A Kalman-Filter Approach to Estimating the Natural Rate of Unemployment

by

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Abstract: In order to be able to conduct effective monetary policy as the unemployment rate falls, policymakers must be armed with some estimate of the natural rate of unemployment. Of course, relatively more accurate estimates of the natural rate afford additional room for the effectiveness of monetary policy at warding off accelerating inflation during periods of economic growth. Using an annual series (1948-1996) of unemployment rate data from the Bureau of Labor Statistics' *Current Population Survey*, this paper employs the Kalman filter in order to estimate the natural rate of unemployment in the United States in 1996. In addition, the Kalman filter estimation yields smoothed estimates of the time series of the natural rate over the period 1948-1996. An especially appealing feature of the model is that it is consistent with a broad variety of macroeconomic models since it follows directly from a standard macro-principles decomposition of actual unemployment into its frictional, structural, and cyclical components.

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I. INTRODUCTION

If nothing else, the natural rate of unemployment--or, more formally, the nonaccelerating inflation rate of unemployment (NAIRU, for short)--is controversial. For recent evidence of this, one need look no further than the Winter 1997 issue of the *Journal of Economic Perspectives*. The six articles which constitute that issue's symposium represent a spectrum of opinion concerning the NAIRU that ranges form strongly supportive to outright hostile. Controversy surrounding the NAIRU proceeds along three general lines: Does such a thing as the natural rate of unemployment exist at all? Can it be estimated? If it can be estimated, are the estimates useful for the conduct of monetary policy?

Eisner (1994), Galbraith (1997), and Gordon (1987, 1988)--among others--have waged empirical arguments that there is no such thing as a natural rate of unemployment. In particular, they contend that unemployment can indeed be reduced, without inflationary repercussions, below any preconceived natural rate greater than zero. Recent counter arguments for the existence of a natural rate have been given by Stiglitz (1997) and Gordon (1997).

Even among those who believe in the existence and theoretical significance of the natural rate, controversy exists surrounding its estimation and usefulness for policy purposes. Robert Gordon (1997) has recently estimated a time path of the natural rate that he believes may indeed be useful for monetary policy. Stiglitz (1997) also champions its usefulness. Staiger, Stock, and Watson (1997), however, contend that precise estimates of the natural rate are unnecessary; widely different estimates of the natural rate perform equally well in forecasting inflation.

Beyond the preceding discussion, why are some economists--at a gut level--so uncomfortable with the notion of a natural rate of unemployment? There exist at least two possible reasons. First, the concept of the natural rate is theoretically tied to another controversial construct: the Phillips curve. Many of the critics of the natural rate concept do not accept the Phillips curve in the first place (Eisner (1994), Galbraith (1997)). Second, recent unemployment-inflation experience in the U.S. has helped to undermine confidence in the natural rate as a useful notion for the conduct of policy: unemployment rates are quite low, yet inflation does not appear to be a danger lurking anywhere nearby.

The purpose of this paper is to estimate the natural rate of unemployment while attempting to assuage the two sources of discomfort given in the preceding paragraph. First, unlike the majority of estimates of the natural rate of unemployment (Gordon (1997), Weiner (1993), etc.), the analysis does not directly employ any form of Phillips curve in estimating the natural rate. Rather, the model relies upon a macroeconomic principles level decomposition of the actual rate of unemployment into its structural, frictional, and cyclical components to arrive at an estimate of the part of the actual unemployment rate that is due to frictional and structural unemployment only. That part of the actual unemployment rate--the part that consists only of noncyclical unemployment--is the resulting estimate of the natural rate. Second, regarding the recent phenomenon of combined low unemployment and low inflation, one should bear in mind that the current experience mirrors well the experience of the U.S. economy in the 1970's of high unemployment and high inflation; then, too, some began to have doubts about the unemployment-inflation tradeoff, but those doubts were largely very short lived.

My analysis proceeds as follows. In the next section, I briefly review the broad variety of methods commonly used for estimating the natural rate of unemployment. In the following section, I present the model that I use as the basis for the present estimation of the natural rate. Having articulated the model, I then demonstrate how the Kalman filter may be employed in order to obtain an estimate of the natural rate of unemployment in 1996 using only a time series of actual unemployment rate data. Since the Kalman filter may also be employed in order to obtain smoothed estimates of an unobservable series, I proceed to illustrate how the Kalman filter may be used not only to estimate some current natural rate, but to estimate the time path of the natural rate as well. Finally, I provide a few concluding comments.

