REGIONAL RESEARCH INSTITUTE WORKING PAPER SERIES
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Labor Market Size and Unemployment Duration: A Theoretical Note

Peter V. Schaeffer, Professor
Tesa G. Gebremedhin, Professor

RESEARCH PAPER 2004-8
[revised March 21, 2007]

Division of Resource Management
West Virginia University
P.O. Box 6108
Morgantown, WV 26506-6108
Phone: (304) 293-4832, ext. 4459
Fax: (304) 293-3752
E-mail: Peter.Schaeffer@mail.wvu.edu

Faculty Research Associates
Regional Research Institute, West Virginia University
www.rri.wvu.edu

Abstract: When job prospects are uncertain, labor market size matters even when labor and jobs, respectively, are homogenous. The expected unemployment duration and its standard deviation may then differ systematically with labor market size.

JEL Classification: J64, J68, R12

Acknowledgments: We acknowledge comments by Gebremeskel Gebremeriam, Robert Gibbs, Haixiao Huang, Scott Loveridge, and Anthony E. Smith. The usual caveat applies.

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1. Introduction

Modern economics has traditionally been interested in the relationship between market size and economic performance. Adam Smith famously observed that the division of labor is limited by the extent of the market (Smith, 1776/1965) and Marshall (1890/1920) established the importance of localization and urbanization economies (agglomeration economies). Contributions to the literature concerning the size of labor markets since then have focused mostly on the effects of agglomeration economies, but the division of labor as one of the foundations of modern economics (Myers, 1983) is implicit in all of them. In both traditions, heterogeneity of labor plays an important role. Although the division of labor could exist if labor were homogenous, in practice its existence and social acceptance, particularly given significant differences in pay, work conditions, and prestige, can more easily be justified by appealing to differences in talents and interests.

In this article, we analyze the relationship between labor market size and expected unemployment duration. Unemployment duration is an important measure of labor market performance. Fields (1976) persuasively argues that it compares favorably to the unemployment rate: *"A priori, we might suppose that unemployment rates are not very satisfactory measures of economic opportunities for potential migrants, who.....would presumably be more concerned with the probabilities of acquiring and retaining employment than with the average employment rate among all workers in the market"* (Fields, 1976: 407). Although he aimed this statement at migrants, it also applies to other workers. Constraints on the life-time duration of unemployment benefits became more stringent with the passage of the Personal Responsibility and Work

Opportunity Reconciliation Act of 1995 (U.S. Library of Congress, 1995; see also Bartik, 2002; Dilger et al., 2004), and employers might interpret long unemployment spells as negative signals about workers' quality (Berkovitch, 1985; Lockwood, 1991). Hence, workers have good reasons to be concerned about the duration of their unemployment and to consider it when making location decisions and/or choosing a profession.

A number of scholars have studied the relationship between unemployment duration and labor market size. Gan and Zhang's (2006) contribution is of particular interest here because they are also investigating the relationship between market size and unemployment duration. They find that unemployment duration in large labor markets is shorter than in small ones. Theirs is a search theoretic model and the result depends on the presence of agglomeration economies which facilitate search and improve job matches. An earlier contribution by Lockwood (1991) shares a similar modeling philosophy, but does not include market size. On the other hand, it includes information about workers' skills. Since firms can screen workers, previous unemployment duration sends a negative signal about a worker's skill level. Other related empirical studies (e.g., Arulampalam and Stewart, 1995; Blanchard and Diamond, 1994) also exclude labor market size. Many other related studies of geographical variations in unemployment are using unemployment rates and not duration (e.g., Neumann and Topel, 1991; Karanassou and Snower, 1998; Bande and Karanassou, 2007). All of them have in common, however, that they attribute differences in unemployment rates or unemployment duration to the heterogeneity of labor and/or employers.

