j = x, g$. The latter is locally financed by a tax on corporate profits.[7] The firm's after tax profits accrue entirely to the representative consumer in each region and constitute the only source of income.

Standard corporate profit tax models assume that taxable corporate profits differ from economic profits in many different and complicated ways. As in Gordon and Wilson (1986), we assume for simplicity that this discrepancy is basically determined by the imperfect deductibility of capital expenditures from taxable income. We depart from their approach in that we assume that the regional government can manipulate the deduction policy, i.e. each region has control over the corporate tax base by choosing different deduction policies. Specifically, the tax base in region i is the value of output minus a fixed share μ^i of the true capital cost.[8] The deductibility

[4]A similar model, in a slightly different context, can be found in Haufler and Schjelderup (2000). The framework is useful as a benchmark case.

[5]Since we assume that regions are identical, restricting the number to two locations does not affect the conclusions of the paper. The results will remain the same if we considered many identical regions.

[6]It is assumed that there is an implicit fixed factor that gives rise to pure profits in each region. We can think of this factor as being entrepreneurial services.

[7]In this model, the regional government does not have any other tax instrument available. It can only use a tax on corporate profits to finance the publicly provided good.

[8]The value of μ^i will depend on the details of the tax rules and tax laws in region i, and it will have the potential to influence the firm's behavior, in particular, the location of its business activity. Hines (1997, 1999) provides a critical survey of the literature

of capital expenses for tax purposes can either be above or below its true costs. Most tax systems allow only incomplete deduction of capital costs, or $\mu^i < 1$, which means that a positive tax is also levied on capital. For $\mu^i = 1$, the corporate tax falls only on pure profits, and when $\mu^i > 1$ capital is subsidized. Thus, taxable income as computed by region i, Ω^i, and economic profits Π^i are related as follows:

$$\Omega^i = f(k^i) - \mu^i r k^i = \Pi^i + (1-\mu^i) r k^i. \tag{2}$$

Taxes paid by the firm to region i, $T^i = t^i \Omega^i$, are

$$T^i = t^i[\Pi^i + (1-\mu^i) r k^i], \tag{3}$$

while net profits $N^i = \Pi^i - T^i$ can be expressed as

$$N^i = (f^i - rk^i)(1-t^i) - t^i(1-\mu^i) r k^i. \tag{4}$$

2.1 The firm's problem

A firm in region i chooses[9] the level of k^i which maximizes net profits for a given tax policy. The problem consists of maximizing N^i with respect to k^i. From the FOC we obtain the expression

$$f^i_k - r = \frac{t^i(1-\mu^i) r}{(1-t^i)} \tag{5}$$

that implicitly defines k^i as a function of t^i, μ^i, and r. It is then straightforward to derive the firm's reaction to changes in t^i and μ^i:

$$\frac{\partial k^i}{\partial t^i} = \frac{(1-\mu^i) r}{f_{kk}(1-t^i)^2}, \quad \text{and} \tag{6}$$

$$\frac{\partial k^i}{\partial \mu^i} = \frac{-t^i r}{f_{kk}(1-t^i)}. \tag{7}$$

An increase in the tax rate will drive capital out of the region if $\mu^i < 1$, and will attract capital if $\mu^i > 0$. For $0 < t^i < 1$, a higher value of μ^i will create an inflow of capital to region i.

2.2 The government's problem

The regional or state government determines the values of t^i and μ^i that maximize the consumer's utility $U^i \equiv U(x^i, g^i)$, given that $x^i = N^i$ and $g^i = t^i \Omega^i$.[10] The optimal levels of the choice

that studies behavioral responses to different regional tax rules, focusing on multinational firms.

[9] As consumer i owns the firm, this is essentially a decision made by her.

[10] We will only consider corporate tax rates that satisfy $0 < t^i < 1$.

variables satisfy the following FOC conditions:

$$\partial U^i/\partial t^i \equiv U_x^i(\partial x^i/\partial t^i) + U_g^i(\partial g^i/\partial t^i) = 0, \tag{8}$$

$$\partial U^i/\partial \mu^i \equiv U_x^i(\partial x^i/\partial \mu^i) + U_g^i(\partial g^i/\partial \mu^i) = 0, \tag{9}$$

where

$$\partial x^i/\partial t^i = -\Omega^i < 0, \qquad \partial x^i/\partial \mu^i = t^i r k^i > 0,$$

$$\partial g^i/\partial \mu^i = \Omega^i + f_{kk}^i(1-t^i)(\partial k^i/\partial t^i)^2, \quad \partial g^i/\partial \mu^i = -t^i r k^i(1+\varepsilon^i),$$

