Testing for spatial dependence in a two-way fixed effects panel data model

Ming He\textsuperscript{a,*}, Kuan-Pin Lin\textsuperscript{b}

\textsuperscript{a}Department of Economics, Vanderbilt University, VU Station B #351819, 2301 Vanderbilt Place, Nashville, TN 37235-1819, United States

\textsuperscript{b}Department of Economics, Portland State University, 1721 SW Broadway, Cramer Hall, Suite 241, Portland, OR 97201, United States

Abstract

In this paper, we derive a full set of Lagrange multiplier (LM) and likelihood ratio (LR) test statistics in a spatial panel data model with both individual and time effects, which is proposed by Lee and Yu (2010). Monte Carlo experiments are carried out to show their satisfactory finite sample performance. We apply our test statistics to the growth convergence example of 48 contiguous U.S. states, our empirical finding suggests positive spatial error autocorrelation and weakly positive spatial lag dependence.

Keywords: Spatial dependence, Lagrange multiplier, Likelihood ratio, Two-way fixed effects panel data model, Convergence

1. Introduction

Spatial econometric models have been rapidly developed during the last two decades after the pioneering work by Cliff and Ord (1973), Anselin (1988a). These models take into account the fact that different spatial units are related to each other, so that variables in different locations must be correlated. The standard approach is to use spatial weights matrices to capture the overall neighborhood effects. There are two popular forms of spatial correlation. The first is the spatial autoregressive process (SAR) which assumes correlation of dependent variables among neighbors. The second is the spatial error autocorrelation which allows the unobserved error terms to be spatially correlated. Spatial econometric models have been used in estimating the effects of geographical and social interaction. Recent applications include environmental Kuznets curve (Maddison, 2006), growth convergence (Ertur and Koch, 2007; Yu and Lee, 2012), spatial trade pattern and gravity model (Behrens et al., 2012), among others.

Hypothesis testing in the spatial econometric framework has been developing fast in the literature. For tests of spatial dependence in cross section models, see Anselin (1988a,b), Anselin et al. (1996), Anselin and Bera (1998), Anselin (2001), Yang (2010), Born and Breitung (2011), Lee and Yu (2011), Qu and Lee (2012). For tests of spatial dependence in panel data models, see Baltagi et al. (2003), Baltagi et al. (2007), Baltagi and Liu (2008),

\*Corresponding author. Tel: +1 503 863 9798.

Email addresses: ming.he@vanderbilt.edu (Ming He), link@pdx.edu (Kuan-Pin Lin)
Baltagi et al. (2013), He and Lin (2011) for various LM and LR test statistics for spatial random effects panel data models. For spatial panel data fixed effects models, Lee and Yu (2010) propose two specifications, one is with only individual effects (one-way effects), and the other one is with both individual and time effects (two-way effects). Debarsy and Ertur (2010) derive the LM and LR test statistics in the one-way fixed effects model, while the generalized LM and LR tests for the two-way fixed effects model have not been developed yet. To test for spatial dependence in the two-way fixed effects model, one simple approach is to introduce time dummy variables and apply the formulae in Debarsy and Ertur (2010). However, the asymptotic properties of parameter estimators in Lee and Yu (2010) are for either finite or large $T$, and the number of time dummy variables will increase as $T$ increases. Thus when $T$ is comparable to or larger than $n$, there will be incidental parameter problem and the estimators for the time dummy variables will be inconsistent. Consequently, this problem will have an impact on the performance of the tests. In our Monte Carlo study, we find severe size distortion in this time dummy variable approach when $T$ is comparable to or larger than $n$. Therefore, for the two-way fixed effects model, we suggest to derive the LM and LR test statistics directly from the transformed model based on Lee and Yu (2010).

The rest of the paper proceeds as follows: model specification and the estimation strategy in Lee and Yu (2010) are briefly reviewed in Section 2. In Section 3, we present formulae of the LM and LR test statistics corresponding to six different hypotheses, while the derivations are relegated to the Appendix. Monte Carlo experiment is conducted in Section 4 in order to evaluate the finite sample performance of these LM and LR tests. In Section 5, we provide an empirical example of growth convergence among 48 contiguous U.S. states to illustrate the use of these test statistics. Conclusion is made in Section 6.

2. Model and Hypotheses

2.1. The Model

The SAR panel data model with two-way fixed effects and SAR disturbances in Lee and Yu (2010) is

$$
\begin{align*}
Y_{nt} &= \lambda_0 W_n Y_{nt} + X_{nt}\beta_0 + c_{n0} + \alpha_{nt} l_n + U_{nt} \\
U_{nt} &= \rho_0 M_n U_{nt} + V_{nt}, \quad t = 1, 2, \ldots, T
\end{align*}
$$

(1)$$

where $Y_{nt} = (y_{1t}, y_{2t}, \ldots, y_{nt})'$, $U_{nt} = (u_{1t}, u_{2t}, \ldots, u_{nt})'$ and $V_{nt} = (v_{1t}, v_{2t}, \ldots, v_{nt})'$ are $n \times 1$ vectors and $v_{it}$ is i.i.d. across $i$ and $t$ with zero mean and variance $\sigma^2_0$. $X_{nt}$ is a $n \times k$ matrix of non-stochastic time varying regressors, $\beta_0 = (\beta_{10}, \ldots, \beta_{k0})'$ is the slope parameter vector. $c_{n0}$ is a $n \times 1$ vector representing the individual effect, while $\alpha_{nt}$ represents the time effect, $l_n$ is a $n \times 1$ vector of ones. $W_n, M_n$ are two possibly different non-stochastic spatial weights matrices, with zero diagonals and normalized rows. $\lambda_0, \rho_0$ represent the magnitudes of spatial lag dependence and the spatial error autocorrelation, respectively. Lee and Yu (2010) show that the maximum likelihood method based on (1) does not consistently estimate $\sigma^2_0$. To solve this problem, they suggest the transformation approach. The transformed model can be written as

$$
\begin{align*}
Y_{nt}^{**} &= \lambda_0 (F_{n,n-1}' W_n F_{n,n-1}) Y_{nt}^{**} + X_{nt}^{**}\beta_0 + U_{nt}^{**} \\
U_{nt}^{**} &= \rho_0 (F_{n,n-1}' M_n F_{n,n-1}) U_{nt}^{**} + V_{nt}^{**}, \quad t = 1, 2, \ldots, T - 1
\end{align*}
$$

(2)$$
where $F_{T,T-1}, F_{n,n-1}$ are transformation matrices whose columns are the orthonormal eigenvectors corresponding to the eigenvalue ones of $J_T = I_T - \frac{1}{T}l_Tl_T'$ and $J_n = I_n - \frac{1}{n}l_nl_n'$, respectively. $I_T, I_n$ are identity matrices, and $l_T$ is a $T \times 1$ vector of ones. ($Y_{n1}^{**}, Y_{n2}^{**}, ..., Y_{n(T-1)}^{**}$) = $(F_{T,T-1}^{'} \otimes F_{n,n-1}^{'})(Y_{n1}', Y_{n2}', ..., Y_{nT}'),$ and $X_{nt}^{**}, U_{nt}^{**}, V_{nt}^{**}$ are similarly defined, (see Lee and Yu (2010) for details of the transformation procedure and regularity assumptions). They show that the maximum likelihood estimator based on (2) is consistent. Let $\theta = (\lambda, \rho, \beta', \sigma)$, then the log-likelihood function can be written as (see Lee and Yu, 2010)

$$
\ln L_{n,T}(\theta) = -\frac{(n-1)(T-1)}{2} \ln(2\pi\sigma^2) + (T-1)[\ln|S_n(\lambda)| + \ln|R_n(\rho)|] - (T-1)[\ln(1-\lambda) + \ln(1-\rho)] - \frac{1}{2\sigma^2} \sum_{t=1}^{T-1} V_{nt}^{**'}V_{nt}^{**} \tag{3}
$$

where $V_{nt}^{**} = R_n^{*}(\rho)[S_n^{*}(\lambda)Y_{nt}^{**} - X_{nt}^{**}\beta], S_n(\lambda), R_n(\rho), S_n^{*}(\lambda)$ and $R_n^{*}(\rho)$ are defined in Appendix A.

