An optimal depletion CGE model: A systematic framework for energy-economy analysis in resource-based economies

By

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Abstract
Numerical economic models of energy fall into two general categories: models analyzing within energy sector issues and models examining the interaction between the energy sector and the rest of the economy. The first category are mostly partial equilibrium models with a very detailed and disaggregated representation of the energy sector. Although very useful for sector planning purposes this class of models essentially neglect the interdependence of the energy sector and the rest of the economy. The second category, appropriately called energy-economy interaction models, are multisectoral and general equilibrium models focusing on the relationship between the energy sector and the rest of the economy. These models offer a rich economy-wide picture but are not as detailed as the first category in their specification of the energy sector. Models employed for energy-economy interaction analysis include input-output, macro-econometric, and computable general equilibrium (CGE), as well as hybrid of these types. With advances in computation capabilities, however, CGE models have become the standard tool and dominate the mainstream of the economic discipline. The model presented in this paper belongs to the optimal depletion category of computable general equilibrium models. It is an optimization model that solves the inter-temporal depletion problem subject to workings of a multi-sector market economy, where relative prices play a crucial role. Such a formulation establishes general equilibrium linkages between the optimal depletion of the resource and the rest of the economy and thus it provides a systematic framework to analyze energy-economy interactions in resource-based economies.

Contents:

1 Introduction

2 Computable General Equilibrium Models

3 An Optimal Depletion CGE Model

4 The Dynamics of the Model

4-1 Objective Function

4-2 Oil Depletion

4-3 Savings and Investment Allocation

5 The Static Model

5-1 Production And Factor Market

5-2 Income Generation And Product Market

Consumption Demand
Government Demand
Investment Demand
Intermediate Demand

5-3 Foreign Trade

5-4 Market Equilibrium

6 Conclusion

Appendix: System Of Notations

1 Introduction

Numerical economic models of energy fall into two general categories: models analyzing within energy sector issues and models examining the interaction between the energy sector and the rest of the economy. The first category are mostly partial equilibrium models with a very detailed and disaggregated representation of the energy sector. Although very useful for sector planning purposes this class of models essentially neglect the interdependence of the energy sector and the rest of the economy. These models are surveyed in Bergman (1988) and Deverajan (1989). The second category, appropriately called energy-economy interaction models, are multisectoral and general equilibrium models focusing on the relationship between the energy sector and the rest of the economy. These models offer a rich economy-wide picture but are not as detailed as the first category in their specification of the energy sector. The early references of this class of models include Hudson and Jorgenson(1974), and Manne (1977). More recent examples include Jorgenson and Wilcoxen (1992), Blitzer et al. (1994), Boyd (2001), and Bohringer (2004).

For energy-economy interaction analysis a number of models have been employed, including input-output, macro-econometric, and computable general equilibrium (CGE), as well as hybrid of these types. With advances in computation capabilities CGE models have become the standard tool and dominate the mainstream of the economic discipline. The model proposed here belongs to the optimal depletion category of computable general equilibrium models. It is an optimization model that solves the inter-temporal depletion problem subject to workings of a multi-sector market economy, where relative prices play a crucial role. Such a formulation establishes general equilibrium linkages between the optimal depletion of the resource and the rest of the economy by working through both factor and product markets.

Section 2 briefly describes the class of multisectoral models known as computable general equilibrium (CGE) models. Section 3 introduces optimal depletion category of CGE models and discusses the important dynamic choices of government. The following section describes important dynamic specifications of the model. In section 5 a full description of all equations in the static portion of the model is explained and finally, the paper concludes with few remarks regarding the proposed modeling framework.

2 Computable General Equilibrium Models

The models commonly referred to as "applied general equilibrium models" (AGEM) or "computable general equilibrium" (CGE) are large multisectoral, economy-wide nonlinear equilibrium models that are closely related to the Walrasian model of a competitive economy. The basic ideas of a multisectoral general equilibrium growth model were laid out by Johansen (1959). His model of the Norwegian economy is one of the first empirical implementation of a general equilibrium model. "General equilibrium" typically refers to Walrasian competitive equilibrium model where all economic agents are price takers who maximize profits or utility, and prices freely adjust to clear markets. This framework simply implies that supply equals demand. CGE models attempt to incorporate the fundamental links among production structure, pattern of demand and incomes of various institutions.

These models are also called price-endogenous models because they are based on the presumption that prices are free to adjust until there is a consistency among the decisions made on the productive side of the economy and decisions made by households and other autonomous decision makers on demand side. General equilibrium and autonomous decision making are two concepts central to the CGE modeling framework. However, according to Dervis, et al. (1982) the CGE framework does not insist on perfect competition, instantaneous market clearing, and absence of government intervention. On the contrary "...imperfectly competitive behavior, quantity or price adjustment lags, and widespread government interventions are compatible with the CGE framework." The main function of these models is to simulate the effects of economic policies; as such the government or public sector is normally incorporated into the model.

In comparing CGE models with input-output and linear programming, the dominant models of the 1950s and 1960s, Dervis, et al. (1982)[1] suggest that CGEs are better suited to planning and policy analysis in mixed-market economies where autonomous decision-making by various economic agents and market mechanisms have an important impact on resource allocation. The input-output and linear programming methods reflect a pure command economy where a central authority fully controls the resources and has to make optimal decisions only subject to technological and physical constraints. Robinson(1989) considers CGE models as a "natural outgrowth" of the earlier input-output and linear programming models that "...add neoclassical substitutability in production and demand, as well as explicit system of market prices and a complete specification of the income flows in the economy." Bell and Srinivasan (1984), suggest that CGE models were developed in response to three limitations of standard input-output models: "1- fixed coefficients and the assumption that changes in prices have no effect except for any income impacts; 2- constant returns to scale and highly elastic supplies of factors and the assumption that the relative prices of commodities will not change; and 3- the absence of algorithms to solve large-scale systems for quantity and price simultaneously."