and ε^i denotes the elasticity of capital with respect to t^i evaluated at the optimal values of $\{t^i, \mu^i\}$, i.e., $\varepsilon^i = (t^i/k^i)(\partial k^i/\partial t^i)$. In order for a solution to exist, $|\varepsilon^i| < 1$, otherwise both $(\partial x^i/\partial \mu^i)$ and $(\partial g^i/\partial \mu^i)$ would be positive and $\partial U^i/\partial \mu^i > 0$. The optimality conditions can be rewritten as follows:

$$MRS^i = -\frac{\partial x^i/\partial t^i}{\partial g^i/\partial t^i} = -\frac{\partial x^i/\partial \mu^i}{\partial g^i/\partial \mu^i} \tag{10}$$

where $MRS^i = U_g^i/U_x^i$. The marginal cost of increasing g^i is measured in terms of units of x^i that should be sacrificed. The optimal combination $\{t^i, \mu^i\}$ is achieved when these costs are equalized for every tax instrument.

An important implication of this model is that investment decisions should not be distorted. In other words, μ^i should be set equal to one.[11] Substituting the corresponding expressions into (10) and simplifying we obtain

$$\Omega^i(t^i/k^i)(\partial k^i/\partial t^i) = f_{kk}^i(1-t^i)(\partial k^i/\partial t^i)^2. \tag{11}$$

This last equation is only satisfied when $\partial k^i/\partial t^i = 0$, which implies that $\mu^i = 1$. Thus, from (5), it becomes clear that $f_k = r$, so investment decisions are not distorted. The following proposition summarizes our findings so far.

Proposition 1. *When there is no cross-border ownership, μ^i should be set equal to one, so that investment decisions are not distorted at the optimal combination $\{t^i, \mu^i\}$.*

3 The Formula Apportionment Model

We now assume that x is produced by a multi-regional firm (i.e., a firm that operates in both regions), and that the consumer residing in region i owns a proportion θ^i of that firm. The

[11] For this conclusion to hold we also need $0 < t^i < 1$.

two identical regions or countries A and B constitute a common market. The only difference with respect to the previous analysis is that the multi-regional firm can shift capital across regions. Total output (or total sales) is given by $f(k^A) + f(k^B)$, and gross profits by $\Pi = f(k^A) + f(k^B) - rk$, where $k = k^A + k^B$. As before, each regional government provides a good that is financed with a tax on corporate profits. The firm's after tax profits are entirely distributed among consumers according to the "ownership" shares, and constitutes the only source of income.

The multi-regional firm pays profit taxes to each regional government where it operates. Both regions adopt a FA method to calculate the share of the firm's activities in each jurisdiction. These shares determine the distribution of tax revenues across regions. The regions do not necessarily adopt formulas of the same type, i.e., each region may employ different shares, in addition to different corporate tax rates.[12] Moreover, they may not even add up to one. The analytical framework analyzes FA tax regimes that consider sales, property, or a combination of the two factors as proxies of the firm's activities in each state. Hence, these shares ultimately depend on k^A and k^B. Let $\alpha(k^A, k^B)[\beta(k^A, k^B)]$ denote the share of the firm's activities in region A[B]. In addition, we assume that tax shares add up to unity, i.e., $\alpha(k^A, k^B) + \beta(k^A, k^B) = 1$.[13] For instance, if only capital shares are considered in the apportionment formula, then $m^{AK} = 1$ in equation (1), so $\alpha(k^A, k^B)$ will become

$$\alpha^K = k^A/k, \tag{12}$$

where α^K is the capital (or property) share in region A. Moreover, $\beta^K = (1 - \alpha^K)$ denotes the corresponding proportion of capital in region B. On the other hand, if the apportionment formula exclusively weighs output shares, $m^{AF} = 1$, and $\alpha(k^A, k^B)$[14] is defined by

$$\alpha^F = f(k^A)/[f(k^A) + f(k^B)] \tag{13}$$

where α^F is the output share in region A and $\beta^F = (1 - \alpha^F)$ denotes the corresponding share in region B. If plants produce for the local markets where they are located, outputs and sales are equivalent, and $\alpha^F[\beta^F]$ also represents sales shares.[15]

As in the previous section, the tax base in region i is the value of output minus a fixed

[12] The choice of the specific formula can be determined on the basis of strategic considerations or bilateral agreements. Our model takes the choice of the FA as given and analyzes the consequences of using different FA systems when the corporate profit tax rates and tax bases are strategically determined.