2.2. Hypotheses

Six hypotheses in the following are considered:

1. $H_{a0}^0: \lambda = \rho = 0$, this is the joint hypothesis. Under the alternative, at least one spatial autoregressive parameter is not zero;

2. $H_{b0}^0: \lambda = 0$ ($\rho = 0$), this is a marginal hypothesis, and there is only spatial lag dependence in the alternative model;

3. $H_{c0}^0: \rho = 0$ ($\lambda = 0$), this is a marginal hypothesis, and there is only spatial error autocorrelation in the alternative model;

4. $H_{d0}^0: \lambda = 0$ ($\rho \neq 0$), this is a conditional hypothesis. We are testing the existence of spatial lag dependence, allowing for the existence of spatial error autocorrelation;

5. $H_{e0}^0: \rho = 0$ ($\lambda \neq 0$), this is a conditional hypothesis. We are testing the existence of spatial error autocorrelation, allowing for the existence of spatial lag dependence;

6. $H_{f0}^0: \lambda = \rho$, we are testing whether spatial lag dependence and spatial error autocorrelation are of the same magnitude.

3. LM and LR Test Statistics

3.1. LM and LR Tests for $H_{a0}^0$

In practice, researchers first need to consider the joint null hypothesis $H_{a0}^0$ in order to decide whether it is necessary to include spatial effects in the model. Under $H_{a0}^0$, both spatial lag dependence and spatial error autocorrelation are absent, the restricted model is the standard two-way fixed effects panel data model. We are testing the existence of both
spatial lag dependence and spatial error autocorrelation parameters based on the alternative model (2). The LM test statistic is

$$LM_a = \hat{\kappa}_{3,a} \hat{z}_{\lambda,a}^2 + \hat{\kappa}_{1,a} \hat{z}_{\rho,a}^2 - 2 \hat{\kappa}_{2,a} \hat{z}_{\lambda,a} \hat{z}_{\rho,a}$$

The advantage of LM test is that we only need to estimate the restricted model. For the LR test, both restricted and unrestricted models need to be estimated, and it is

$$LR_a = 2 \left[ \ln L_{n,T}(\hat{\theta}_a) - \ln L_{n,T}(\tilde{\theta}_a) \right]$$

Exploring the first order condition of the log-likelihood function with respect to $\sigma^2$ (see Appendix A), we can rewrite the LR test statistic as

$$LR_a = -(n-1)(T-1) \ln \left( \frac{\sigma^2_a}{\hat{\sigma}^2_a} \right) + 2(T-1) \ln \left( \frac{|\tilde{S}_{n,a} \tilde{R}_{n,a}|}{(1-\tilde{\lambda}_a)(1-\tilde{\rho}_a)} \right)$$

Under the joint null hypothesis $H^a$, $LM_a$ and $LR_a$ converge in distribution to $\chi^2_2$.

3.2. LM and LR Tests for $H^b_0$

If the researcher has information that there is no spatial error autocorrelation, and wants to test if spatial lag dependence exists, then marginal tests corresponding to $H^b_0$ should be used. Under $H^b_0$, we assume there is no spatial lag dependence. We are testing the existence of spatial lag dependence based on the alternative model $S^*_n(\lambda)Y^*_n = X^*_n \beta + V^*_n$. The LM test statistic for this case is

$$LM_b = \frac{1}{b^*_1,b + \hat{\omega}_b} z_{\lambda,b}^2$$

For the LR test, both restricted and unrestricted models need to be estimated, and it is

$$LR_b = 2 \left[ \ln L_{n,T}(\hat{\theta}_b) - \ln L_{n,T}(\tilde{\theta}_b) \right]$$

Similarly, by exploring the first order condition of the likelihood function with respect to $\sigma^2$, we can rewrite the LR test statistic as

$$LR_b = -(n-1)(T-1) \ln \left( \frac{\sigma^2_b}{\hat{\sigma}^2_b} \right) + 2(T-1) \ln \left( \frac{|\tilde{S}_{n,b}|}{1-\hat{\lambda}_b} \right)$$

Under the marginal null hypothesis $H^b_0$, $LM_b$ and $LR_b$ are asymptotically distributed as $\chi^2_1$.

---

1Throughout, we use $\tilde{\theta}$ to denote unrestricted estimator, and $\hat{\theta}$ to denote restricted estimator. In the main text, we do not present the formulae for relevant quantities in the expression of test statistics for the sake of compactness, and they can be found in Appendix A. The subscripts “$a$”, “$b$”, “$c$”, “$d$”, “$e$”, “$f$” stand for relevant statistics and quantities for cases $a, b, c, d, e, f$, respectively.
3.3. LM and LR Tests for $H^c_0$

On the other hand, if the researcher has information that no spatial lag dependence exists, and wants to test whether $\rho = 0$, then the marginal tests corresponding to $H^c_0$ should be used. Under $H^c_0$, we assume there is no spatial error autocorrelation. We are testing the existence of spatial error autocorrelation based on the alternative model $Y_{nt}^{**} = X_{nt}^{**}\beta + R_n^*(\rho)^{-1}V_{nt}^{**}$. The LM test statistic is

$$LM_c = \frac{1}{b^2_{3,c}}\hat{z}^2_{\rho,c}$$

For the LR test, it is

$$LR_c = 2 \left[ \ln L_{n,T}(\tilde{\theta}_c) - \ln L_{n,T}(\hat{\theta}_c) \right]$$

Similarly, by exploring the first order condition of the likelihood function with respect to $\sigma^2$, we can write the LR test statistic as

$$LR_c = -(n - 1)(T - 1) \ln \left( \frac{\hat{\sigma}^2_c}{\tilde{\sigma}^2_c} \right) + 2(T - 1) \ln \left( \frac{|\tilde{R}_n|}{1 - \tilde{\rho}} \right)$$

Under the marginal null hypothesis $H^c_0$, $LM_c$ and $LR_c$ are asymptotically distributed as $\chi^2_1$.