The other popular economy-wide models are econometric models, which normally serve to analyze economic policy questions by relying on the estimates of econometric relationships. In area of development planning CGE models have wider range of application and are more suitable in the case of developing countries. De Melo (1988) provides three reasons for the inadequacy of econometric models in developing countries: 1-paucity of reliable time-series data for sufficiently long periods; 2-inappropriateness of data when available; and 3-short time span available for hypothesis testing (as a result of rapid and significant changes in policy regimes).

CGE models are essentially applied general equilibrium models. With advances in solution algorithms and computing power these models have proliferated in more recent years. Availability of data and development of powerful yet low cost computers have made the CGE models a very attractive tool, particularly in addressing more complex economy-wide issues.

There is a growing trend to use CGE models both in developed and developing countries. Applications of CGE models in developed economies are mostly microeconomic, focusing on estimating the welfare impact of alternative tariff and tax structures or energy policies. Shoven and Whalley (1984) present an introduction and a survey of CGE models of taxation and international trade applied to developed economies; a more recent survey is Pereira, and Shoven (1988). CGE models applied to energy issues in developed countries include Jorgensen (1982), Jorgensen and Wilcoxen (1990), Bergman 1988 and 1990. In developing countries CGE models have been applied to a wider range of medium to long-term macro and microeconomics issues. Dervis et. al (1982) and Robinson (1989) provide comprehensive surveys of the characteristics and applications of CGE models in developing countries. A concise survey of applications of CGE models is presented in Decaluwe, B and A. Martens (1988). Devarajan (1988) reviews the CGE applications to natural resources and taxation issues in developing countries, and de-Melo(1988) surveys CGE models for trade policy analysis in developing countries.

More recently CGE modeling has been used at the regional level to examine a broad range of problems including growth and development issues of urban system, regional impacts of national changes, and development issues in a multi-region framework. Examples of applying CGE models to regional issues include Jones and Walley (1989), Harrigan and McGregor (1989), and Kraybill et al (1992).

3 An Optimal Depletion CGE Model

The main focus of the model presented in this paper is on the optimal rate of depleting an exhaustible resource, the optimal level of savings, and the optimal allocation of total investment funds in the economy. The extensive literature concerned with optimal depletion of an exhaustible resource, with only a few exceptions, ignores the economy-wide and sectoral distribution effects of resource depletion. Typically, capital accumulation and consumption are discussed within the limited framework of the one-sector neoclassical growth models (Aarrestad 1978). These models do not consider the role of prices in influencing production and consumption decisions of firms and households, and undermine the significance of inter-sectoral interaction on the optimal depletion profile. The treatment of the optimal depletion of an exhaustible resource independently of the rest of the economy is justified when perfect capital markets prevail.[2] Clearly, in the case of economies where well functioning capital markets do not exist, the rate of resource depletion is closely related to activities in the rest of the economy. In any realistic circumstance, the intensity of interaction among various sectors and markets across the economy has significant bearing on the depletion program, as does the level of domestic and international prices. Private and public consumption and savings decisions as well as the investment allocation mechanism of a country directly affect its level of resource extraction. In these instances a general equilibrium approach that fully captures the economy-wide effects of resource depletion is the appropriate tool.

In a survey of the application of computable general equilibrium models to questions of natural resources in developing countries, Devarajan (1988) identifies three categories of models:

> 1- "Energy Management Models" generally focus on energy-economy interactions. These models provide a detailed treatment of supply and demand of the energy sector while non-energy sectors of the economy are dealt with in an aggregate form and are often taken as exogenous to the model.
>
> 2- "Dutch Disease Models" are those that study the effects of an export boom on the rest of the economy. These models have been applied to countries that rely heavily on oil income.[3]
>
> 3- "Optimal Depletion Models" take into account the exhaustibility of the resource and establish optimal extraction of the resource in a multisectoral context.

Devarajan (1988) sketches out the formal structure of the last two classes of models and presents some results from the application of these models. In particular, he describes results of an optimal depletion model applied to Egypt by Martin and van Wijnbergen (1986) in which they calculate the optimal path of the real exchange rate.[4]

The model to be proposed here belongs to the optimal depletion category of computable general equilibrium models. It is an optimization model that determines the optimal development path of the economy, hence, the inter-temporal depletion problem subject to workings of a multi-sector market economy. Such a formulation establishes general equilibrium linkages between the depletion profile of the resource and the rest of the economy by working through both factor and product markets.

In the proposed framework the government plays a central role in the economy. Notwithstanding its pivotal role, the government does not work within the environment of a command economy. It strives to achieve societal objectives within the more realistic environment of a mixed economy in which market also plays an important role. Thus, the government is an optimizing agent that faces the institutional constraints posed by the workings of a market economy, where producers and households independently pursue profit or utility maximization. The core of government's decisions, and the focus of this study, are optimal rate of depletion of resource, optimal level of investment and investment allocation.

The government as the owner of both physical and natural capitals in the oil sector receives returns to these factors. Oil revenues are the major source of government revenues and significantly affect activities in the rest of the economy. Given domestic prices, world prices of both imports and exports, and international trade elasticities, the government, as the owner of the oil resource, at the intra-temporal level manages the oil sector as a short-run profit maximizing firm. At the inter-temporal level, however, the government determines the magnitude of the physical capital in the oil sector, hence, the rate of resource extraction. The government also influences household savings decision through its tax policies and other instruments, which are not explicitly modeled. In other words, the economy-wide savings is determined by the government's choice of the rate of private savings as it optimizes a social welfare function.