[13] Pethig and Wagener (2003) call such regimes "uniform FA tax systems".

[14] For notational simplicity, we will just write it α hereafter.

[15] This weight is consistent with a tax at the origin. Given that in our setting regions are identical, in a symmetric equilibrium exports and imports will be zero.

share μ^i of the true capital cost. Hence, the firm is allowed to deduct a proportion μ of its capital costs, where μ is defined as $\mu = \alpha^K \mu^A + \beta^K \mu^B$. Denoting by t^i region i's statutory corporate profit tax rate, and $t = \alpha t^A + \beta t^B$ the effective tax rate, then the firm's total tax burden under FA is given by

$$\begin{aligned} T &= t[f(k^A) + f(k^B) - \mu r k] \\ &= t[\Pi + (1-\mu) r k] \\ &= t\Omega, \end{aligned} \tag{14}$$

where Ω is the tax base. As a result, after-tax profits $N = \Pi - T$ are

$$N = (1-t)\Pi - t(1-\mu) r k. \tag{15}$$

The sequence of events is as follows: (i) the regional governments announce their tax policy, i.e. they announce the corporate tax rate (t^i) and tax base (determined by μ^i) for a given FA system; (ii) the firm observes the tax policy chosen by each state and allocates capital across regions; (iii) g^i and x^i are determined and payoffs are realized.

3.1 Firm's Profit Maximization

It will result very useful to compare the impact of a change in t^i on k^A, k^B and k under the special FAs mentioned before, i.e., when α is α^K, α^F, or any combination of the two factors. To do so, we first determine how the multi-regional firm allocates capital between jurisdictions. Consider the case of a general apportionment formula.The firm maximizes (15) by choosing k^A and k^B taking the tax policy as given. The FOCs for an interior solution are

$$f_k^A - r = \frac{t(1-\mu^A)}{(1-t)} r + \frac{\alpha_A (t^A - t^B)}{(1-t)} \Omega, \tag{16}$$

$$f_k^B - r = \frac{t(1-\mu^B)}{(1-t)} r + \frac{\beta_B (t^B - t^A)}{(1-t)} \Omega, \tag{17}$$

where $\alpha_A = (\partial \alpha / \partial k^A) > 0$, and $\beta_B = (\partial \beta / \partial k^B) > 0$. If $\mu^A = \mu^B = \mu = 1$, the previous conditions become $(f_k^i - r) = \alpha_i \Pi[(t^i - t^j)/(1-t)]$ (for $i, j = 1, 2$). Compared to the results in Section 2, there is now an additional term which affects capital allocation between jurisdictions when different corporate tax rates are adopted by local authorities. In this case, capital will be optimally allocated between regions only if $t^A = t^B = t$.

The comparative static results under different FA systems with respect to tax rates are hard to obtain because the firm does not only influence the tax base, but also the effective tax rate t by changing its capital allocation across regions. For instance, $(\partial t/\partial k^A) = (t^A - t^B)\alpha_A$, so the effective tax rate increases with k^A if $t^A > t^B$, and decreases if $t^A < t^B$. In this section we calculate the firm's response to different t^is under full symmetry, i.e., when the two regions are completely identical, and they choose the same tax rates ($t^A = t^B = t$) and tax bases ($\mu = \mu^A = \mu^B$). These assumptions imply that $k^A = k^B = k/2$ and $f_{kk} = f^A_{kk} = f^B_{kk}$. As a result,

$$\frac{\partial k^A}{\partial t^A} = \frac{1}{(1-t)f_{kk}}\left[\frac{\alpha(1-\mu)}{(1-t)}r + \alpha_A\Omega\right], \quad (18)$$

$$\frac{\partial k^B}{\partial t^A} = \frac{1}{(1-t)f_{kk}}\left[\frac{\alpha(1-\mu)}{(1-t)}r - \beta_B\Omega\right]. \quad (19)$$

Similar expressions can be obtained for t^B. In a symmetric equilibrium $\alpha = \beta = 1/2$, and $\alpha_A = \beta_B$. Note that when $\mu = 1$, $|(\partial k^A/\partial t^A)| = (\partial k^B/\partial t^A) > 0$.[16] If $\mu < 1$, total capital is reduced as t^A increases, $(\partial k/\partial t^A) < 0$.