3.4. LM and LR Tests for $H^d_0$

In practice, the information of $\lambda = 0$ or $\rho = 0$ is seldom available. In order to test the existence of one parameter, researcher must take into account the estimate of the other parameter. For instance, to test if $\lambda = 0$, the case that $\rho \neq 0$ must be allowed. Under the conditional null hypothesis $H^d_0$, we assume there is no spatial lag dependence. We are testing the existence of spatial lag dependence based on the alternative model (2). The LM test statistic is

$$LM_d = \hat{\kappa}^2_{3,d}\hat{\lambda}^2_{\lambda,d}$$

For the LR test, it is

$$LR_d = 2 \left[ \ln L_{n,T}(\tilde{\theta}_d) - \ln L_{n,T}(\hat{\theta}_d) \right]$$

Similarly, by exploring the first order condition of the likelihood function with respect to $\sigma^2$, we can rewrite the LR test statistic as follows

$$LR_d = -(n - 1)(T - 1) \ln \left( \frac{\hat{\sigma}^2_d}{\tilde{\sigma}^2_d} \right) + 2(T - 1) \ln \left( \frac{|\tilde{S}_{n,d}\tilde{R}_n|}{(1 - \tilde{\lambda}_d)(1 - \tilde{\rho}_d)} \right) - \ln \left( \frac{|\tilde{R}_n|}{1 - \tilde{\rho}_d} \right)$$

Under the conditional null hypothesis $H^d_0$, $LM_d$ and $LR_d$ are asymptotically distributed as $\chi^2_1$. 

5
3.5. LM and LR Tests for \( H_0^e \)

Similarly, in order to test the existence of spatial error autocorrelation, the case that \( \lambda \neq 0 \) must be allowed. Under the conditional null hypothesis \( H_0^e \), we assume there is no spatial error autocorrelation. We are testing the existence of spatial error autocorrelation based on the alternative model (2). The LM test statistic is

\[
LM_e = \hat{\kappa}_{1,e} \hat{\kappa}_{\rho,e}^2
\]

For the LR test, it is

\[
LR_e = 2 \left[ \ln L_{n,T}(\tilde{\theta}_e) - \ln L_{n,T}(\hat{\theta}_e) \right]
\]

Similarly, by exploring the first order condition of the likelihood function with respect to \( \sigma^2 \), we can write the LR test statistic as

\[
LR_e = -(n-1)(T-1) \ln \left( \frac{\hat{\sigma}^2_{\rho,e}}{\sigma^2_{\rho,e}} \right) + 2(T-1) \left[ \ln \left( \frac{|\tilde{S}_{n,e} \tilde{R}_{n,e}|}{(1-\hat{\lambda}_e)(1-\hat{\rho}_e)} \right) - \ln \left( \frac{|\tilde{S}_{n,e}|}{1-\hat{\lambda}_e} \right) \right]
\]

Under the conditional null hypothesis \( H_0^e \), \( LM_e \) and \( LR_e \) are asymptotically distributed as \( \chi^2_1 \).

3.6. LM and LR Tests for \( \lambda = \rho \)

Another hypothesis of interest in empirical research is to test if the spatial lag dependence effect and spatial error autocorrelation effect are of the same magnitude. Specifically, we want to test the null hypothesis \( H_f^0 : \lambda = \rho \). The alternative model is the full specification in (2). The LM test statistic is

\[
LM_f = \hat{\kappa}_{3,f} \hat{\kappa}_{\lambda,f}^2 + \hat{\kappa}_{1,f} \hat{\kappa}_{\rho,f}^2 - 2 \hat{\kappa}_{2,f} \hat{\kappa}_{\lambda,f} \hat{\kappa}_{\rho,f}
\]

For the LR test, it is

\[
LR_f = 2 \left[ \ln L_{n,T}(\tilde{\theta}_f) - \ln L_{n,T}(\hat{\theta}_f) \right]
\]

Similarly, by exploring the first order condition of the likelihood function with respect to \( \sigma^2 \) and making use of the fact that \( \hat{\lambda}_f = \hat{\rho}_f \), we can write the LR test statistic as follows

\[
LR_f = -(n-1)(T-1) \ln \left( \frac{\hat{\sigma}^2_f}{\sigma^2_f} \right) + 2(T-1) \left[ \ln \left( \frac{|\tilde{S}_{n,f} \tilde{R}_{n,f}|}{(1-\hat{\lambda}_f)(1-\hat{\rho}_f)} \right) - \ln \left( \frac{|\tilde{S}_{n,f} \tilde{R}_{n,f}|}{(1-\hat{\lambda}_f)^2} \right) \right]
\]

Under the conditional null hypothesis \( H_0^f \), \( LM_f \) and \( LR_f \) are asymptotically distributed as \( \chi^2_1 \).

4. Monte Carlo Simulation

In this section, we conduct a small Monte Carlo simulation to evaluate the size and power performance of the proposed test statistics. The data generating process is

\[
Y_{nt} = \lambda_0 W_n Y_{nt} + X_{nt1} \beta_{10} + X_{nt2} \beta_{20} + c_{n0} + \alpha_{t0} l_n + (I_n - \rho_0 M_n)^{-1} V_{nt}, \quad t = 1, \ldots, T
\]
where \( V_{nt} = (v_{1t}, v_{2t}, ..., v_{nt})' \), \( v_{it} \sim i.i.n. (0, \sigma^2_0) \), with \( \sigma^2_0 = 5 \). For the individual effects, each element of \( c_{nt0} \) is generated independently from a uniform distribution on \([-5, 5]\). For the time effects, \( \alpha_{(t+1)0} = 1.05\alpha_{t0}, t = 1, ..., T - 1, \alpha_{10} \) is generated from a uniform distribution on \([0, 10]\). Each element of \( X_{nt1} \) is generated independently from \( N(0, 16) \), while each element of \( X_{nt2} \) is generated independently from a uniform distribution on \([0, 10]\). For the spatial weights matrices, \( W_n \) is the first order rook contiguity matrix, and \( M_n \) is the first order queen contiguity matrix. For the population parameter values, we set \( \beta_{10} = 0.5, \beta_{20} = 0.7 \), both \( \lambda_0 \) and \( \rho_0 \) take values from \([-0.8, 0.8] \), with increment 0.2. For each combination of parameter values, two sample sizes are chosen, namely, \( n = 49, t = 4 \) and \( n = 16, t = 40 \), with 1000 repetitions performed. The nominal size is set to be 0.05.

The simulation results of the joint \( LM \) and \( LR \) test statistics are reported in Table 1 and Table 2, with the empirical size highlighted in boldface. For the sake of compactness, we only report the results for \( \lambda = 0, \pm 0.4, \pm 0.8 \), the rest of the results are similar. For the (49, 4) sample, the empirical size of \( LM_a \) is 0.049 and that of \( LR_a \) is 0.044. For the (16, 40) sample, the empirical size of \( LM_a \) is 0.045 and that of \( LR_a \) is 0.042, these empirical sizes are all within the 95% confidence interval of the frequency of rejection (FR).