Once the savings level is determined the next question is how investment funds are allocated among sectors. The government concerned with the long run social welfare decides the investment share of the oil sector. The remainder of the investment fund is distributed among non-oil sectors. This residual investment is allocated such that the more productive and profitable sectors of the economy receive a larger share. An alternative to the present formulation is one in which the government determines the investment shares for all sectors. This formulation would imply a much larger role for the government in the economy. Clearly, the greater command of the government would result in a different optimal path for the economy, including a different oil depletion path. Another approach to investment allocation is to introduce perfect foresight for individual firms. Each firm would make its investment decisions to maximize its net present worth. This approach to dynamic behavior of producers is more recent and less widely adopted in multisectoral models.[5]

The following sections fully describe an optimal depletion CGE model and discusses its unique characteristics. The notation conventions used in presenting the model is in Appendix.

4 The Dynamics of the Model

The following sections present the equations of the dynamic model and discuss in detail the objective function and the two important intertemporal linkages in this model: depletion of the exhaustible resource oil, and optimal savings and investment allocation. A full description of the equations of the static sub-model are presented in Section 6.

4-1 The Objective Function

In our model, we maximize the welfare of the representative household, which includes the present value of the utility of consumption over time and the present value of end-of-planning-horizon capital stock and oil reserves:

$$\text{MAX } J = \int U(C_t) * e^{\Delta t} dt + \left[\overline{PK} * \sum_i K_{i,T} + \overline{PR} * RSRV(T) \right] * e^{-\Delta T}$$

Here, (C_t) represents Cobb-Douglas aggregation of consumption of $CD_{i,t}$ of goods from sector i in time period t with fixed consumption shares ch_i :

$$C_t = CD_{1t}^{ch_1} * CD_{2t}^{ch_2} \ldots\ldots CD_{nt}^{ch_n} \qquad \text{where} \qquad \sum_{i=1}^{n} ch_i = 1$$

and $\overline{PK}$ is the price of terminal capital stock; $\overline{PR}$ is the price of resource at terminal period; and Δ is the social discount rate. The utility function is concave, reflecting diminishing marginal utility of consumption. In other words, as the society gets richer the value of an additional unit of consumption declines. The general form of the utility function is $U(c) = \frac{1}{1-\Phi} c^{1-\Phi}$ with $\Phi \neq 1$, where a higher constant elasticity of marginal utility (Φ) implies a higher degree of consumption smoothing over time. The positive social discount rate (Δ) implies that when faced with the choice between a unit of consumption today or the same unit tomorrow, the society chooses the first option.

The statement of our problem, with the objective function written in a discrete form, is summarized as:

(1) Objective function

$$\text{MAX } J = \sum_{t=0}^{T} \frac{1}{(1+\Delta)^t} * \frac{1}{(1-\Phi)} * \left[\prod_i \left(CD_{i,t} \right)^{ch_i} \right]^{1-\Phi} + \frac{1}{(1+\Delta)^T} * \left[\overline{PK} * \sum_i K_{i,T} + \overline{PR} * RSRV(T) \right]$$

Subject to: equations 2-47, to be described in the following sections.

4-2 Optimal Depletion of the exhaustible resource

The major focus of this study is characterizing the extraction path for an exhaustible resource in a multisectoral framework. The optimal path is identified for a given planning period during which the economy enjoys substantial oil reserves. Our interest is with the economy-wide effects of oil extraction; namely: the optimal intertemporal pattern of extraction constrained by workings of a market economy, the optimal intertemporal pattern of accumulating physical capital, and the allocation of investment funds. The issues related to terminal depletion of oil and the switch to the non-oil era, albeit interesting and important, are not considered in this study.

The dynamic updating of the oil reserves, in discrete form, as shown below, enters into the computer program that solves the model:

(2) Oil reserve updating

$$S_{t+1} = S_t - XD_{'oil',t}$$

4-3 Savings and Investment Allocation

One important feature of the present model is its explicit treatment of the dynamic inter-period market equilibrium. The government chooses the private marginal propensity to save *(MPS)* and the rate of investment in the oil sector *($ISHR_{'oil'}$)* so as to maximize the social welfare function as represented in equation (1). The non-oil sectors receive the remainder of investment funds based on their relative profitability in past and current periods. This specification of investment allocation assumes that non-oil sectors have myopic expectations (Dervis et al. 1982). Specifically, each non-oil sector's share of investment funds, $ISHR_{in}$, is equal to its share in aggregate capital income, SP_{in}, adjusted upward if the sector's profit rate is higher than the average profit rate and adjusted downward otherwise:

(3) Investment shares in non-oil sectors

$$ISHR_{in,t+1} = SP_{in,t} + \Omega * SP_{in,t} * \left[\frac{RP_{in,t} - AVGRP}{AVGRP} \right]$$

where RP_{in} is the sectoral profit rate, *AVGRP* is the average profit rate for the economy as a whole, and Ω is an investment mobility parameter, a measure of the responsiveness of capital markets to sectoral profit rates.[6] The following three equations show how profit shares, SP_{in}, profit rate, RP_{in}, and average profit rate, *AVGRP*, are determined. Note that the profit rate, RP_{in} includes R_{in}, rate of return on capital as well as capital gains (d_i is the sectoral depreciation rate).

(4) Share in overall profits

$$SP_{in} = R_{in} * K_{in} \Big/ \sum_{jn} R_{jn} * K_{jn}$$

(5) Determination of profit rates

$$RP_{in,t+1} = R_{in,t+1} + \left[PK_{in,t+1} - (1 + d_{in}) * PK_{in,t} \right] \Big/ PK_{in,t}$$

(6) economy wide profit rate

$$AVGRP = \left[\sum_{in} RP_{in} * K_{in} \right] \Big/ \sum_{in} K_{in}$$

The investment funds in each sector augment the sector's capital stock but at a decreasing rate as shown below:

(7) Dynamic capital equation

$$K_{i,t+1} = K_{i,t} * (1 - d_i) + \theta_i * K_{i,t} \left[1 - \left[1 + \frac{DK_{i,t}}{2 * \theta_i * K_{i,t}} \right]^{-2} \right]$$

where θ is the investment cost adjustment coefficient. This specification embodies an absorptive capacity constraint, i.e. the marginal efficiency of sectoral investment declines if investment grows too rapidly.[7] As the rate of investment, $\frac{DK}{K}$, rises, the return to additional *DK* declines. Technically, with such an absorptive capacity constraint, the rate of increase in capital stock, *K*, would be smaller than the rate of increase in investment as a percentage of capital stock, *DK/K*.