There is, of course, no doubt that heterogeneity of labor and employers causes different labor market outcomes, and that the study of such heterogeneity is critical to our understanding of labor markets and the design of labor market policies. However, in this paper we are asking if the expected unemployment duration would be the same even if labor markets were structurally identical in every respect except size, and even if labor and employers, respectively, were both homogenous. We will show that the answer may be no.

It is easier to understand the intuition behind the result if we consider a job opening as an opportunity to purchase a lottery ticket. The odds of having a winning ticket are determined by the number of competitors for the job. The expected length of time before a player will get a winning ticket is determined by (1) the frequency with which lottery tickets are being offered and (2) the probability that a ticket is a winner. Large labor markets offer more opportunities to apply for job openings, but also have more competitors for each opening (lower probability) than small markets. In other words, small markets can compensate for having fewer job opening during a given time period by offering better odds (having fewer competitors for those openings). This tradeoff opportunity is exhausted, however, once the probability of winning reaches 1. At this point size begins to matter.

The rest of the paper is organized as follows. In the next section we present the model. This is followed in section three the analysis of the effect of labor market size on expected unemployment duration. In section four we take a brief look at the standard deviation of the expected unemployment duration as a measure of risk. Section five provides a short summary and conclusions.

2. The Model

Success in obtaining a job depends on the frequency with which jobs become available and the number of other workers competing for the same job. In this model the workers' qualifications do not matter because we assume that are all identical. Hence, employers are indifferent who they hire. The purpose of this assumption is to focus solely on the effect of labor market size on unemployment duration. We assume that job openings are uniformly distributed over time, so that the expected time between them is fixed. This assumption is justified since there is nothing in the model to suggest that jobs are seasonally or otherwise unevenly distributed over time. Since all jobs are identical, an applicant who is offered a job will immediately accept it and an available job is filled instantaneously.

We will use the following notation.

λ expected length of time between job offers. This is a stochastic variable. Its distribution is assumed to be uniform over time.

t number of periods of length λ; t=1,2,3,...

Λ time before a position is obtained (unemployment duration). This expected unemployment duration, $E[\Lambda]$, will be defined below.

U number of unemployed workers competing for jobs. We assume that the economy is in a steady state and hence that this number is constant and reflects the natural rate of unemployment.

L measure of labor market size, consisting of those currently employed plus those looking for employment. As we did for unemployment, we assume that L is constant.

p joint probability that an applicant is offered a vacant job and that the offer is accepted. Because of the assumptions made, p is constant.

We make the following assumptions about functional relationships.

$$(1) \qquad (\quad), \frac{\quad}{\quad} \quad , \frac{\quad}{\quad} \qquad \text{for } 0 \le p < 1$$

The justification for the assumption is that with higher unemployment, competition for available job openings is stiffer. The sign of the second derivative follows because the probability declines and gets close to zero for very large values of U.

$$(2) \qquad \lambda = \lambda(L), \frac{d\lambda}{dL} < 0, \frac{d^2\lambda}{dL^2} > 0$$

Job openings occur as a result of job turnover because of retirement. Thus, large labor markets will have more frequent job openings. This is our justification for the negative derivative in equation (2). The second derivative follows directly from the assumption about constant turnover.

$$(3) \qquad U = U(L), \frac{dU}{dL} > 0, \frac{d^2U}{dL^2} \ge 0$$

Finally, the absolute number of unemployed (U) is assumed to be positively related to the total number of labor market participants (L). In a real world setting where differences among workers and jobs provide incentives to search, larger labor markets, particularly those that are also spatially clustered, may be more efficient and hence may have relatively less unemployment. This justifies the sign of the second derivative in equation (3). However, strictly speaking, given the highly restrictive assumptions of our model made for the purpose explained in the introduction, such considerations are beyond the scope of our model.