Since we are testing the joint presence of spatial lag dependence parameter \( \lambda \) and spatial error autocorrelation parameter \( \rho \), then \( LM_a \) and \( LR_a \) are supposed to deviate from \( \chi^2_2 \) distribution asymptotically as either \( \lambda \) or \( \rho \) deviates from 0, and this is confirmed from our simulation results. FR increases rapidly when either \( \lambda \) or \( \rho = 0 \) deviates from 0. For example, when \( \lambda = 0.4, \rho = -0.2 \), FRs of \( LM_a, LR_a \) are 0.957, 0.970 for the (49, 4) sample. FRs are almost uniformly 1 for the (16, 40) sample, exhibiting very good power performance. In practice, researcher should first conduct the joint tests to determine if spatial effects actually exist. If the joint null hypothesis \( \lambda = \rho = 0 \) is rejected, then certain econometric specification with spatial effects should be used.

The experiment results of the marginal \( LM \) and \( LR \) test statistics are in Table 3 and Table 4. For these two marginal hypotheses, we also present results based on the time dummy variables approach using the test statistics in Debarsy and Ertur (2010), so \( LM^{DE}_b, LR^{DE}_b, LM^{DE}_c, LR^{DE}_c \) stand for the test statistics using their formulae. In Table 3, we evaluate the performance of test statistics corresponding to \( H^0_b \). For the (49, 4) sample, the empirical sizes of \( LM_b, LM^{DE}_b, LR_b, LR^{DE}_b \) are 0.063, 0.068, 0.062, 0.069, respectively. Although \( LM^{DE}_b, LR^{DE}_b \) are a little oversized, they are still reasonable. The powers of the four tests in this case are generally comparable. For the (16, 40) sample, however, in which case \( T \) is larger than \( n \), the empirical sizes of \( LM_b, LR_b \) are 0.046, 0.045, while those of \( LM^{DE}_b, LR^{DE}_b \) are 0.303, 0.317. Thus test statistics based on the time dummy variables approach suffer from size distortion, and this distortion is even more severe for \( LM^{DE}_c \) and

\[ \text{FR} \approx \left( -1.96 \sqrt{\frac{0.05 \times 0.95}{1000}} + 0.05, 1.96 \sqrt{\frac{0.05 \times 0.95}{1000}} + 0.05 \right) \approx (0.036, 0.064). \]
$LR_c^{DE}$ as given in Table 4. For the (16, 40) sample, the empirical sizes of $LM_c^{DE}, LR_c^{DE}$ are 0.770, 0.476, which could be very misleading. Also, when $\rho$ deviates from 0 to 0.2, there is an unreasonable decrease in the powers of $LM_c^{DE}$ and $LR_c^{DE}$. Therefore, in practice, we suggest researchers to use the test statistics derived from the transformation approach as presented in this paper, instead of taking the time dummy variables approach.

$\sim \sim \sim <$ - - - - - - Table 5 and 6 Approximately Here - - - - - - $>

Simulation results of the conditional $LM$ and $LR$ test statistics are summarized in Table 5 and Table 6. To save space, for each conditional hypothesis, we only report the results for two values of the conditional parameters. In Table 5, when $\rho = -0.5$, the empirical sizes of $LM_d, LR_d$ are 0.046, 0.067 for the (49, 4) sample, while they are 0.056, 0.062 for the (16, 40) sample. The results for $\rho = 0.5$ are similar. In Table 6, when $\lambda = -0.5$, the empirical sizes of $LM_e, LR_e$ are 0.038, 0.045 for the (49, 4) sample, although $LM_e$ is a little under sized, but 0.038 is still within the FR’s 95% confidence interval. Similarly, the empirical sizes of $LM_e, LR_e$ are 0.047, 0.046 for the (16, 40) sample. The results for $\lambda = 0.5$ are similar. Conditional tests are important in practice since rejection of joint hypothesis does not give the researcher direction about which spatial effects actually exist, while marginal tests requires additional information about the data generating process. For example, marginal test $LM_b$ or $LR_b$ tend to over reject the null hypothesis $\lambda = 0$ when $\rho \neq 0$, and the model selection could be misleading. In this case, the conditional tests $LM_d$ or $LR_d$ should be used instead.

$\sim \sim \sim <$ - - - - - - Table 7 and 8 Approximately Here - - - - - - $>

The simulation results for the null hypothesis $H_0^f$ are summarized in Table 7 and Table 8, only results for the (49, 7) sample is reported for the sake of compactness. Under the null hypothesis $H_0^f : \lambda = \rho$, both $LM_f$ and $LR_f$ are asymptotically distributed as $\chi^2_1$. As a result, the main diagonals of Table 7 and Table 8 demonstrate the empirical sizes of $LM_f$ and $LR_f$, respectively. Most of the empirical sizes fall into the interval (0.036, 0.064), with only a few exceptions. For example, when $\lambda = \rho = -0.8$, $LM_f$ is a little undersized, with FR to be 0.034. On the other hand, when $\lambda = \rho = -0.4$, $LR_f$ is a little oversized, with FR to be 0.071. When $\lambda$ and $\rho$ deviate from each other, the power function increases rapidly. In both Table 7 and Table 8, we can see the pattern that FR is larger the further away from the main diagonals. Moreover, the frequency of rejection of $LM_f$ and $LR_f$ are very close to each other, demonstrating their asymptotic equivalence. FR for the (16, 40) sample is not provided here to save space, the performances of test statistics are generally better than that for the (49, 7) sample.

For a robustness check, we use an alternative way to generate the innovation error terms. We generate $v_{it}$ from a mixed distribution, with probability 0.5 from a log-normal distribution, and with probability 0.5 from a Weibull distribution. Specifically,

$$v_{it} = r_{it} - E[r_{it}], \quad r_{it} = (1 - W_{it})Z_{1,it} + W_{it}Z_{2,it}$$

where $W_{it}$ is a Bernoulli random variable with success probability $p = 0.5$, $\log Z_{1,it} \sim N(-2, 4)$, $Z_{2,it}$ is generated from a Weibull distribution with scale parameter 5 and shape parameter 1. $W_{it}, Z_{1,it}, Z_{2,it}$ are independent of each other. $r_{it}$ is demeaned by the population
mean so that \( E[v_t] = 0 \). To save space, we only summarize the results corresponding to \( H_0^d \) and \( H_0^e \) in Table 9 and Table 10. Generally, the performances of \( LM_d, LR_d, LM_e, LR_e \) are satisfactory even under non-normal distribution of the innovation error terms, although the power curves climb a little slower than those in the normal error case.