When $\alpha^K[\beta^K]$ and $\alpha^F[\beta^F]$ are the shares in the formula,

$$\alpha^K_A = \frac{k^B}{k^2}, \qquad \beta^K_B = \frac{k^A}{k^2}, \quad (20)$$

$$\alpha^F_A = \frac{f^B f^A_k}{F^2}, \qquad \beta^F_B = \frac{f^A f^B_k}{F^2}, \quad (21)$$

and in a symmetric equilibrium,

$$\alpha^K_A = \beta^K_B = \frac{1}{2k}, \quad (22)$$

$$\alpha^F_A = \beta^F_B = \frac{f_k}{4f}. \quad (23)$$

Given that production exhibits decreasing returns to scale, average product is always greater than marginal product, or $f/(k/2) > f_k$, which implies that $\alpha^K_A > \alpha^F_A$. Thus, $|\partial k^A/\partial t^A|_{\alpha^K} > |\partial k^A/\partial t^A|_{\alpha^F}$, so that capital in A is less responsive to a change in t^A when $\alpha = \alpha^F$. Moreover, if net profits are positive, i.e., $N > 0$,[17] and $\alpha = \alpha^K$, then

$$\frac{\alpha^K(1-\mu)}{(1-t)}r - \beta^K_B\Omega < 0, \quad (24)$$

and $(\partial k^B/\partial t^A)_{\alpha^K} > 0$. On the other hand, the expression $(\partial k^B/\partial t^A)_{\alpha^F}$ is in general ambiguous, but $|(\partial k^B/\partial t^A)_{\alpha^F}| < (\partial k^B/\partial t^A)_{\alpha^K}$ always hold, so the impact of a change in t^A on capital in region B is largest (in absolute value) under a FA which gives full weight to the property share.

The comparative static results with respect to μ^A (and μ^B) are less complicated to derive.

[16] Remember that with no cross-border ownership and no strategic competition, $\partial k^i/\partial t^i = 0$ when $\mu = 1$.

[17] Net profits should be positive in our setup because this is the only source of income of the representative consumer.

It can be shown that, under full symmetry,

$$\frac{\partial k^A}{\partial \mu^A} = -\frac{tr}{f_{kk}(1-t)} > 0, \quad \text{and} \tag{25}$$

$$\frac{\partial k^B}{\partial \mu^A} = 0. \tag{26}$$

An increase in μ^A attracts more capital to A and does not affect the amount of capital allocated to region B. Remember that higher values of μ^A imply a lower tax on capital (or eventually a subsidy on capital if μ becomes greater than one) due to the fact that capital costs cannot be fully deducted from taxable income.

4 The Government's Problem

Two different cases will be examined. First, it is assumed that each state considers the same taxable income, i.e., $\mu^A = \mu^B = \mu$, but strategically determines the corporate tax rate t^i taking the tax rate chosen by the other region as given. Next, we allow regional governments to also compete on the level of μ^i. The analysis always focuses on symmetric Nash equilibria.

4.1 Strategic determination of corporate tax rates

Each regional government or country i maximizes the utility of its representative consumer by choosing the statutory corporate tax rate t^i treating the tax rate in the other jurisdiction as given. For the moment, we assume that both regions follow the same tax policy regarding tax bases, i.e., $\mu^A = \mu^B = \mu$. The tax revenue is used to finance the provision of a regional good, g^i. For the tax authority in region A, the problem consists of maximizing $U^A \equiv U(x^A, g^A)$ with respect to t^A subject to the government budget constraint, $g^A = t^A \alpha \Omega$, given that $x^A = \theta N$, and viewing t^B as parametric.[18] From the FOC, we obtain the reaction function of the local government in region A:

$$MRS^A = -\frac{\partial x^A/\partial t^A}{\partial g^A/\partial t^A}. \tag{27}$$

The latter implicitly defines t^A as a function of t^B. A similar expression can be obtained for region B. These two conditions jointly determine the equilibrium values of t^A and t^B. In a

[18] Given that we focus on a symmetric equilibrium, $\theta = \theta^i$, for $i = A, B$.

symmetric equilibrium,

$$\frac{\partial x^A}{\partial t^A} = -\theta\alpha\Omega, \tag{28}$$

$$\frac{\partial g^A}{\partial t^A} = \alpha\Omega + t(1-t)f_{kk}\left[\left(\frac{\partial k^A}{\partial t^A}\right)^2 + \left(\frac{\partial k^B}{\partial t^A}\right)^2\right]. \tag{29}$$

As a consequence,

$$MRS^A = \theta\gamma \tag{30}$$

where

$$\gamma = \frac{\alpha\Omega}{\alpha\Omega + t(1-t)f_{kk}\left[(\partial k^A/\partial t^A)^2 + (\partial k^B/\partial t^A)^2\right]} > 1.$$

The previous condition holds for any FA system, i.e, α can be substituted by α^K or α^F, or any convex combination of them. The tax rates are inefficiently low, or alternatively, there is underprovision of the publicly provided good under FA if $\theta\gamma > 1$. On the other hand, there is overprovision if $\theta\gamma < 1$.