Given the assumptions, the hiring outcome in each interval is a random event. Since workers are identical, previous failure to obtain a job has no bearing on the probability of future success because it contains no information about a worker's qualifications, but is purely the result of a stochastic process. The expected unemployment duration has a binomial distribution because of the assumption that job openings are distributed uniformly over time. Therefore, the expected time until employment is secured is given by

$$(4) \qquad E[\Lambda] = \sum_{t=1}^{\infty} \lambda t\, p\,(1-p)^{t-1} = \frac{\lambda}{p} .$$

For a job seeker the unemployment duration is positively related to the frequency of job openings (λ) and inversely related to the joint probability of being offered and accepting a job (p). This result represents in mathematical form the intuitive statement that small markets can compensate for relative lack of opportunities, represented by λ, by increasing the probability of success, p.

3. <u>Labor Market Size and Unemployment Duration</u>

Since λ and p are functions of labor market size, the expected unemployment duration depends on L.

$$(5) \qquad \frac{dE[\Lambda]}{dL} = \frac{\frac{d\lambda}{dL} p - \frac{dp}{dL} \lambda}{p^2}$$

The sign of the derivative is indeterminate because ${d\lambda}/{dL} < 0$ and ${dp}/{dL} = {dp}/{dU}\,{dU}/{dL} < 0$ for $0 < p < 1$. This shows that small labor markets may be able to compensate for the longer expected time between job offers with fewer applicants competing for the same jobs. However, this tradeoff is limited by the constraint $p \leq 1$.

P*roposition 1: In a steady state economy with homogenous labor and homogenous jobs, respectively, and with unemployment, the expected unemployment duration in a particular labor market is a function of labor market size.*

Proof: Follows from equation (5).

Since equation (5) does not permit a firm conclusion regarding the effect of market size on expected unemployment duration, we take different tack to further investigate further this relationship. Consider a period of a given length T (e.g., a month), and let n denote the expected number of job openings occurring during this period. For a given value of p, the larger the number of job openings during the period, the less likely it is that the worker will still be unemployed at the end of the period. Let us compare two labor markets identical in every respect except the number of offers. Since λ is negatively related to L (see equation (2)), equation (4) shows that to remain equally attractive, a small labor market must offer unemployed workers a higher probability to compensate for the smaller number of job openings. To show the tradeoff, we equate the probabilities of failing to get a job during the period of length T in a small ($\mathbb{S}$) and in a large ($\mathbb{L}$) market, respectively: $(1-p_{\mathbb{S}})^n = (1-p_{\mathbb{L}})^{\alpha n}$. We can restate n as: $n = T/\lambda_{\mathbb{S}}$; $\alpha > 1$ indicates by what factor the size of labor market $\mathbb{L}$ exceeds that of labor market $\mathbb{S}$. Taking the n^{th} root[1] on both sides and rearranging terms, we end up with

(6) $$p_{\mathbb{S}} = 1-(1-p_{\mathbb{L}})^{\alpha}.$$

This gives us the condition under which failure (and hence success) is equally likely in markets $\mathbb{S}$ and $\mathbb{L}$, respectively, Equation (6) shows that for a large value of α, $p_{\mathbb{S}}$ has to approach 1 to compensate for the market's relative lack of size, even for a small value of p_{L}.

As difficult as meeting this condition may be, equation (6) still understates the challenge to a small market to be competitive. The analysis is for a period of length $\lambda_{\mathbb{S}}$ when there is only one job opening in market $\mathbb{S}$. During this same period there will be α job openings in market $\mathbb{L}$. Thus, the probability that a worker in market $\mathbb{L}$ will be employed before the end of the period is positive, and the probability that she will have obtained a job by the end of the period is as high as that of a counterpart in market $\mathbb{S}$ (by equation (6)). This makes market $\mathbb{L}$ more attractive for someone who is unemployed. We have just demonstrated that the unemployment duration may have weaknesses for comparing the desirability of job markets. This serves to qualify Fields' (1976) strong endorsement of this measure.