II. ESTIMATING THE NATURAL RATE: A BRIEF REVIEW

While a broad variety of methods have been used for estimating the natural rate of unemployment, these methods may be divided into two basic categories: smoothing methods and econometric methods.[1] Smoothing methods generally consist of fitting a trend through actual unemployment or through its troughs. For example, Mankiw (1994) uses a 21-year moving average of actual unemployment as an estimate of the natural rate in his intermediate textbook. Of course, the major drawback of such methods of estimation is their ad hoc nature.

Alternatively, econometric methods of estimating the natural rate of unemployment are just that: they are methods of estimating the natural rate of unemployment that are based upon some sort of economic model. Typically, such estimates employ either a single equation or a system of

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[1] The discussion in this section is due largely to the excellent survey provided in Adams and Coe (1990).

equations.

There are four equations which are frequently used to estimate the natural rate. First, the most commonly used method of estimating the natural rate of unemployment is to infer it from the parameters of an estimated Phillips curve. Weiner (1993) and Gordon (1997) are recent examples of this approach.

A second single-equation method of estimating the natural rate relies upon the structural determinants of unemployment. In this approach, an equation such as

$$U = \alpha_0 + \alpha_1(y - y^{tr}) + f(STRUCTURAL) + \alpha Z^{u} + e_{u} \qquad \textbf{(1)}$$

is estimated, where the unemployment rate U is related to a cyclical variable that is represented by the deviation of actual output y from its trend y^{tr}, a set of structural variables, and a set of other relevant variables. Then, fixing the output gap at its cyclically neutral level in order to distinguish between cyclical and noncyclical unemployment, the natural rate of unemployment may be estimated as

$$UNAT = \hat{\alpha} + f(STRUCTURAL). \qquad \textbf{(1')}$$

Coe (1990) and Adams and Coe (1990) make use of such an approach.

The remaining two single-equation econometric methods of estimating the natural rate are not, in fact, ways of estimating the natural rate of unemployment at all. Rather, they are ways in which the level of potential output may be estimated as a proxy for the natural rate; of course, this reasoning is grounded in Okun's (1962) law. In particular, one of these two methods employs directly an expression of Okun's law; the other makes use of Okun's law only implicitly, estimating a production function and then calculating potential output as the output level achieved when all factors are at their predetermined "natural" or "normal" levels. Examples of these methods include U.S. Congress (1988), Masson et. al. (1988), and Torres and Martin (1989).

Systems techniques of estimating the natural rate attempt to improve upon single-equation methods by combining two or more of the aforementioned equations to create a larger, richer model. Adams and Coe (1990) estimate such a model.

III. THE MODEL

The actual rate of unemployment, which is observed frequently and easily, may be thought of as the sum of two distinct components. One component, the rate of cyclical unemployment, captures the unemployment associated with changes in business conditions--primarily recessions. The other component, which I will refer to as the natural rate component, includes frictional as well as structural unemployment. Hence, at any given time t, this relationship may be expressed as

$$U_t = UNAT_t + \beta_t, \qquad \textbf{(2)}$$

where U_t is the observed actual unemployment rate, $UNAT_t$ is the natural rate of unemployment, and β_t is the rate of cyclical unemployment.

To complete the model, two additional assumptions must be made regarding the evolution of both the natural rate and the cyclical rate. Following Blanchard and Quah (1989), I introduce two disturbances, one that is temporary and one that is permanent. Specifically, I assume that the natural rate of unemployment $UNAT_t$ evolves as a random walk and that the cyclical rate of unemployment β_t exhibits serial correlation; that is, I assume that shocks to cyclical unemployment are temporary and that shocks to the natural rate of unemployment are permanent. Therefore, a given policymaker's best approximation to his stochastic environment may be characterized as:

$$U_t = UNAT_t + \beta_t, \tag{2}$$

$$UNAT_t = UNAT_{t-1} + \epsilon_t, \tag{3}$$

$$\beta_t = \rho\beta_{t-1} + \eta_t, \tag{4}$$

where ρ is between 0 and 1 and where ϵ_t and η_t are independently distributed error terms with

$$\epsilon_t \sim N\left(0,\ \sigma_\epsilon^2\right),\ \eta_t \sim N\left(0,\ \sigma_\eta^2\right).$$

IV. THE MODEL AND THE KALMAN FILTER

The model given in the preceding section is a form of unobservable components model; a policymaker observes the actual unemployment rate U_t which consists of an unobserved natural rate component $UNAT_t$ plus a cyclical rate component β_t. Such unobservable components models are especially amenable to representation via the Kalman filter.