5. An Example: Growth Convergence Revisited

5.1. Conditional Growth Hypothesis

In this section, we illustrate the application of our developed \( LM \) and \( LR \) test statistics by revisiting the growth convergence hypothesis. In the neoclassical growth theory (Solow, 1956), absolute convergence refers to the hypothesis that economies with the same preferences and technology have the same steady state income per capita, and each economy will reach the steady state level of income per capita in the long run. Besides absolute convergence, the concept of conditional convergence has also been proposed to control for the difference in the steady state per capita income. The convergence or a negative relation between the initial level of income per capita and growth rate has been found in several studies (Barro, 1991; Barro and Sala-i-Martin, 1992; Mankiw et al., 1992; Islam, 1995). Islam (1995) argue that analysis of growth convergence should use panel data to control for unobserved country specific heterogeneity, and they found higher rate of convergence than that in Mankiw et al. (1992). Ertur and Koch (2007) build a spatial augmented Solow model, they estimate cross section convergence model and find spatial dependence. Yu and Lee (2012) estimate a spatial dynamic panel data model and find even higher rate of convergence than that in Islam (1995). Here for the illustration purpose, we estimate the following growth convergence model for the 48 contiguous U.S. states,

\[
\begin{align*}
\ln y_t - \ln y_{t-1} &= \lambda_0 W_n (\ln y_t - \ln y_{t-1}) + \gamma_0 \ln y_{t-1} + c_{n0} + \alpha_{t0} ln + U_{nt} \\
U_{nt} &= \rho_0 M_n U_{nt} + V_{nt}, \quad t = 1, \ldots, T
\end{align*}
\]

where \( \ln y_t = (\ln y_{1t}, \ln y_{2t}, \ldots, \ln y_{nt})' \), \( \ln y_0 \) is a \( n \times 1 \) vector of initial income per capita. \( c_{n0} \) represents the state specific effects, which incorporate the unobserved heterogeneous initial technology level, saving rate, growth rate of population and technology, capital depreciation rate, among others. \( \alpha_{t0} \) represents the time effects. From the conditional growth theory, \( \gamma_0 = -(1 - e^{-\phi \tau}) \), where \( \phi \) is the annual rate of convergence, and \( \tau \) is the length of time interval in years.

5.2. Data

We obtain annual data of nominal state per capita income (SPI) since 1930 from the Bureau of Economic Analysis (BEA). We obtain the all urban consumers price index (CPI-U) from the Bureau of Labor Statistics (BLS), and deflate income per capita by CPI-U. We decided to use the data from 1950 to 2010 to avoid the impact of great recession and World War II. Following Islam (1995), we use the five-year span to calculate growth rate since a short time span is inappropriate for two reasons. First, by using a longer time span, the innovation term \( V_{nt} \) will be less influenced by business cycle fluctuations and is less likely to be serially correlated than in a yearly data setup. Second, spatial spillovers might take
years to happen so that we may not have enough variation in a short time period. Thus \( \tau = 5 \) in our analysis, and \( \ln y_0 = \ln y_{1950}, \ln y_1 = \ln y_{1955}, \ldots, \ln y_{12} = \ln y_{2010} \). For the spatial weights matrices, we set \( W_n = M_n \) and first use the inverse of great circle distance (with different cutoff distances) between state capitals. We also use the \( q \)–nearest neighbors weights matrices for robustness check.

5.3. Results and Discussion

The calculated \( LM \) and \( LR \) test statistics are summarized in Table 11. The first row of Table 11 shows the values of test statistics corresponding to \( H_a^0 \). As can be seen, no matter which weights matrix we use, the joint null hypothesis \( \lambda = \rho = 0 \) is always rejected, with right tail probability to be 0. Thus, given that the general model is the spatial panel model with both individual and time effects, either spatial lag dependence or spatial error autocorrelation must exist. Without further information, however, we cannot conclude which type of spatial effects exist. For marginal tests corresponding to \( H_b^0 \), testing results from using different weights matrices are consistent. For instance, \( LM_b = 184.33, LR_b = 116.65 \) in the first two columns. If we believe that there is no spatial error autocorrelation, i.e. \( \rho = 0 \), then these are strong evidences for \( \lambda \neq 0 \). Nevertheless, we cannot rely only on these marginal tests since the assumption \( \rho = 0 \) may not be true, in which case we need to refer to the conditional tests for \( H_d^0 \). The results corresponding to \( H_d^0 \) are somewhat mixed. When the cutoff distance is 800 or 1000 miles, the test statistics are significant at the 10\% level. For example, \( LM_d = 5.49, LR_d = 3.13 \) when the cutoff distance is 800 miles, with \( p \)-values to be 0.019 and 0.077, respectively. But when the cutoff distance is 600 miles or \( q = 6.8 \) when we use the \( q \)–nearest neighbors weights matrices, the test statistics are small, leading to the acceptance of no spatial lag dependence. On the other hand, for testing \( \rho = 0 \), \( LM_c \) and \( LR_c \) are statistically significant, suggesting that spatial error autocorrelation exists if \( \lambda = 0 \). Similarly, we need to refer to the conditional tests. The values of test statistics for \( H_e^0 \) are large regardless of the choice of spatial weights matrices. For example, \( LM_e = 33.22, LR_e = 21.62 \) when the cutoff distance is 1000 miles, so we reject the null hypothesis that \( \rho = 0 (\lambda \neq 0) \). Finally, we are interested in testing whether the two types of spatial effects are of the same magnitude, i.e. \( \lambda = \rho \), and the results are somewhat inconclusive. In all five cases, \( LM_f \) rejects \( H_0^f \) at the 5\% significance level, while \( LR_f \) do not reject \( H_0^f \) at the 10\% significance level in the cases that cutoff distances are 800 or 1000 miles. This inconsistency might be due to finite sample error or different finite sample behavior of \( LM \) and \( LR \) tests.

Considering all of these tests, we conclude that in the post 1950 economic growth of U.S. states, there exist spatial error autocorrelation and weak spatial lag dependence. We thus estimate the full model as well as the model with restriction \( \lambda = 0 \) (i.e. the spatial error model). Estimation results are summarized in Table 12 and Table 13 for five choices of spatial weights matrices. In Table 12 for the full model, results are similar regardless of the choice of spatial weights matrices. For instance, when cutoff distance is 800 miles, the estimated spatial lag coefficient is 0.295. This suggests that the growth rate of one state is
positively related to the growth rate of its neighboring states, although it is not statistically significant. Also, the spatial error component is positively correlated, one explanation might be the technological spillovers. The estimated coefficient of initial income per capita is negative, with value $-0.313$ when the cutoff distance is 800 miles, and the implied annual convergence rate is 0.075. Our estimates of annual convergence rate are similar to that in Yu and Lee (2012), and are larger than that in Islam (1995). The results for spatial error model are similar to those of the full model. However, by comparing log likelihood values of these two models, the spatial error model is preferred.

6. Conclusion

In this paper, we consider hypothesis testing in a SAR panel data model with individual effects, time effects and SAR disturbances, which is proposed in Lee and Yu (2010). We argue that in this model framework, although test can be performed by including time dummy variables and use the formulae in Debarsy and Ertur (2010), there is incidental parameter problem when $T$ becomes large or comparable to $n$. Since the estimates for the parameters of dummy variables are inconsistent, test statistics based on the one-way fixed effects model will perform poorly. We then employ the transformation approach in Lee and Yu (2010) to swipe out both individual and time effects. We first derive LM and LR tests for the joint null hypothesis that there is neither spatial lag dependence nor spatial error autocorrelation. Secondly, two marginal LM and LR tests for the SAR variable and SAR disturbances are derived, respectively. We next derive two conditional LM and LR tests for SAR variable and SAR disturbances. Finally, we derive LM and LR test to detect whether the two types of spatial autoregressive effects are of the same magnitude.