5 The Static Model

The static portion of the model is a multisectoral general equilibrium model of a Walrasian competitive economy. Apart from the peculiar effects of dynamics of the oil sector, the static model shares many of the features of the family of CGE models constructed for developing countries by Dervis, de Melo, and Robinson (1982)[8] -- such as imperfect substitution in trade and imperfections in factor markets. The following sections present a detailed discussion of the equations of the static portion of the model. We present first the supply side of the economy by describing equations that characterize production and factor markets. The next section devotes itself to the demand side of the economy and the equations describing the mapping of value added into institutional income as well as demand blocks in product market. The subsequent section presents equations that specify imports and exports. Finally, the market equilibrium and macro closure equations are presented.

An overall schematic view of the major components of the model is depicted in Figure 1. The figure includes factors, products rates, and prices as well as the various functional forms that link the parts together.

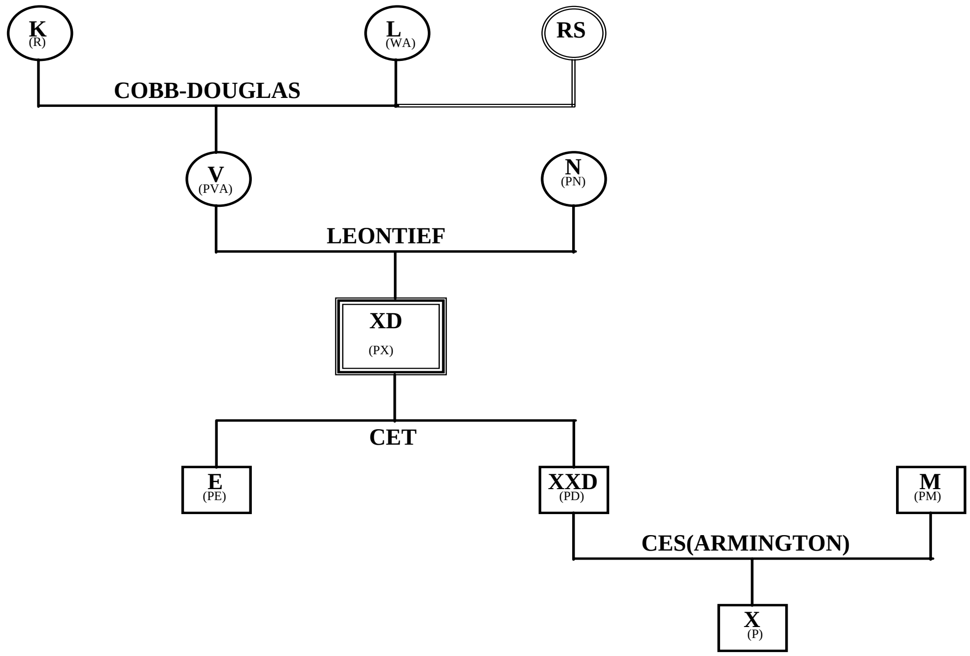

Figure 1. Factors, Prices, and Products in the CGE

Factors & Products:		**Rates & Prices**	
K:	man-made capital	R:	rate of return on capital
L:	labor	WA:	wage rate
RS:	natural capital (resource)	ω:	shadow price of resource
V:	value added	PV:	value added price
N:	intermediate inputs	PN:	price of intermediates (incl. tax)
XD:	domestic output	PX:	average sales (output) price
E:	exports	PE:	domestic price of exports
XXD:	domestic sales of domestic goods	PD:	domestic prices
M:	imports	PM:	domestic price of imports
X:	composite good	P:	price of composite good

5-1 Production and Factor Markets

The gross output of non-oil sectors is related to inputs according to a Cobb-Douglas production function in the following general form:

(8) Production function for non-oil sectors

$$XD_{in} = ad_{in} * L_{in}^{\alpha_{in}} * K_{in}^{1-\alpha_{in}}$$

where the index "in" refers to non-oil sectors. Parameters ad_{in} and α_{in} are constants and reflect the production technology. It must be noted that in addition to labor and capital, intermediate inputs are also required to produce each sector's output. This amounts to a two level production where at one level capital and labor produce the real value added which in the next level combines with intermediate inputs according to input-output fixed coefficients to produce output (see Figure 15). But it has become a common practice in CGE models to simplify the production technology by leaving out the intermediate inputs while properly taking them into account when defining value added price (equation 10).[9]

With labor and physical capital as the primary inputs, the production technology is a constant-returns-to-scale technology. In this specification of technology the number of firms in the sector does not matter and the whole sector can be seen as a single large firm that takes output and input prices as given.

The production specification for the oil sector is different. The oil produced over the years is ultimately going to be limited by total recoverable reserves. Oil is an exhaustible resource and its cost of production depends crucially on the stock of reserves. The smaller the remaining stock the larger is the cost of extracting a unit (a full discussion of our assumptions and specification of optimal extraction of oil was provided earlier in the section that presented the dynamic model). The production function in the oil sector also has a Cobb-Douglas functional form with constant-returns-to-scale with capital and labor as inputs: In symbols this function is as follows:

(9) Production function for oil sector

$$XD_{'oil'} = A(S) * L_{'oil'}^{\alpha_{'oil'}} * K_{'oil'}^{1-\alpha_{'oil'}}$$

where *XD*, *L*, and *K* are output, labor input and capital stock respectively; constant parameter α is the labor share in output. The scale factor *A(S)* depends on *S*, the total stock of resource remaining in the ground at each period. Therefore, *A(S)*, decreases over time as the stock of oil is depleted, reflecting the increase in marginal cost of extraction as seen in the cost function. Specifically, we assume:

$$A(S) = S^{\Sigma} * Z$$

where *Z* is a positive constant parameter reflecting the technology and Σ is the stock elasticity of resource output.