As Mitchell (1982) so succinctly states, "[a] Kalman filter is simply a way of revising expected values of unobservable variables . . . by employing observations of other variables that are correlated with them" (368). Originally developed within control engineering by Kalman (1960) and Kalman and Bucy (1961), economists such as Rosenberg (1973) and Garbade (1977) began to incorporate the Kalman filter into economics during the 1970's. In the context of the model given in **(2)-(4)**, the

Kalman filter provides an optimal updating scheme for the unobservable natural rate of unemployment $UNAT_t$ based upon observations of the actual rate of unemployment U_t as they sequentially become available.

Under the assumption of normality, standard application of the Kalman filter to the model given here produces the following conditional distribution of the natural rate of unemployment $UNAT_t$:

$$UNAT_t|U_t \sim N\left(E_{t-1}UNAT_t + \left[\frac{Cov_{t-1}(UNAT_t,\ U_t)}{Var_{t-1}(U_t)}\right](U_t - E_{t-1}U_t),\right.$$

$$\left. \Sigma_{t-1}^{UNAT} + \sigma_\epsilon^2 - \left[\frac{(Cov_{t-1}(UNAT_t,\ U_t))^2}{Var_{t-1}(U_t)}\right]\right), \tag{5}$$

where

$$\Sigma_t^{UNAT} \equiv E_t(UNAT_t - E_tUNAT_t)^2$$

defines the variance at time t of $UNAT_t$ conditional on current information. The conditional variance and covariance terms in **(5)** follow directly from the model given in **(2)-(4)**; Balvers and Cosimano (1994, 727) have shown that these terms are

$$Cov_{t-1}(UNAT_t,\ U_t) = \Sigma_{t-1}^{UNAT} + \sigma_\epsilon^2 + \rho\Sigma_{t-1}^{UNAT,\ \beta} \tag{6}$$

and

$$Var_{t-1}(U_t) = \sum_{t-1}^{UNAT} + \sigma_{\epsilon}^{2} + 2\rho\sum_{t-1}^{UNAT,\,\beta} + \rho^{2}\sum_{t-1}^{\beta} + \sigma_{\eta}^{2}. \tag{7}$$

How does the preceding relate to the problem of learning about the natural rate of unemployment? As mentioned earlier, the Kalman filter constitutes an optimal updating procedure. However, the Kalman filter itself does not estimate the unknown parameters--in this case, the natural rate of unemployment--of a model. Rather, the Kalman filter consists of a set of recursive algorithms that permits calculation of the one-step-ahead prediction errors and their variance-covariance matrix, conditional on the model's unknown parameters. Then, when these calculations of the one-step-ahead prediction errors and their variance-covariance matrix are used in conjunction with the prediction error decomposition of the likelihood function, conventional maximization methods yield estimates of the unknown parameters (Cuthbertson, Hall, and Taylor 1992, 222). Hence, via the Kalman filter, it is possible for a policymaker to be able to learn about the natural rate of unemployment by observing only a set of sequential observations of the relatively easily observed actual rate of unemployment.

To gain insight into the Kalman-filter updating process, consider the following simplified example. Suppose, for the sake of illustration, that in some given year s the natural rate of unemployment is exactly equal to the natural rate of unemployment--that is, suppose that $UNAT_s = U_s$--and that the policymaker knows this is true. How, then, would the Kalman filter yield an estimate of the natural rate of unemployment in year $s + 1$ given an observation of the actual rate of unemployment in year $s + 1$? The conditional distribution of the natural rate of unemployment given in (5) indicates that

$$E_{s+1}(UNAT_{s+1}|U_{s+1}) = E_s(UNAT_{s+1}) + \left[\frac{Cov_s(UNAT_{s+1}, U_{s+1})}{Var_s(U_{s+1})}\right](U_{s+1} - E_sU_{s+1}). \quad \textbf{(8)}$$