Proposition 2: In a steady state economy with homogenous labor and homogenous jobs, respectively, and with unemployment, having the same expected unemployment duration as labor market $\mathbb{L}$ may still leave labor market $\mathbb{S}$ at competitive disadvantage to labor market $\mathbb{L}$.

Proof: Preceding discussion.

Choose a period of length δ, such that $\delta < \lambda_{\mathbb{S}}$. At random pick a time τ as the starting point to compare the job application outcomes in markets $\mathbb{S}$ and $\mathbb{L}$, respectively. The probability that there will be a job opening in the small market during the random time interval $[\tau, \tau + \delta]$ is equal to $\delta/\lambda_{\mathbb{S}} < 1$. Therefore, even if $p_{\mathbb{S}} = 1$, the probability that a worker in a small market will be offered a job in a random interval of length δ is less than 1, whereas, if α is large, the same probability in market $\mathbb{L}$ could approach 1, even if $p_{\mathbb{L}}$ is small (Figure 1). Figure 1 illustrates the tradeoff between p and α.

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1 Since n falls out of the equation after we take the n^{th} root, we need not worry about n being a natural number.

Figure 1 about here

Figure 1 shows the expected result that when the economy booms and demand for labor is strong so that $p \approx 1$, size is less important. Similarly, if the economy is so depressed that $p \approx 0$ ("nobody" gets a job), size also does not matter much. Obviously, if the economy breaks down and $p = 0$, size no longer matters. On the other hand, as we have shown, when $p = 1$, size may still matter. Nonetheless, at the extreme values, market size is less important. It is at intermediate values of p that large labor markets offer job seekers the greatest advantage over smaller ones. Figure 1 also shows that if we wish to hold $(1-p)^{\alpha}$ constant, p has to drop less as we move from α =2 to α =3 than from α =1 to α =2, showing that there are diminishing returns to the advantage of labor market size.

4. Standard Deviation of Expected Unemployment Duration

In comparing labor markets, it is not only the expected duration of unemployment, but also the standard deviation that should be considered, as the latter provides a rough measure of risk.

$$(7) \qquad \sigma(\Lambda) = \sqrt{1-p}\,\frac{\lambda}{p}$$

Of course, if $p=1$, then there is no risk, though in a small market the wait for the next job opening that will take the worker off the unemployment roll may be relatively long and an unemployed worker in a larger labor market may find a job sooner, even without being guaranteed a job. In the more relevant case of $0 < p < 1$, the derivate of the standard deviation of Λ with respect to L is indeterminate because it consists of two terms, the first negative and the second positive:

(8) $$\frac{d\sigma}{dL} = 2\lambda \frac{1-p}{p} \underbrace{\frac{d\lambda}{dL}}_{\text{neg.}} + \lambda^2 \underbrace{\frac{p-2}{p^3}}_{\text{neg.}} \underbrace{\frac{dp}{dL}}_{\text{neg.}} .$$

What we can say is that, in general, the standard deviation in two labor markets of different size will not be the same, even if $E[\Lambda_S] = E[\Lambda_L]$.

Proposition 3: *In a steady state economy with homogenous labor and homogenous jobs, respectively, and with unemployment, the standard deviation of the expected unemployment duration varies with labor market size.*

Proof: Follows from equation (8).

5. <u>Summary and Conclusions</u>

The model presented here shows that in an uncertain world there may be benefits to agglomeration even if labor and employers are homogenous. Note that the model only considers size, but not a labor market's spatial concentration. The importance of the latter for, for example, labor market pooling, has been explored by Rosenthal and Strange (2001). They found that labor market pooling provides the most robust explanation of agglomeration. Our model lacks a spatial component and can therefore not address the issue of concentration or density, only of size.

It is important that we acknowledge the limitations of the model so as not to overstate its implications. In a nutshell, the model suggests the possibility that expected unemployment duration and associated standard deviations may systematically differ with labor market size, even if labor markets are identical in every respect except size, and even if labor and employers are both homogenous. The finding that differences among either workers or jobs are not necessary to obtain such a result is not obvious and this article provides a "proof by example."