There are some limitations to the use of Cobb-Douglas production function for the oil sector that must be mentioned. Under this functional form for any strictly positive stock of resource and physical capital, and any strictly positive wage rate and oil price, there exists a profitable, strictly positive extraction level. In other word, with Cobb-Douglas function it is not profitable to leave any oil in the ground or abandonment of oil extraction is not possible. The reason is that the marginal product of labor rises toward infinity as labor approaches zero (see the necessary conditions for equation 11). Since we are sure that there exists a positive amount of physical capital in the sector (in form of oil rigs), therefore, as long as there is a positive amount of resource in the ground it is profitable to continue to extract. Not being able to abandon the oil production poses no problem in this model since we are looking at a window of time where we always have positive oil reserves and expect oil production to be profitable. Impossibility of abandonment would be a problem in a context where it is optimal to leave positive reserves in the ground as extraction costs become too high.

The amount of capital in each sector, *K*, is assumed to be fixed within each period. This implies that current investments will add to capacity only in future periods.

Capital is a composite good assumed to consist of fixed proportions of different investment goods. These proportions are summarized in the capital composition matrix, where an element b_{ij} is the amount of capital good originating from sector "*i*" that will be used to make up one unit of real capital in sector "*j*". The parameters "*ad*" and "*z*" reflect technological progress in each sector and are constant within a period. A Leontief input-output technology is assumed for intermediate inputs that implies intermediate inputs are demanded in fixed proportion to the level of output.

Competitive profit-maximizing behavior in all sectors implies that in each sector the value of the marginal product of each factor must equal its price. Thus, total factor payments in each sector are equal to the total value added by that sector. The (physical) marginal product of labor for each sector is simply the derivative of its production function (equations 8 and 9) with respect to labor. Before we can find the (money) values of these marginal products we need to define net price or value added price. The value added price, *PV*, is the price that producers use to make their output level and factor demand decisions and is defined as the value of output at producer's price minus the cost of the composite intermediate input. Sectoral value added price is given by:

(10) Definition of value added prices

$$PV_i = PX_i(1 - tn_i) - \sum_{j=1}^{n} P_j * a_{ji}$$

where:

PV_i : value added price for sector i

PD_i : domestic price of sector i's output

tn_i *:* indirect tax rate

P_i *:* price of composite good

a_{ij} *:* input-output coefficients

Profits are then the difference between revenues (output at value-added prices, which excludes the cost of intermediate inputs) and capital and labor costs. Thus, the profit maximization conditions, both for oil and non-oil sectors, that wages equal the value of the marginal product of labor can be written as:

(11) Labor demand function

$$WA * wd_i * L_i = XD_i * PV_i * \alpha_i$$

where *WA* is the economy-wide average wage rate of labor and *wd* is wage distortion parameter that measures the extent to which sectoral wage rate, *WAS*, deviates from the average, *WA*. Note that this formulation permits market distortions in the labor market. These distortions are measured by parameter *wd*, which is defined as $wd_i = WAS_i / WA$ and is normally fixed over time.

The return to capital in each sector is found as the residual of value added net of payments made to labor. The sectoral capital demands are determined by the following equation:

(12) Capital demand function

$$R_i * K_i = XD_{in} * PV_{in} - WA * wd_i * L_i$$

where *R* is the rate of return on capital.

5-2 Income Generation and Product Markets

The demand side of the economy consists of four basic blocks: consumption demand, government demand, investment demand, and intermediate demand.

<u>1- Consumption Demand</u>

There is a single representative household in the economy that owns the capital in the non-oil sectors as well as the total supply of labor in the economy, and receives payments made to these factors. Thus household income is total value added less the sum of depreciation expenditures, *DEPR*, and the total payments made to physical and natural capital in the oil sector, *OILREV:*

(13) Household income

$$Y = \sum_i PV_i * XD_i - DEPR - OILREV$$

The household saves a portion of its disposable income (total income less direct taxes, *DIRTAX*) and spends the remainder. Household saving is given below in which *MPS* is the household's marginal propensity to save and is determined through optimizing a social welfare function, as discussed in Section 3-8-2).

(14) Household savings

$$HHSAV = MPS * (Y - DIRTAX)$$

The single household is assumed to have a fixed structure of consumption where it purchases products of various sectors by a fixed expenditure share. This demand specification is a variation of Stone's linear expenditure system and is derived from a Cobb Douglas utility function to be discussed later. The fixed consumption shares imply unitary income and price elasticities:

(15) Household consumption behavior

$$CD_i = \left[ch_i * ((1 - MPS) * Y - DIRTAX)\right] / P_i$$

where CD_i is total consumption demand for output of sector *i*; and ch_i is fixed consumption share.

<u>2- Government Demand</u>

The sources of government revenue include direct and indirect taxes, tariff, and the revenues from the oil sector, *OILREV*. The government revenue *GR* is specified by the following budget equations:

(16) Government revenue

$$GR = DIRTAX + INDTAX + TARIFF + OILREV$$

(17) Direct taxes

$$DIRTAX = td * Y$$

(18) Indirect taxes on domestic production

$$INDTAX = \sum_i tn_i * PD_i * XD_i$$

(19) Tariff revenues

$$TARIFF = \sum tm_i * M_i * \overline{PWM_i} * ER$$

(20) Oil revenues

$$OILREV = XD_{'oil'} * PV_{'oil'} - WA * wd_{'oil'} * L_{'oil'} - DEPRO$$

where td and tn_i are direct and indirect tax rates, ER is the exchange rate between US dollars and the Iranian Rials, tm_i is the sectoral tariff rate, and *DEPRO* is the depreciation expenditure in the oil sector.