We conduct Monte Carlo experiments to evaluate finite sample performances of the suggested test statistics. These tests exhibit very good finite sample empirical size and power. An empirical example is provided to illustrate application of these $LM$ and $LR$ test statistics. We use this two-way fixed effects spatial panel data model to study the growth convergence for the 48 contiguous U.S. states. We find positive spatial error autocorrelation, possibly due to spatial technological spillovers. We also find weakly positive spatial lag dependence, suggesting that the growth rate of one state is positively correlated to that of its neighboring states. The implied annual rate of convergence in our estimation is between 0.075 and 0.086, which is similar to that in Yu and Lee (2012) and larger than that in Islam (1995).
Appendix A

A.1. Summary for Notations

Notation Set 1

$$S_n(\lambda) = I_n - \lambda W_n, R_n(\rho) = I_n - \rho M_n, G_n(\lambda) = W_n S_n(\lambda)^{-1}, H_n(\rho) = M_n R_n(\rho)^{-1}$$

$$W_n^* = F_n^* - \lambda W_n, M_n^* = F_n^* - \lambda M_n, S_n^*(\lambda) = I_n - \lambda W_n^*, R_n^*(\rho) = I_n - \rho M_n^*, G_n^*(\lambda) = W_n^* S_n^*(\lambda)^{-1}, H_n^*(\rho) = M_n^* R_n^*(\rho)^{-1}$$

Notation Set 2

$$V_{nt}^{**} = R_n^*(\rho)[S_n^*(\lambda)Y_{nt}^{**} - X_{nt}^{**}\beta],$$ for restricted estimation, let $$\hat{V}_{nt}^{**} = \hat{R}_n^*[\hat{S}_n^*Y_{nt}^{**} - X_{nt}^{**}\hat{\beta}]$$,

$$\hat{\sigma}^2 = \frac{\sum_{t=1}^{T-1} \hat{V}_{nt}^{**}'\hat{V}_{nt}^{**}}{(n-1)(T-1)},$$ where $$\hat{R}_n^* = R_n^*(\hat{\rho}), \hat{S}_n^* = S_n^*(\hat{\lambda})$$. Similarly, for the unrestricted estimation, let $$\tilde{V}_{nt}^{**} = \tilde{R}_n^*[\tilde{S}_n^*Y_{nt}^{**} - X_{nt}^{**}\tilde{\beta}]$$,

$$\tilde{\sigma}^2 = \frac{\sum_{t=1}^{T-1} \tilde{V}_{nt}^{**}'\tilde{V}_{nt}^{**}}{(n-1)(T-1)},$$ where $$\tilde{R}_n^* = R_n^*(\tilde{\rho}), \tilde{S}_n^* = S_n^*(\tilde{\lambda})$$.

Notation Set 3

$$z_\lambda = -(T-1)\text{tr}[G_n^*(\lambda)] + \frac{1}{\hat{\sigma}^2} \sum_{t=1}^{T-1} \left(V_{nt}^{**}' R_n^*(\rho) W_{nt}^* Y_{nt}^{**}\right)$$

$$z_\rho = -(T-1)\text{tr}[H_n^*(\rho)] + \frac{1}{\hat{\sigma}^2} \sum_{t=1}^{T-1} \left(V_{nt}^{**}' M_n^* R_n^*(\rho)^{-1} V_{nt}^{**}\right)$$

$$\nu = \frac{1}{\sigma^2} \sum_{t=1}^{T-1} \left[R_n^*(\rho)G_n^*(\lambda)X_{nt}^{**}\beta\right]'\left[R_n^*(\rho)G_n^*(\lambda)X_{nt}^{**}\beta\right], \quad \Lambda = \frac{1}{\sigma^2} \sum_{t=1}^{T-1} \left[R_n^*(\rho)G_n^*(\lambda)X_{nt}^{**}\beta\right]'\left[R_n^*(\rho)X_{nt}^{**}\right]$$

$$\Delta = \frac{1}{\sigma^2} \sum_{t=1}^{T-1} \left[R_n^*(\rho)X_{nt}^{**}\right]'\left[R_n^*(\rho)X_{nt}^{**}\right], \quad \omega = \nu - \Lambda \Delta^{-1} \Lambda'$$

and let $$\hat{z}_\lambda, \hat{z}_\rho, \hat{\nu}, \hat{\Lambda}, \hat{\Delta}, \hat{\omega}$$ denote the restricted maximum likelihood estimators of corresponding quantities.

Notation Set 4

$$b_1 = (T-1)\text{tr} \left[G_n^{*2}(\lambda)\right] + (T-1)\text{tr} \left[(R_n^*(\rho)G_n^*(\lambda)R_n^*(\rho)^{-1})'(R_n^*(\rho)G_n^*(\lambda)R_n^*(\rho)^{-1})\right]$$

$$b_2 = (T-1)\text{tr}[G_n^*(\lambda)H_n^*(\rho)] + (T-1)\text{tr} \left[H_n^*(\rho)R_n^*(\rho)G_n^*(\lambda)R_n^*(\rho)^{-1}\right]$$

$$b_3 = (T-1)\text{tr}[H_n^*(\rho)(H_n^*(\rho) + H_n^*(\rho))]$$

$$b_2^* = b_2 - \frac{2(T-1)}{n-1}\text{tr}[G_n^*(\lambda)]$$

$$b_3^* = b_3 - \frac{2(T-1)}{n-1}\text{tr}[H_n^*(\rho)]$$

$$\kappa_1 = \frac{b_1^* + \omega}{b_1^* b_3^* - b_2^* b_3^* \omega}, \quad \kappa_2 = \frac{b_2^*}{b_1^* b_3^* - b_2^* b_3^* \omega}, \quad \kappa_3 = \frac{b_3^*}{b_1^* b_3^* - b_2^* b_3^* \omega}$$
and let \( \hat{b}_1, \hat{b}_2, \hat{b}_3, \hat{b}_4, \hat{b}_5, \hat{\kappa}_1, \hat{\kappa}_2, \hat{\kappa}_3 \) denote the restricted maximum likelihood estimators of corresponding quantities.

### A.2. Score Vector

Let the order be \( \theta = (\lambda, \rho, \beta', \sigma^2)' \), then

\[
\frac{\partial \ln L_{n,T}(\theta)}{\partial \theta} = \left( z_\lambda, z_\rho, \left[ \frac{1}{\sigma^2} \sum_{t=1}^{T-1} X_{nt}^{**} R_n' V_{nt}^{**} \right] \right)
\]

\[
\left( \frac{-(n-1)(T-1)}{2\sigma^2} + \frac{1}{2(\sigma^2)^2} \sum_{t=1}^{T-1} V_{nt}^{**} V_{nt}^{**} \right)
\]

### A.3. Concentrated Log-likelihood Function

Let \( \frac{\partial \ln L_{n,T}(\theta)}{\partial \beta} = 0, \frac{\partial \ln L_{n,T}(\theta)}{\partial \sigma^2} = 0 \), we get

\[
\hat{\beta} = \left( \sum_{t=1}^{T-1} X_{nt}^{**} R_n' \hat{R}_n X_{nt}^{**} \right)^{-1} \left( \sum_{t=1}^{T-1} X_{nt}^{**} \hat{R}_n' \hat{R}_n X_{nt}^{**} \right), \quad \hat{\sigma}^2 = \frac{\sum_{t=1}^{T-1} \hat{V}_{nt}^{**} \hat{V}_{nt}^{**}}{(n-1)(T-1)}
\]

plug it into the log-likelihood function, we obtain the concentrated log-likelihood function

\[
\ln L_{n,T}^c(\theta) = -\frac{(n-1)(T-1)}{2} \left[ \ln(2\pi) + 1 \right] + (T-1) \left[ \ln |S_a(\lambda)| + \ln |R_n(\rho)| \right]
\]

\[- (T-1) \left[ \ln(1-\lambda) + \ln(1-\rho) \right] - \frac{(n-1)(T-1)}{2} \ln(\hat{\sigma}^2)\]