Within the extremely restrictive assumptions of our model, small labor markets can respond by offering better odds in the lottery for employment. Once they have exhausted this option, they stop being competitive. The model does not allow for adjustments in other variables, such as wages, probability of future layoffs, etc. However, in the real world, we would expect market participants to respond creatively. Thus, it is possible that institutional arrangements and responses may differ by market size. This possibility has some empirical support from Wojan's (2000) study of rural manufacturing firms during market downturns. His findings suggest these firms they behaved differently vis-à-vis a subset of their workers than did their counterparts in urban (larger) markets.

The results from our model complement previous reasons given for the existence for agglomerations. While heterogeneity in labor markets is certainly the most compelling reason, we show that even in its absence incentives for agglomeration can emerge.

References

Bartik, Timothy J. (2002). "Spillover Effects of Welfare Reforms in State Labor Markets." *Journal of Regional Science* 42(4): 667-701.

Berkovitch, Elazar (1985). "Reputation Effect in Equilibrium Search and Bargaining-A Stigma Theory of Unemployment Duration." Northwestern University, Center for Mathematical Studies in Economics and Management Science, Discussion Paper No. 668.

Dilger, Robert J., Eleanor Blakely, Melissa Latimer, Barry Locke, L. Christopher Plein, Lucinda A. Potter, and David Williams (2004). "WV Works" Perspectives of Former Recipients Who Have Exhausted Their 60 Months of Program Eligibility." *West Virginia Public Affairs Reporter* 21(2, Summer): 2-20.

Fields, Gary S. (1976), "Labor Force Migration, Unemployment and Job Turnover." *Review of Economics and Statistics* 58(4, November): 407-415.

Gan, Li and Qinghua Zhang (2006), "The Thick Market Effect on Local Unemployment Fluctuations." *Journal of Econometrics* 133: 127-152.

Lockwood, Ben (1991). "Information Externalities in the Labour Market and the Duration of Unemployment." *Review of Economic Studies* 58(4, June): 733-753.

Marshall, Alfred (1920). *Principles of Economics* (8^{th} edition). London: Macmillan (first published 1890).

Myers, M.L. (1983). *The Soul of Modern Economic Man. Ideas of Self-Interest: Thomas Hobbes to Adam Smith*. University of Chicago Press, Chicago.

Neumann, George R. and Robert H. Topel (1991). “Employment Risk, Diversification, and Unemployment.” *Quarterly Journal of Economics* 106: 1341-1365.