Government, analogous to households, is assumed to have a fixed expenditure structure such that it purchases goods and services in fixed proportions, cg_i:

(21) Government expenditure pattern

$$GD_i = cg_i * GR / P_i$$

where GD_i is the government's demand for the output of sector i. Government savings, *GSAV*, is found as a residual;

(22) Government savings

$$GSAV = GR - \sum_i P_i * GD_i$$

3- Investment Demand

We assume that the level of investment demand is determined by the level of total savings available to the economy. Total savings includes private and government savings, depreciation, and foreign savings;

(23) Total savings

$$SAVINGS = HHSAV + GSAV + DEPR + FSAV * ER$$

Foreign savings, *FSAV*, is given by:

(24) Foreign savings

$$FSAV * ER = \Psi * \sum_i PV_i * XD_i$$

where Ψ is the share of capital account in GDP. The sum of depreciation expenditures contributes to total investment in the next period;

(25) Total depreciation expenses

$$DEPR_{t+1} = \sum_i d_i * PK_{i,t} * K_{i,t+1}$$

where d_i is the given rate of depreciation in sector *i*, PK_i is the price of a unit of capital employed in sector *i* defined as:

(26) Definition of capital goods prices

$$PK_i = \sum_j P_j * b_{ji}$$

And b_{ij} is an element of the capital coefficient matrix and represents the amount of capital good originating from sector *i* that will be used to make up one unit of real capital used in sector *j*.

The inventory investment in each sector, IV_i, is assumed to be a fixed proportion, *riv*, of the sector's output (in the base run sectoral inventory investments for all periods are assumed to be constant and equal to their base year value in real terms). Sectoral productive investments are determined assuming that investable funds available to sector *i* is a given proportion, $ISHR_i$, of total productive investment which is total savings less total inventory investment, *TOTIV*.

(27) Sectoral inventory investment

$$IV_i = riv_i * XD_i$$

(28) Total inventory investment

$$TOTIV = \sum IV_i * P_i$$

(29) Investment by sector of destination (oil sector)

$$DK'_{'oil'} = (ISHR_{'oil'} * (SAVINGS - TOTIV)) / PK_{'oil'}$$

(30) Investment by sector of destination (non-oil sectors)

$$DK_{in} = (ISHR_{in} * (SAVINGS \; - TOTIV \; - DK_{'oil'} * PK_{oil})) / PK_{in}$$

In equations (29) and (30) DK_i is the volume of investment by sector of destination and $ISHR_i$ is the sector share of investment. The investment share for the oil sector $ISHR_{'oil}$is optimally determined, as explained in Section 3.8, and the non-oil investment proportions are in a way measures of profitability of each sector and their determination was also explained in Section 3.8. Notice that DK_i is investment "to" sector *i* but we are interested in finding investment demand "from" sector *i*. This is referred to as "investment by the sector of origin", ID_i, and it is determined using the capital composition matrix, b_{ij};

(31) Investment by sector of origin

$$ID_i = \sum_j b_{ij} * DK_j$$

<u>4- Intermediate demand</u>

As a result of the fixed coefficients assumption, intermediate demand is derived as follows:

(32) Intermediate demand

$$INT_i = \sum_j a_{ji} * XD_j$$

5-3 Foreign Trade

Products of sectors are either internationally traded or nontraded. Traded sectors are those that have either imports or exports or both. We start with the discussion of imports but before doing that a word on notation is in order. In the following equations the index "it" identifies traded sectors, while the index "itn" refers to non-traded sectors. The union of subsets "it" and "itn" is "i" the set of all sectors. The index "in", as before, identifies non-oil sectors.

Imports

Imports are assumed to be imperfect substitutes for domestically produced goods. Following Armington's formulation we define a composite commodity, X, to be a CES aggregation of the imported goods, M, and the domestically produced goods, XXD (the relationships between X, XD, XXD, M, and E are shown schematically in Figure 1. The aggregation function is:

(33) Composite good aggregation for traded sectors

$$X_{it} = ac_{it} * \left[\delta_{it} * M_{it}^{-\rho_{it}} + (1-\delta_{it}) * XXD_{it}^{-\rho_{it}}\right]^{-\frac{1}{\rho_{it}}}$$

where ac_{it} is a shift parameter; δ_{it}, is the share of imported good in the composite commodity; and ρ_{it}, the function's exponent parameter is related to the trade substitution elasticity σ by the expression: $\sigma_{it}=1/1+\rho_{it}$. The trade elasticity of substitution, σ, is a measure of the ease with which domestic product and imports can be substituted for each other. If no substitution is possible (σ=0), then composite good aggregation takes place with fixed proportions and relative price changes cannot directly affect the demand for imports. If, on the other hand, domestic product and imports are perfect substitutes (σ=∞) the price ratio is the same for all ratios of imports to domestic products. So the greater the substitution elasticity the easier it is to substitute the two goods. We use values of the elasticity of substitution greater than zero and less than infinity so that a finite variation in the ratio of price results in a finite variation in *M/XXD* ratio. Clearly, for sectors such as agriculture σ is large, whereas for capital goods it is quite low.