(8) shows that the expectation of the natural rate of unemployment in year $s + 1$ is the sum of the expectation of the natural rate formed in the prior year, plus that fraction of any "surprise" change in the actual unemployment rate from the preceding year that may be attributed to a change in the natural rate of unemployment rather than a change in the rate of cyclical unemployment. Under the current assumption that $UNAT_s = U_s$, and that the policymaker knows this, **(8)** reduces to

$$E_{s+1}(UNAT_{s+1}|U_{s+1}) = UNAT_s + \left[\frac{Cov_s(UNAT_{s+1}, U_{s+1})}{Var_s(U_{s+1})}\right](U_{s+1} - U_s), \qquad \textbf{(8')}$$

which reveals that the expectation in year $s + 1$ of the natural rate given the contemporaneous observation of the actual rate of unemployment is simply last year's known natural rate plus that portion of the change in the actual rate that may be attributed to a change in the natural rate component of the actual unemployment rate.

V. DATA AND ESTIMATION

A. The Data

The series of data that I use in order to estimate the natural rate of unemployment via the Kalman filter is the civilian unemployment rate taken from the Bureau of Labor Statistics's Current Population Survey. The data are annual, ranging form 1948 to 1996.

B. Estimation

In carrying out my estimation of the natural rate of unemployment via the Kalman filter, I employ the KALMAN command in TSP™ Version 4.4. Since the value of the parameter ρ is unknown in the model, I conduct a grid search over the interval [0, 1], maximizing the value of the log-likelihood function as a means of estimating the value of ρ. Using this method, I obtain an estimate of ρ that is equal to 0.58. This value seems quite reasonable for annual data, as it corresponds to a business cycle half-life of about 15 months. The estimate that ρ equals 0.58 also accords well with Blanchard and Quah's (1989) finding that temporary shocks peak after 2 to 4 quarters, then decline to vanish after about 3 to 5 years.

Hence, the model becomes

$$U_t = UNAT_t + \beta_t, \tag{2}$$

$$UNAT_t = UNAT_{t-1} + \epsilon_t, \tag{3}$$

$$\beta_t = 0.58\beta_{t-1} + \eta_t. \tag{4'}$$

Rewriting the model in its state-space decomposition[2] gives the measurement equation,

$$U_t = X'\delta_t, \tag{9}$$

and the transition equation,

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[2] For more about the Kalman filter and state-space models, see Cuthbertson, Hall, and Taylor (1992) or Harvey (1989).

$$\delta_t = T\delta_{t-1} + \gamma_t, \tag{10}$$

where

$$X' = (1,\ 1);\ \delta_t = (UNAT_t,\ \beta_t)';\ T = \begin{pmatrix} 1 & 0 \\ 0 & 0.58 \end{pmatrix};\ \gamma_t = (\epsilon_t,\ \eta_t)'.$$

1. *Estimating the Natural Rate in the Current Period*

After selecting a subjective prior value for $UNAT_{1947}$ such that $UNAT_{1947} = 4$, straightforward application of the preceding model to the series of annual unemployment data described above yields an estimate of the natural rate of unemployment for the "current period"-- that is, for the last period in the data series which, in the present analysis, is 1996. The results of the Kalman-filtering estimation are summarized in Table 1.

Dependent Variable is U_t
Sample: 1948 1996
Included Observations: 49

Variable	Estimated Coefficient	Standard Error	*t*-statistic
$UNAT_t$	5.68527	0.774051	7.34482
β_t	-0.285268	0.774051	-0.368538
	Sum of Squared Residuals	25.4553	
	Variance of Residuals	0.519497	
	Log Likelihood	-73.0162	

Table 1. *Kalman-Filter Estimate of* $UNAT_t$ *for* $t = 1996$

As Table 1 indicates, Kalman filtering of the actual unemployment series suggests that the natural rate of unemployment in 1996 is 5.69%. Since actual unemployment in 1996 is 5.4%, the analysis indicates that the actual unemployment in the U.S. is currently slightly below the natural rate.

While the Kalman filter is indeed useful in providing such an optimal updating scheme for the unobservable natural rate of unemployment, the Kalman filter can also be used to produce smoothed estimates of an unobservable series. The next section demonstrates such an application.