We will now consider the effect of employing different FA systems. In a symmetric equilibrium, the FAs considered in this paper determine the same equilibrium capital level in both regions.[19] Let γ^l ($l = F, K$) be the value of γ in (30) that only uses production shares ($l = F$) or property shares ($l = K$) in the formula. Given that $(\partial k^B/\partial t^A)_{\alpha^K} > |(\partial k^B/\partial t^A)_{\alpha^F}|$, and $|\partial k^A/\partial t^A|_{\alpha^K} > |\partial k^A/\partial t^A|_{\alpha^F}$, it is then evident that $\gamma^K > \gamma^F$ as the denominator gets smaller when $\alpha = \alpha^K$. So for a given θ, $MRS^A|_{\alpha^K} > MRS^A|_{\alpha^F}$.

Consider a situation where tax rates are inefficiently low. Then, the degree of underprovision is more severe when $\alpha = \alpha^K$ compared to $\alpha = \alpha^F$ as $MRS^A|_{\alpha^K} > MRS^A|_{\alpha^F} > 1$. Thus, a shift towards a (symmetric) FA under output shares actually constitutes an improvement for the economy as a whole. In fact, it dominates, under the previous conditions, any convex combination of capital and output shares. The reason is that capital becomes less responsive to changes in tax rates when α^F is used as opposed to α^K. Therefore, we have the following proposition.

Proposition 2. *When corporate tax rates are strategically chosen by regional governments, a symmetric Nash Equilibrium is consistent with either efficient or inefficient levels of the publicly provided good. If there is underprovision of the good when the FA is based exclusively on production shares, then there will also be underprovision under a FA which employs only property*

[19]When $t^A = t^B$, conditions (16) and (17) are independent of the FA system chosen.

shares, or any combination of the two. However, the underprovision will be less severe if the FA gives full weight to production shares.

4.2 Strategic determination of both corporate tax rates and tax bases

Now suppose that the regional government can choose both the levels of t^i and μ^i simultaneously taking the tax instruments decided by the other jurisdiction as given. The local government of region A determines the values of $\{t^A, \mu^A\}$ which maximize $U^A \equiv U(x^A, g^A)$, where $g^A = t^A \alpha \Omega$, and $x^A = \theta N$ viewing $\{t^B, \mu^B\}$ as parametrically. The FOCs of this problem are:

$$\partial U^A/\partial t^A \equiv U_x^A(\partial x^A/\partial t^A) + U_g^A(\partial g^A/\partial t^A) = 0, \quad (31)$$

$$\partial U^A/\partial \mu^A \equiv U_x^A(\partial x^A/\partial \mu^A) + U_g^A(\partial g^A/\partial \mu^A) = 0. \quad (32)$$

These two equations implicitly define t^A and μ^A as a function of t^B and μ^B. Similar expressions can be obtain for region B. The four conditions determine a Nash equilibrium $\{t^A, t^B, \mu^A, \mu^B\}$. Rearranging (31) and (32), we obtain

$$MRS^A = -\frac{\partial x^A/\partial t^A}{\partial g^A/\partial t^A} = -\frac{\partial x^A/\partial \mu^A}{\partial g^A/\partial \mu^A}, \quad (33)$$

which is equivalent to the condition derived in Section 2. The expressions for $(\partial x^A/\partial t^A)$ and $(\partial g^A/\partial t^A)$ in a symmetric Nash Equilibrium were obtained before.[20] The corresponding expressions for $(\partial x^A/\partial \mu^A)$ and $(\partial g^A/\partial \mu^A)$ are

$$\partial x^A/\partial \mu^A = \theta t r k^A > 0, \quad (34)$$

$$\partial g^A/\partial \mu^A = -t r k^A (\alpha + \varepsilon), \quad (35)$$

where $\varepsilon = (t/k^A)(\partial k^A/\partial t^A) < 0$ is the elasticity of capital with respect to the tax rate in equilibrium. Notice that a solution for μ^A will exist if in equilibrium $\alpha > |\varepsilon|$, i.e., in equilibrium $|\varepsilon|$ should be relatively small, in which case $\partial g^A/\partial \mu^A < 0$. As a result,