The CES formulation implies that consumers will choose a mix of domestic goods, *XXD*, and imported goods, *M*, on the basis of their relative prices. Consumers are assumed to minimize the cost of obtaining a "unit of utility":

(34) Value of domestic sales

$$P_{it} * X_{it} = PD_{it} * XXD_{it} + PM_{it} * M_{it}$$

subject to (33). The solution to this problem yields the ratio:

(35) FOC for composite good

$$\frac{M_{it}}{XXD_{it}} = \left[\frac{PD_{it}}{PM_{it}}\right]^{\sigma_{it}} * \left[\frac{\delta_{it}}{1-\delta_{it}}\right]^{\sigma_{it}}$$

where *P* is the price of the composite good *X*, *PD* and *PM* are the prices, in domestic currency, of domestic and imported goods respectively. With this specification *PD* is determined endogenously and is no longer equal to *PM*, which is fixed exogenously and is linked to the world price *PWM* by:

(36) Definition of domestic import price

$$PM_{it} = \overline{PWM_{it}} * ER * (1 + tm_{it})$$

For sectors with no imports the composite good is equal to domestic sales of domestically produced goods *XXD:*.

(37) Composite good aggregation for sectors with no imports

$$X_{itn} = XXD_{itn}$$

<u>Exports</u>

Similarly, on the export side we allow the domestic prices to diverge from the world price by utilizing product differentiation concepts. Specifically, a Constant Elasticity of Transformation (CET) function allocates domestic output, *XD*, between domestic use, *XXD*, and exports[10], *E:*

(38) CET function

$$XD_{it} = at_{it}\left[\gamma_{it} * E_{it}^{\phi_{it}} + (1-\gamma_{it}) * XXD_{it}^{\phi_{it}}\right]^{\frac{1}{\phi_{it}}}$$

where at_{it} is a shift parameter; γ_{it} is the share of exports in domestic output; and the exponent θ_{it} is related to φ the elasticity of transformation by the expression φ=1/ϕ-1. Producers can either export or sell in the domestic market. Their problem is to maximize revenue from a given level of output subject to the CET transformation function.

(39) Value of domestic output

$$PX_{it} * XD_{it} = PD_{it} * XXD_{it} + PE_{it} * E_{it}$$

The first-order condition represents export supply and is a function of the relative export price to domestic price, the elasticity of transformation between the two uses and the share parameters in the CET function.

(40) Export supply for traded sectors

$$E_{it} = XXD_{it}\left[\frac{PE_{it}}{PD_{it}} * \frac{1-\gamma_{it}}{\gamma_{it}}\right]^{\frac{1}{\phi_{it}-1}}$$

Note that implicit assumption in this specification is that there is always a positive amount of export for any positive world price of export. In other words, each traded sector always exports at least some of its output, thus a complete discontinuation of exports is not possible. Therefore, if one wanted to incorporate the possibility of full depletion of oil reserves, hence zero oil exports, one must drop CET formulation in favor of a more suitable specification.

For sectors with no exports domestic supply *XD* is equal to domestic sales *XXD:*

(41) Domestic sales for non-traded sectors

$$XD_{itn} = XXD_{itn}$$

The world market price of exports PWE_{it} is linked to domestic price PD_{it} by te_{it} the fixed export duty and *ER*, the foreign exchange rate.

(42) Definition of domestic export prices

$$\overline{PWE_{it}} * ER = PD_{it} * (1 + te_{it})$$

Notice that the underlying assumption here is that all export demand is for domestically produced goods rather than for the composite commodity. Put differently, exports are netted out of domestically produced commodities, *XD*, before the remainder, *XXD*, plus imports, *M*, produce the composite domestically traded good, *X*.

5-4 Market Equilibrium

We have established thus far the dependence of the different components of demand and supply on commodity and factor prices. The equilibrium condition in the product market is given by equation (43). The supply side consists of a composite good, *X*, which is an aggregation of imports and the portion of domestically produced good that is not exported, *XXD*. The demand side includes: demand for private consumption (*CD*), demand for public consumption (*GD*), investment (*ID*), inventory demand (*IV*), and finally demand for intermediate inputs (*INT*).

(43) Product market equilibrium

$$X_i = CD_i + GD_i + ID_i + IV_i + INT_i$$

Total labor supply grows at a constant rate, Γ; it is also assumed that the labor market clears. These conditions are shown in the following two equations:

(44) Labor supply updating

$$LS_{t+1} = LS_t * (1 + \Gamma)$$

(45) Labor market equilibrium

$$LS = \sum_i L_i$$

Finally the current account balance defines foreign savings as the difference between the values of imports and exports, or:

(46) Current account balance

$$\sum \overline{PWM_i} * M_i = \sum \overline{PWE_i} * E_i + FSAV$$

Walras' law states that the sum of the nominal values of excess demands of all product and factor markets must equal zero. However, in this model, the system of equations for intra-temporal equilibrium are not independent and thus not sufficient to determine the unknowns. Since all demand and supply functions in the model are homogenous of degree zero in all prices and the wage rate we can specify an additional constraint. This constraint defines the numeraire price index and will not affect any real magnitude in the system.

(47) Definition of market price index

$$\overline{P} = \sum_i P_i * \lambda_i$$

where P is price index and λs are weights in the price index.

6- Conclusion

Combining elements from exhaustible resources and computable general equilibrium literatures we presented a dynamic multisectoral optimization model. This model belongs to the optimal depletion category of computable general equilibrium models. It solves the inter-temporal depletion problem subject to workings of a multi-sector market economy, where relative prices play a crucial role. Such a formulation establishes general equilibrium linkages between the optimal depletion of the resource and the rest of the economy by working through both factor and product markets. The model provides a systematic framework to analyze various questions and policy issues related to the interaction of energy sector in economies that enjoy an abundance of a valuable exhaustible resource such as oil, gas, and coal. In addition to addressing the important questions of optimal depletion, optimal savings, and investment allocation in a resource based economy the model can be used to analyze a much wider array of developmental issues. For example, the model can be used to simulate economy-wide effects of various scenarios of world oil prices, export quotas, and changes in tax and tariff policies in a systematic and efficient way.

Appendix

System of Notation

The notation conventions used in presenting the model as well as a complete list of parameters and variables of the model are presented in this appendix. The parameters and variables are grouped in various categories for easy reference.