### A.4 Information Matrix

The information matrix is

\[
I = \left( \begin{array}{cccc}
I_{\lambda\lambda} & * & * & * \\
I_{\rho\lambda} & I_{\rho\rho} & * & * \\
I_{\beta\lambda} & I_{\beta\rho} & I_{\beta\beta} & * \\
I_{\sigma^2\lambda} & I_{\sigma^2\rho} & I_{\sigma^2\beta} & I_{\sigma^2\sigma^2}
\end{array} \right) = \left( \begin{array}{cccc}
b_1 + \nu & * & * & * \\
b_2 & b_3 & * & * \\
N' & 0 & \Delta & * \\
\frac{T-1}{\sigma^2} \text{tr}[G_n^*(\lambda)] & \frac{T-1}{\sigma^2} \text{tr}[H_n^*(\rho)] & 0 & \frac{(n-1)(T-1)}{2(\sigma^2)^2}
\end{array} \right)
\]

### Appendix B

#### B.1 The Joint Test Statistic \( LM_a \)

The joint null hypothesis we consider is \( H_0^a : \lambda = \rho = 0 \), and the restricted model is \( Y_{nt}^{**} = X_{nt}^{**} \beta + V_{nt}^{**} \). The restricted estimator is the standard OLS estimator, i.e., \( \hat{\beta}_a = \left[ \sum_{t=1}^{T-1} X_{nt}^{**} X_{nt}^{**} \right]^{-1} \left[ \sum_{t=1}^{T-1} X_{nt}^{**} Y_{nt}^{**} \right] \). We are testing the existence of both spatial lag dependence and spatial error autocorrelation parameters in the full model (2). Under the row normalization assumption of \( W_n, M_n \), we have \( \text{tr}(J_n W_n) = \text{tr}(J_n M_n) = -1 \). Using this fact, the score vector and information matrix under \( H_0^a \) and evaluated at the OLS estimator are

\[
\left. \frac{\partial \ln L_{n,T}(\theta)}{\partial \theta} \right|_{H_0^a} = \left( \begin{array}{ccc}
\hat{z}_{\lambda,a} & \hat{z}_{\rho,a} & 0 \\
0 & 0 & 0
\end{array} \right), \quad I|_{H_0^a} = \left( \begin{array}{cccc}
\hat{b}_{1,a} + \hat{\nu}_a & * & * & * \\
\hat{b}_{2,a} & \hat{b}_{3,a} & * & * \\
\hat{\Delta}_a & \hat{\Delta}_a & * & * \\
\frac{1-T}{\sigma^2} & \frac{1-T}{\sigma^2} & 0 & \frac{(n-1)(T-1)}{2(\sigma^2)^2}
\end{array} \right)
\]
Partition $\mathcal{I}_{|H_0^a}$ so that

$$\mathcal{I}_{|H_0^a} = \begin{pmatrix} \mathcal{I}_{11}^a & \mathcal{I}_{12}^a \\ \mathcal{I}_{21}^a & \mathcal{I}_{22}^a \end{pmatrix}_{|H_0^a}$$

where $\mathcal{I}_{11}^a = \begin{pmatrix} \hat{b}_{1,a} + \hat{\nu}_a & \hat{b}_{2,a} \\ \hat{b}_{2,a} & \hat{b}_{3,a} \end{pmatrix}$

Then

$$\left(\mathcal{I}_{11}^a - \mathcal{I}_{12}^a\mathcal{I}_{22}^{-1}\mathcal{I}_{21}^a \right)_{|H_0^a}^{-1} \left( \begin{array}{c} \hat{b}_{1,a} + \hat{\nu}_a \\ \hat{b}_{2,a} \\ \hat{b}_{3,a} \end{array} \right) = \frac{1}{\hat{b}_{1,a}^* \hat{b}_{3,a}^* - \hat{b}_{2,a}^2 + \hat{b}_{3,a}^* \hat{\omega}_a} \left( \begin{array}{c} \hat{b}_{3,a} \hat{b}_{2,a}^* \\ -\hat{b}_{2,a}^* \hat{b}_{1,a}^* - \hat{\omega}_a \end{array} \right)$$

and the joint LM test statistic is

$$LM_a = \tilde{\kappa}_{3, \lambda,a} \tilde{z}_{\lambda,a} + \tilde{\kappa}_{1, \rho,a} \tilde{z}_{\rho,a} - 2\tilde{\kappa}_{2, \lambda,a} \tilde{z}_{\lambda,a} \tilde{z}_{\rho,a}$$

Under the joint null hypothesis $H_0^a$, $LM_a \xrightarrow{d} \chi^2_2$.

B.2 The Marginal Test Statistic $LM_b$

The first marginal hypothesis is $H_0^b : \lambda = 0$ ($\rho = 0$), and the restricted model is $Y_{nt}^{**} = X_{nt}^{**} \beta + V_{nt}^{**}$. Therefore, we are testing the existence of the spatial lag dependence parameter in the alternative model $S_n^a(\lambda)Y_{nt}^{**} = X_{nt}^{**} \beta + V_{nt}^{**}$. The score vector and information matrix under $H_0^b$ and evaluated at the OLS estimator are

$$\frac{\partial \ln L_n,T(\theta)}{\partial \theta}_{|H_0^b} = \left( \begin{array}{c} \tilde{z}_{\lambda,b} \\ 0 \\ 0 \end{array} \right)$$

$$\mathcal{I}_{|H_0^b}^b = \begin{pmatrix} \mathcal{I}_{11}^b & \mathcal{I}_{12}^b \\ \mathcal{I}_{21}^b & \mathcal{I}_{22}^b \end{pmatrix}_{|H_0^b}$$

where $\mathcal{I}_{11}^b = \hat{\beta}_{1,b} + \bar{\nu}_b$

Then

$$\left(\mathcal{I}_{11}^b - \mathcal{I}_{12}^b\mathcal{I}_{22}^{-1}\mathcal{I}_{21}^b \right)_{|H_0^b}^{-1} = \frac{1}{\hat{b}_{1,b}^* + \hat{\omega}_b}$$

and the LM test statistic is

$$LM_b = \frac{1}{\hat{b}_{1,b}^* + \hat{\omega}_b} \tilde{z}_{\lambda,b}^2$$

Under the marginal null hypothesis $H_0^b$, $LM_b \xrightarrow{d} \chi^2_1$.