$$MRS^A = \frac{\theta}{\alpha + \varepsilon}. \quad (36)$$

Remember that $\alpha = 1/2$ in a symmetric equilibrium. If $\theta = 1/2$, which implies an equal distribution of the multi-region corporate profits among residents, and given that $\varepsilon < 0$, then it is straightforward to see from (36) that $MRS^A > 1$, or equivalently, the good provided by the regional government ends up being underprovided. Moreover, this result holds for any FA, i.e., $\alpha = \alpha^K, \alpha = \alpha^F$, or any convex combination, $\alpha = m^K \alpha^K + m^F \alpha^F$, where $m^K + m^F = 1$. The following proposition summarizes the result.

[20] See equations (28) and (29).

Proposition 3. *In a symmetric Nash equilibrium where corporate profits are equally distributed among residents, i.e., $\theta = 1/2$, there is underprovision of the publicly provided good.*

We showed in Section 2 that when there is no strategic competition between regions, and regional governments choose the optimal levels of t^i and μ^i, full deduction of capital should be allowed, i.e., $\mu^i = 1$. However, when a firm can shift capital between regions, and regional governments determined the corporate tax rate and tax base strategically, then they will choose a value of μ^i that is less than one in any FA. The following proposition states formally this result.[21]

Proposition 4. *If regional governments choose both t^i and μ^i $(i = A, B)$, then in a symmetric Nash equilibrium $\mu^i < 1$ for any FA system.*

Lowering the level of μ^i (in particular, choosing a value less than one), allows the regional government to increase g^i according to (35). On the other hand, we have concluded that a symmetric Nash equilibrium is characterized by an underprovision of the publicly provided good. As a result, the level of t^i should then be lower than the optimal value of t^i adopted when $\mu^i = 1$. Hence, the regional government opts to distort investment decisions by choosing $\mu^i < 1$ in order to reduce corporate tax rates and the incentive to shift the activities to the other region.

Proposition 3 claims that there is underprovision of the publicly provided good. In addition, it was established earlier (Proposition 2) that when there is underprovision, it will be less severe if the FA completely relies on output (or sales) shares. This result also holds here. Let $MRS\mid_{\alpha^F}$ be the MRS under a system of FA that only employs output shares, and $MRS\mid_{\alpha^K}$ the MRS under a FA that relies exclusively on capital or property shares. Then, the following expression is true

$$MRS\mid_{\alpha^K} = \frac{\theta}{\alpha + (t/k^A)(\partial k^A/\partial t^A)_{\alpha^K}} > \frac{\theta}{\alpha + (t/k^A)(\partial k^A/\partial t^A)_{\alpha^F}} = MRS\mid_{\alpha^F} > 1. \tag{37}$$

The first inequality holds because $|(\partial k^A/\partial t^A)_{\alpha^F}| < |(\partial k^A/\partial t^A)_{\alpha^K}|$, and the second results from Proposition 3. Hence, the departure from the optimal provision of the regional goods is even greater in this last scenario. The result is also valid for any FA that uses a linear combination of property and output shares, in particular, the uniform FA suggested by the Multistate Tax Compact in 1967 for the U.S. In our model, a uniform FA equally weighs the property and the output shares, i.e. $\alpha = (1/2)\alpha^K + (1/2)\alpha^F$. Thus, it is clear from our previous analysis that

$$MRS\mid_{\alpha^K} > MRS\mid_{(1/2)\alpha^K + (1/2)\alpha^F} > MRS\mid_{\alpha^F} > 1. \tag{38}$$

[21] The proof is relegated to the Appendix.

Proposition 5. *The degree of underprovision is more important when the FA employs exclusively capital shares or a convex combination of capital and output shares, compared to a FA that only uses sales shares to approximate the firm's activities in the region.*

Therefore, we conclude that the strategic determination of $\{t^i, \mu^i\}$ by regional governments may result in corporate tax rates set at inefficiently low levels and tax systems that only allow incomplete deduction of capital costs from the taxable income (i.e., $\mu^i < 1$). Moreover, the problem will become more serious as the FA relies more heavily on property shares to proxy the firm's activities in the region.