The following notation rules will be observed in presenting the equations of the model

1- Scalars are in upper case Greek letters

2- Indexed parameters are in lower case Roman or Greek letters.

3- Endogenous variables are all denoted in upper case Roman letters.

4- The variables exogenously fixed will have a bar on top.

5- Time subscripts are suppressed for all variables unless there are time lags involved.

6- Indices are also in lower case but are always shown as subscripts.

7- The index "i" refers to all sectors unless otherwise specified.

8- A subset of "i" is "in" which refers to non-oil sectors.

A complete list of parameters and variables with notation used in the text and in the computer program (GAMS) are presented in the table below.

List of Scalars, Parameters and Variables in the Model

TEXT	DESCRIPTION
SCALARS	
Δ	discount rate
Γ	population growth rate
Ω	mobility of investment funds between sectors
Φ	constant elasticity of marginal utility
Ψ	share of capital account in GDP
Σ	stock elasticity of resource output
PARAMETERS INDEXED BY SECTOR	
ad_i	prod. fn shift parameter non-oil sectors
α_i	labor share parameter in non-oil prod. fn
σ_i	elasticity of substitution
δ_i	armington function share parameter
ac_i	armington function shift parameter
ρ_i	armington function exponent
φ_i	elasticity of transformation
γ_i	cet function share parameter
at_i	cet function shift parameter
ϕ_i	cet function exponent
θ_i	capital adjustment cost coefficient
d_i	depreciation rates
riv_i	ratio of inventory investment to output
cg_i	government consumption share
ch_i	private consumption share
tm_i	import tariff rates
te_i	export duty rates
tn_i	indirect tax rates
a_{ij}	input-output coefficients
b_{ij}	capital composition coefficients
λ_i	weights in the price index
wd_i	wage distortion ratio
kd_i	capital rental distortion ratio
VARIABLES	
*** PRICES	
$\overline{P}_t$	price index
ER_t	exchange rate
$PD_{i,t}$	domestic prices

List of Scalars, Parameters and Variables in the Model - continued

TEXT	DESCRIPTION
$PM_{i,t}$	domestic price of imports
$PE_{i,t}$	domestic price of exports
$PX_{i,t}$	average output price
$PK_{i,t}$	price of a unit of capital (sector of dest)
$PV_{i,t}$	value added price
$P_{i,t}$	price of composite goods
$\overline{PWM}_{i,t}$	world market price of imports
$\overline{PWE}_{i,t}$	world market price of exports
*** GOODS	
$X_{i,t}$	composite goods supply
$XD_{i,t}$	domestic output
$XXD_{i,t}$	domestic sales of domestic goods
$E_{i,t}$	exports
$M_{i,t}$	imports
*** FACTORS	
$K_{i,t}$	capital stock
$R_{i,t}$	rate of return on capital
LS_t	labor supply
$L_{i,t}$	employment by sector
WA_t	average wage rate
*** RESOURCE SECTOR	
A_t	prod. fn shift parameter in oil sector
S_t	stock of oil in ground at t
OILREV	return to capital and resource in the oil sector
*** PROFIT VARIABLES	
$SP_{i,t}$	sectoral profit share in capital income
$RP_{i,t}$	sectoral profit rate
$AVGPR_t$	average profit rate
*** DEMAND	
$INT_{i,t}$	intermediates uses
$CD_{i,t}$	final demand for private consumption
$GD_{i,t}$	final demand for government consumption
$ID_{i,t}$	final demand for productive investment
$IV_{i,t}$	inventory investment demand by sector

List of Scalars, Parameters and Variables in the Model - continued

TEXT	**DESCRIPTION**
$TOTIV_t$	total inventory investment demand
*** INCOME ACCOUNTS	
Y_t	private GDP
GR_t	government revenue
$TARIFF_t$	tariff revenue
$INDTAX_t$	indirect tax revenue
$DUTY_t$	export duty revenue
*** SAVINGS AND INVESTMENT	
MPS_t	marginal propensity to save
$HHSAV_t$	household savings
$GSAV_t$	government savings
$DEPR_t$	total depreciation expenditure
$SAVINGS_t$	total savings
$FSAV_t$	foreign savings
$ISHR_{i,t}$	sector share of investible funds
$DK_{i,t}$	volume of investment by sector of destination
*** WELFARE INDICATOR FOR OBJECTIVE FUNCTION	
J	objective function variable

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Notes:

[1] The comprehensive work of Dervis, deMelo and Robinson (1982) presents a systematic formulation of CGE models and their application to the policy problems of developing countries.

[2] See Devarajan (1988) for a brief discussion and references.

[3] For a full discussion of Dutch Disease phenomenon see Corden and Neary (1982). For an example of CGE models that have studied the impact of oil income on the economy see Benjamin, Deverajan, and Wiener (1986).

[4] This model is the only one in the optimal depletion category that the survey refers to.

[5] Pereira and Shoven (1988) suggest one reason for slow adoption of production-side dynamics is the scarcity of accepted theories regarding the dynamic behavior of firms.

[6] For a full explanation and limitations of this approach to modeling the investment allocation see Dervis, et al (1982). For an intertemporal forward looking investment behavior specification see Go (1989).

[7] This is a simplified form of the absorptive capacity function used in Kendrick (1990).

[8] For a more recent exposition and refinement of this class of CGE models see Devarajan, Lewis and Robinson (1991).

[9] See Deverajan, Lewis and Robinson (1991) for a brief discussion of this simplification. Note that the combination of value added and intermediate inputs, is not restricted to be a Leontief fixed coefficient type relation and other two-level relationships are possible. For example Lewis (1991) has specified a production technology with a set of nested CES and Cobb-Douglas functions.

[10] The idea of CET specification is due to Powell and Gruen (1968). The idea of product differentiation between domestic output and exports is very common in CGE models of developing countries.