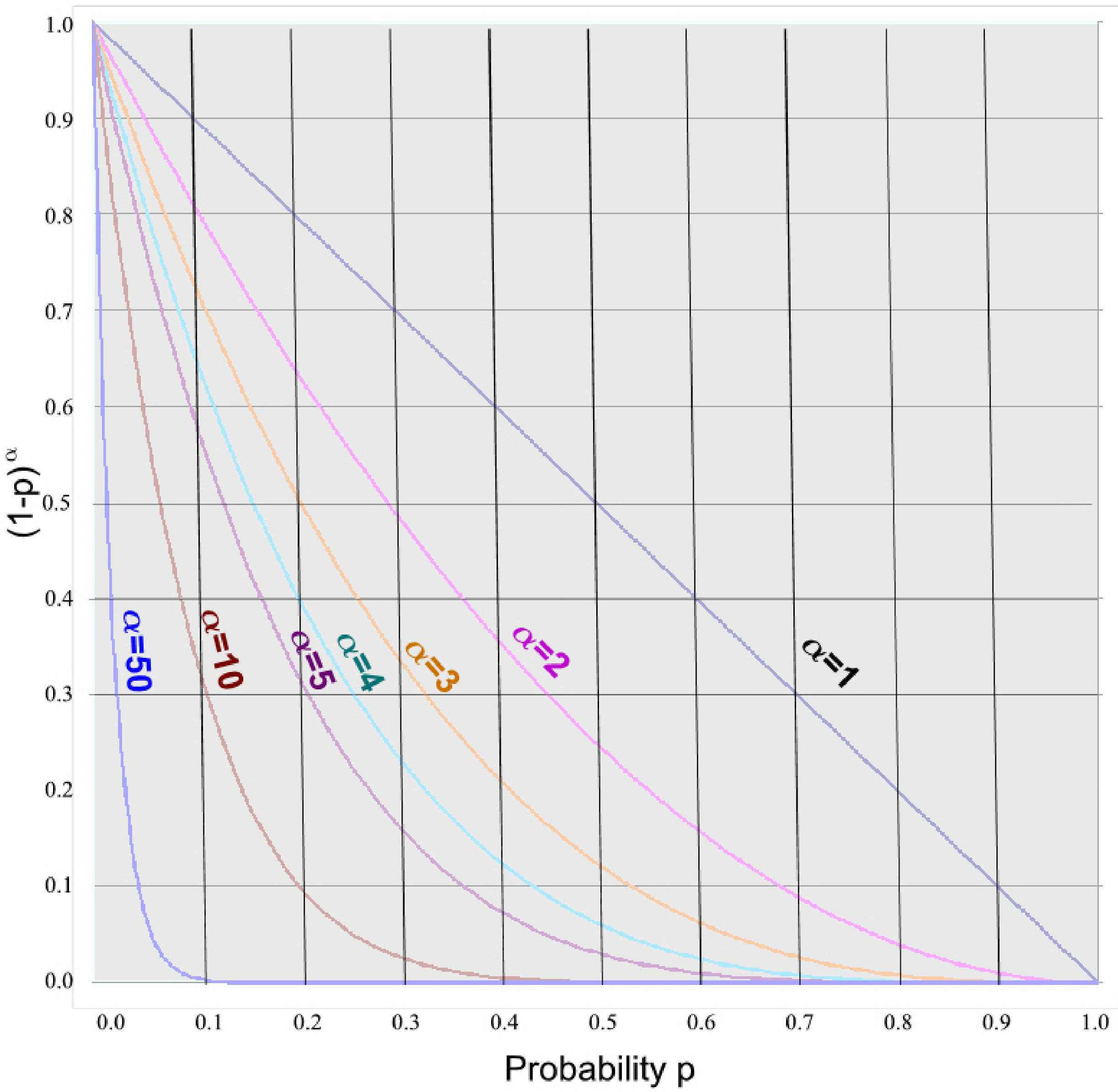
Rosenthal, Stuart S. and William C. Strange (2001). “The Determinants of Agglomeration.” *Journal of Urban Economics* 50(2): 191-229.

Smith, Adam (1965). *An Inquiry into the Nature and Causes of the Wealth of Nations*. New York: The Modern Library. (Originally published in 1776).

U.S. Library of Congress (1995), Thomas Legislative Information on the Internet. H.R.4: *Personal Responsibility and Work Opportunity Act of 1995*. http://thomas.loc.gov/cgi-bin/query/D?c104:6:./temp/~c104UAsLBQ::

Wojan, Timothy R. (2000), “Fragile Virtue: Rural Labor Market Response to a New Competitive Environment.” In Peter V. Schaeffer and Scott Loveridge, editors, *Small Town and Rural Economic Development: A Case Studies Approach*. Chapter 31: 251-259. Westport, CT: Praeger.

Figure 1: Relationship between labor market size and expected unemployment duration



Explanation:
Assume that p=0.2. The distance between the p-axis and the alpha-curve gives the probability of continued unemployment, and the distance from there to the top of the graph shows the probability of obtaining a job within the defined period. In this fgure market size is represented by alpha. The market represented by altha=10 is two and a half times as large as the market represented by alpha=4. Figure 1 shows that probability of successfully securing employment increases quickly as market size increases.