5 Conclusions

States and countries that employ a corporate profit tax to finance part of its activities face the problem of measuring a corporation's tax liability to each region. The use of a FA to allocate income across locations introduces very complicated incentive effects for both the firms that operate in different jurisdictions and for local governments that shape the structure of the corporate tax system. In this paper we consider a tax competition model with respect to corporate profit taxes and tax bases under different FA systems. We show that in a symmetric equilibrium there is underprovision of the locally provided good due to the strategic interaction between the governments. Local authorities choose to distort investment decisions, i.e., firms are only allowed to deduct a portion of capital costs from taxable income, so that they can adopt lower tax rates. This result does not hold in a closed economy with no cross-border ownership. In this case, full deduction of capital costs is allowed and, consequently, investment decisions are not distorted.

We also show that the degree of underprovision is less severe when the formula relies on output shares. In our symmetric setup, these shares are equivalent to sales shares. Thus, we claim that the recent shift by most states in the U.S. to a FA that gives more importance to sales would constitute an improvement in terms of welfare for all regions.

A Appendix.

A.1 Proof of Proposition

Proposition 6. *If local governments choose both t^i and μ^i $(i = A, B)$, then in a symmetric Nash equilibrium $\mu^i < 1$ for any FA system.*

Proof. The FOCs (31) and (32) establish the values of t^A and μ^A. Similar expressions determine t^B and μ^B. Given that we analyze a symmetric equilibrium (i.e., $t^A = t^B = t$ and $\mu^A = \mu^B = \mu$), the following analysis holds for both regions. Suppose that $\mu^A = 1$ and $\partial U^A/\partial t = 0$. We will show that $\partial U^A/\partial \mu^A < 0$, so the FOCs are not satisfied. Thus, the regional government can increase utility by choosing a lower μ^A. From (31)

$$\frac{U_g^A}{U_x^A} = -\frac{\partial x^A/\partial t^A}{\partial g^A/\partial \mu^A}. \tag{39}$$

Hence, $\partial U^A/\partial \mu$ can be written as

$$\partial U^A/\partial \mu = U_x^A(\partial x^A/\partial \mu^A)\left[1 - \left(\frac{\partial x^A/\partial t^A}{\partial g^A/\partial t^A}\right)\left(\frac{\partial g^A/\partial \mu^A}{\partial x^A/\partial \mu^A}\right)\right]. \tag{40}$$

Both U_x^A and $\partial x^A/\partial \mu^A$ are positive, so the sign of $\partial U^A/\partial \mu$ depends on the sign of the expression between brackets. It can be shown that

$$\frac{\partial x^A/\partial t^A}{\partial g^A/\partial t^A} > \frac{\partial x^A/\partial \mu^A}{\partial g^A/\partial \mu^A}, \tag{41}$$

so that the expressions between brackets is negative and $\partial U^A/\partial \mu < 0$. Suppose, by way of contradiction, that the LHS of (41) is less than or equal to the RHS. Replacing, we obtain

$$-\frac{\theta \alpha \Omega}{\alpha \Omega + t(1-t)f_{kk}[(\partial k^A/\partial t^A)^2 + (\partial k^B/\partial t^A)^2} \leq -\frac{\theta}{\alpha + (t/k^A)(\partial k^A/\partial t^A)}. \tag{42}$$

Note that when $\mu^A = 1$, $\partial k^A/\partial t^A = -\partial k^B/\partial t^A = \alpha_A \Pi/(1-t)f_{kk} < 0$. Substituting these results into the last condition and rearranging we reach the following expression

$$0 \geq (1-\alpha) + \frac{t}{k^A}\frac{\partial k^A}{\partial t^A}\left(\frac{2k^A\alpha_A}{\alpha} - 1\right). \tag{43}$$

The first term on the RHS of (43) is positive, and the sign of the second term is determined by the sign of $[(2k^A\alpha_A/\alpha) - 1]$, which is positive for any FA system:
(i) Suppose that the FA gives full weight to property shares, i.e., $\alpha = \alpha^K = 1/2$, and $\alpha_A = \alpha_A^K = 1/2k$. Given that in a symmetric equilibrium $2k^A = k$, then $2k^A\alpha_A/\alpha = 1$, so we reach a contradiction as the RHS of (43) is strictly positive.
(ii) If the FA gives full weight to output shares, i.e., $\alpha = \alpha^F = 1/2$, and $\alpha_A = \alpha_A^F = f_k/4f$, then $2k^A\alpha_A/\alpha = f_k/[f/(k/2)]$. As the function exhibits decreasing returns to scale, $f_k/[f/(k/2)] < 1$, which implies that the second term of (43) is negative, contradicting our original claim.

We then conclude that in a symmetric Nash equilibrium μ^A (and consequently μ^B) is less than one. □

References

Anand, B. N., and R. Sansing (2000), "The Weighting Game: Formula Apportionment as an Instrument of Public Policy," National Tax Journal, 53(2).