2. *Estimating the Time Path of the Natural Rate*

As illustrated in the preceding section, the Kalman filter gives an optimal estimate of a current state vector based on all prior information. In addition, the Kalman filter may also be used to produce smoothed estimates of the time series of the unobservable variable given all information.

How are such smoothed estimates obtained? The final round of the Kalman-filtering procedure described in the preceding section yields an estimate of the current state vector as well as its covariance matrix at time $t = T$. Given this information, smoothing consists of looking backward from time $t = T$ to obtain optimal estimates for time $T - 1$, then for $T - 2$, and so forth. Figure 1 depicts both the time series of actual unemployment and the smoothed estimates of the time series of the natural rate of unemployment from 1948 to 1996.

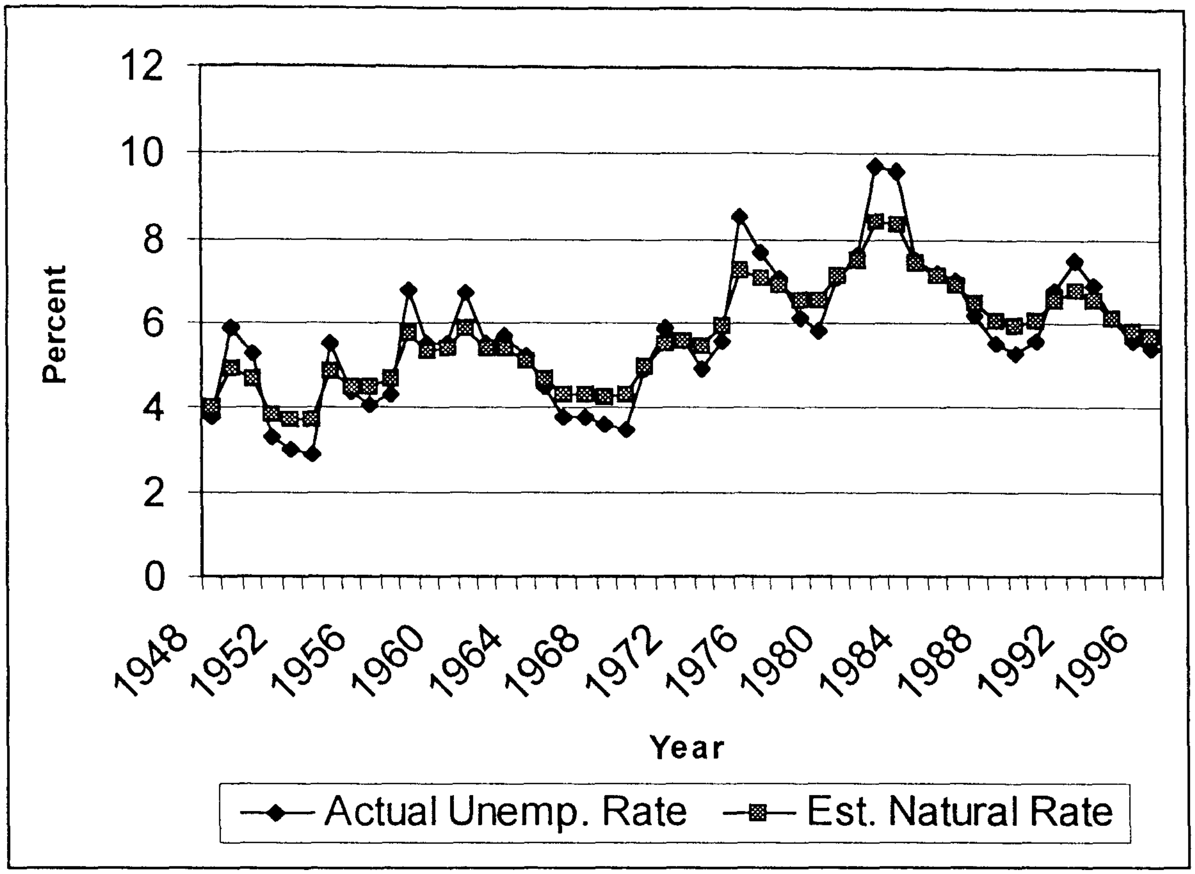

Figure 1. *Smoothed Estimates of the Natural Rate of Unemployment, 1948-1996*

Casual observation of Figure 1 reveals that the estimated time path of the natural rate of unemployment provided by the Kalman filter appears somewhat more variable than is usually suggested either by our more stereotypical thinking about the natural rate[3] or by the estimated time paths provided by more conventional smoothing methods of the natural rate. However, this lack of smoothness may be acceptable for at least four reasons.