Brueckner, J. and L. Saavedra (2001), Do Local Governments Engage in Strategic Property-Tax Competition?. National Tax Journal 54, 203-229.

Edmiston, Kelly D. (1998), "The Manipulation of State Corporate Income Tax Apportionment Formulas as an Economic Development Tool," Fiscal Research Program Report No. 26, Fiscal Research Program, Georgia State University.

Goolsbee, A., and E. L. Maydew (1998), "Coveting Thy Neighbor's Manufacturing: the Dilemma of State Income Apportionment," NBER Working Papers, Working Paper 6614.

Gordon, R. H., and J. D. Wilson (1986), "An examination of multijurisdictional corporate income taxation under formula apportionment," Econometrica 54(6), 1357-73.

Haufler, A, and G. Schjelderup (2000), "Corporate tax systems and cross country profit shifting," Oxford Economic Papers 53, 306-325.

Hines J. R. Jr. (1997) Tax policy and the activities of multinational corporations. In: AJ Auerbach (ed) Fiscal policy: Lessons from economic research. MIT Press, Cambridge, pp 401-445.

Hines J. R. Jr (1999) Lessons from behavioral responses to international taxation. National Tax Journal 52(2): 305-322.

Klassen, K. J., and D. A. Shackelford (1998), "State and Provincial Corporate Tax Planning: Income Shifting and Sales Apportionment Factor Management," Journal of Accounting and Economics 25(3), 385-406.

McClure, C. E. (1980), "The state corporate income tax: lambs in wolves clothing?" In: Aaron, H. J., and M. Boskin (Eds), The Economics of Taxation, The Brookings Institution, Washington, DC.

McClure, C. E. (1981), "The elusive incidence of the corporate income tax: the state case", Public Finance Quarterly 9(4), 395-413.

Mieszkowski and Zodrow (1985), "The Incidence of a Partial State Corporate Income Tax," National Tax Journal 38, 489-96.

Pethig, R., and A. Wagener (2003), "Profit Tax Competition and Formula Apportionment."

Weiner, J. M. (1996), "Estimates of How the Unitary Tax Affects Business Investment," U.S. Treasury Department Working Paper, Washington, D.C.

STATE	Formula	STATE	Formula
ALABAMA	3 Factor	MONTANA	3 Factor
ALASKA	3 Factor	NEBRASKA	Sales
ARIZONA	Double weighted Sales	NEVADA	No State Income Tax
ARKANSAS	Double weighted Sales	NEW HAMPSHIRE	Double weighted Sales
CALIFORNIA	Double weighted Sales	NEW JERSEY (1)	Double weighted Sales
COLORADO	3 Factor	NEW MEXICO	Double weighted Sales
CONNECTICUT	Double weighted Sales	NEW YORK	Double weighted Sales
DELAWARE	3 Factor	NORTH CAROLINA	Double weighted Sales
FLORIDA	Double weighted Sales	NORTH DAKOTA	3 Factor
GEORGIA	Double weighted Sales	OHIO	60 % Sales, 20 % Property/Payroll
HAWAII	3 Factor	OKLAHOMA	3 Factor
IDAHO	Double weighted Sales	OREGON (2)	Double weighted Sales
ILLINOIS	Sales	PENNSYLVANIA	Triple weighted Sales
INDIANA	Double weighted Sales	RHODE ISLAND	3 Factor
IOWA	Sales	SOUTH CAROLINA	Double weighted Sales
KANSAS	3 Factor	SOUTH DAKOTA	No State Income Tax
KENTUCKY	Double weighted Sales	TENNESSEE	Double weighted Sales
LOUISIANA	Double weighted Sales	TEXAS	Sales
MAINE	Double weighted Sales	UTAH	3 Factor
MARYLAND	Double weighted Sales	VERMONT	3 Factor
MASSACHUSETTS	Double weighted Sales	VIRGINIA	Double weighted Sales
MICHIGAN	90 % Sales, 5 % Property/Payroll	WASHINGTON	No State Income Tax
MINNESOTA	75% Sales, 12.5 % Property/Payroll	WEST VIRGINIA	Double weighted Sales
MISSISSIPPI	3 Factor	WISCONSIN	Double weighted Sales
MISSOURI	3 Factor	WYOMING	No State Income Tax
		DIST. OF COLUMBIA	3 Factor

Table 1: Formulas for tax year 2003, as of January 1, 2003.