First, the Kalman-filtering estimate of the time path of the natural rate of unemployment is certainly superior to ad hoc smoothing methods of estimating the natural rate. The Kalman method described above follows directly from standard decomposition of the actual rate of unemployment

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[3] See, for example, the discussion on pages 21-2 of Gordon (1997).

into its frictional, structural, and cyclical components, while smoothing methods are generally conducted either by straightforward data smoothing or by selecting periods when unemployment and output are thought to be at their "natural" levels. While the simplicity of such naive estimates has some appeal, more economic theory is probably warranted. Hence, the somewhat less smooth estimated time path of the natural rate afforded by the Kalman filter may be an improvement over smoother--but less theoretical--estimates.

Second, while the Kalman-filter, unobservable-components method of estimating and modeling the time path of the natural rate follows from relatively more economic theory than traditional smoothing methods, the model does not suffer from too much theory. That is, since the unobservable-components model of the natural rate of unemployment derives from a basic macro-principles decomposition of the unemployment rate into its cyclical, structural, and frictional components, the model accords well with an extremely broad variety of popular macroeconomic models, rather than being founded upon one specific form of macro model.

Third, for monetary policy purposes, the appearance of the estimated time path of the natural rate of unemployment is unimportant. Rather, what matters is how well deviations of the actual rate of unemployment from the estimated natural rate explain unexpected fluctuations in the rate of inflation.

A fourth and final possibility concerns relatively newer macroeconomic models such as efficiency wage models.[4] Such models actually permit the possibility of multiple natural rates due to nominal wage rigidities. Hence, if efficiency wage models are correct, the time path of the natural

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[4] Akerlof and Yellen (1986) is a useful collection of articles concerning efficiency wage theory.

rate might be less smooth than our conventional thinking on the natural rate would suggest.

VI. CONCLUSION

This paper has applied the Kalman filter to an unobservable-components model of unemployment in order to estimate the natural rate of unemployment. Results indicate that the natural rate of unemployment in 1996 is 5.69%, which is slightly greater than the actual rate of unemployment. An annual time series of the natural rate is also estimated for the period 1948-1996. The resulting series suggests that the natural rate is at its lowest in 1952, when it reaches 3.72%; the highest estimated value of the natural rate occurs in 1982 when it rises to a value of 8.39%.

APPENDIX

Year	Unemployment Rate	Smoothed Natural Rate	Year	Unemployment Rate	Smoothed Natural Rate
1948	3.8	4.04654	1973	4.9	5.47616
1949	5.9	4.91091	1974	5.6	5.94477
1950	5.3	4.67371	1975	8.5	7.25946
1951	3.3	3.85266	1976	7.7	7.07742
1952	3	3.71736	1977	7.1	6.8993
1953	2.9	3.73558	1978	6.1	6.55193
1954	5.5	4.83822	1979	5.8	6.51198
1955	4.4	4.50874	1980	7.1	7.13886
1956	4.1	4.48508	1981	7.6	7.4764
1957	4.3	4.68796	1982	9.7	8.38749
1958	6.8	5.77845	1983	9.6	8.34445
1959	5.5	5.35995	1984	7.5	7.42706
1960	5.5	5.40304	1985	7.2	7.16228
1961	6.7	5.8758	1986	7	6.93
1962	5.5	5.37553	1987	6.2	6.46965
1963	5.7	5.3753	1988	5.5	6.07612
1964	5.2	5.08184	1989	5.3	5.93045
1965	4.5	4.70178	1990	5.6	6.03871
1966	3.8	4.34425	1991	6.8	6.52633
1967	3.8	4.30444	1992	7.5	6.79986
1968	3.6	4.24753	1993	6.9	6.518
1969	3.5	4.29963	1994	6.1	6.12007
1970	4.9	4.99163	1995	5.6	5.83452
1971	5.9	5.54703	1996	5.4	5.68527
1972	5.6	5.58581			

Table 2. *Actual Unemployment Rates and Smoothed Natural Rate Estimates, 1948-1996*

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