B.3 The Marginal Test Statistic $LM_c$
The second marginal hypothesis is $H_0^c : \rho = 0$ ($\lambda = 0$), and the restricted model is $Y_{nt}^{**} = X_{nt}^{**} \beta + V_{nt}^{**}$. Therefore, we are testing the existence of the spatial error autocorrelation parameter in the alternative model $Y_{nt}^{**} = X_{nt}^{**} \beta + R_n^*(\rho)^{-1}V_{nt}^{**}$. The score vector and information matrix under $H_0^c$ and evaluated at the OLS estimator are

$$
\frac{\partial \ln L_{n,T}(\theta)}{\partial \theta} \bigg|_{H_0^c} = \left( \hat{z}_{\rho,c} 0 0 \right)^T, \quad \mathcal{I}_{H_0^c}^c = \begin{pmatrix}
\hat{b}_{3,c} & \ast & \ast \\
0 & \hat{\Delta}_c & \ast \\
\frac{1-T}{\sigma^2} & 0 & \frac{(n-1)(T-1)}{2(\sigma^2)^2}
\end{pmatrix}
$$

Partition $\mathcal{I}_{H_0^c}^c$ so that

$$
\mathcal{I}_{H_0^c}^c = \begin{pmatrix}
\mathcal{I}_{11}^c & \mathcal{I}_{12}^c \\
\mathcal{I}_{21}^c & \mathcal{I}_{22}^c
\end{pmatrix} \bigg|_{H_0^c}, \quad \text{where} \quad \mathcal{I}_{11}^c|_{H_0^c} = \hat{b}_{3,c}
$$

Then $(\mathcal{I}_{11}^c - \mathcal{I}_{12}^c \mathcal{I}_{22}^{-1} \mathcal{I}_{21}^c)|_{H_0^c}^{-1} = \frac{1}{\hat{b}_{3,c}}$, and the LM test statistic is

$$
LM_c = \frac{1}{\hat{b}_{3,c}^2} \hat{z}_{\rho,c}^2
$$

### B.4 The Conditional Test Statistic $LM_d$

The first conditional hypothesis is $H_0^d : \lambda = 0$, allowing for the possibility that $\rho \neq 0$. The restricted model is $Y_{nt}^{**} = X_{nt}^{**} \beta + R_n^*(\rho)^{-1}V_{nt}^{**}$. Therefore, we are testing the existence of the spatial lag dependence parameter in the full model (2). The score vector and information matrix under $H_0^d$ and evaluated at the maximum likelihood estimator are

$$
\frac{\partial \ln L_{n,T}(\theta)}{\partial \theta} \bigg|_{H_0^d} = \left( \hat{z}_{\lambda,d} 0 0 \right)^T, \quad \mathcal{I}_{H_0^d}^d = \begin{pmatrix}
\hat{b}_{1,d} + \hat{v}_d & \ast & \ast & \ast \\
\hat{b}_{2,d} & \hat{b}_{3,d} & \ast & \ast \\
\hat{\Lambda}_d & 0 & \hat{\Delta}_d & \ast \\
\frac{1-T}{\sigma^2} & \frac{T-1}{\sigma^2} & \text{tr}(\hat{H}_{n,d}^*) & 0 & \frac{(n-1)(T-1)}{2(\sigma^2)^2}
\end{pmatrix}
$$

Partition $\mathcal{I}_{H_0^d}^d$ so that

$$
\mathcal{I}_{H_0^d}^d = \begin{pmatrix}
\mathcal{I}_{11}^d & \mathcal{I}_{12}^d \\
\mathcal{I}_{21}^d & \mathcal{I}_{22}^d
\end{pmatrix} \bigg|_{H_0^d}, \quad \text{where} \quad \mathcal{I}_{11}^d|_{H_0^d} = \begin{pmatrix}
\hat{b}_{1,d} + \hat{v}_d & \ast \\
\hat{b}_{2,d} & \hat{b}_{3,d}
\end{pmatrix}
$$

Then

$$(\mathcal{I}_{11}^d - \mathcal{I}_{12}^d \mathcal{I}_{22}^{-1} \mathcal{I}_{21}^d)|_{H_0^d}^{-1} = \frac{1}{\hat{b}_{3,d}^2} \left( \frac{\hat{b}_{2,d}^*}{\hat{b}_{3,d}} - \hat{b}_{2,d}^* \hat{b}_{3,d}^* + \hat{b}_{3,d}^* \hat{\omega}_d \hat{b}_{3,d}^* + \hat{\omega}_d \right)
$$

and the LM test statistic is

$$
LM_d = \hat{\kappa}_{3,d} \hat{z}_{\lambda,d}^2
$$
Under the conditional null hypothesis \( H_0^d \), \( LM_d \xrightarrow{d} \chi^2_1 \).

B.5 The Conditional Test Statistic \( LM_e \)

The second conditional hypothesis is \( H_0^e : \rho = 0 \), allowing for the possibility that \( \lambda \neq 0 \). The restricted model is \( S_n^*(\lambda)Y_{nt}^{**} = X_{nt}^{**}\beta + V_{nt}^{**} \). Therefore, we are testing the existence of the spatial error autocorrelation parameter in the full model (2). The score vector and information matrix under \( H_0^e \) and evaluated at the maximum likelihood estimator are

\[
\frac{\partial \ln L_{n,T}(\theta)}{\partial \theta} |_{H_0^e} = (0 \quad \bar{z}_{\rho,e} \quad 0 \quad 0)^T, \quad \mathcal{I}_{|H_0^e}\end{align*}

Partition \( \mathcal{I}_{|H_0^e} \) so that

\[
\mathcal{I}_{|H_0^e} = \begin{pmatrix} \mathcal{I}_{11}^e & \mathcal{I}_{12}^e \\ \mathcal{I}_{21}^e & \mathcal{I}_{22}^e \end{pmatrix}, \quad \text{where} \quad \mathcal{I}_{11}^e |_{H_0^e} = \begin{pmatrix} \hat{b}_{1,e} + \hat{\nu}_e & * \\ \hat{b}_{2,e} & \hat{\lambda}_{e}^t \\ 0 & 0 \end{pmatrix}
\]

Then

\[
(\mathcal{I}_{11}^e - \mathcal{I}_{12}^e \mathcal{I}_{22}^{-1} \mathcal{I}_{21}^e) |_{H_0^e}^{-1} = \begin{pmatrix} \hat{b}_{1,e}^* + \hat{\omega}_e & \hat{b}_{2,e}^* \\ \hat{b}_{2,e}^* & \hat{b}_{3,e}^* \end{pmatrix}^{-1} = \frac{1}{b_{1,e}^* b_{2,e}^* - b_{2,e}^* b_{3,e}^* + b_{3,e}^* \hat{\omega}_e} \begin{pmatrix} \hat{b}_{3,e} - b_{2,e}^* \\ -b_{2,e}^* \end{pmatrix}
\]

and the LM test statistic is

\[
LM_e = \hat{\kappa}_{\rho,e} \bar{z}^2_{\rho,e}
\]

Under the conditional null hypothesis \( H_0^e \), \( LM_e \xrightarrow{d} \chi^2_1 \).

B.6 The Equality Test \( LM_f \)

The null hypothesis is \( H_0^f : \lambda = \rho \). Using \( \lambda \) to denote the common value of \( \lambda \) and \( \rho \), then the restricted model is \( S_n^*(\lambda)Y_{nt}^{**} = X_{nt}^{**}\beta + R_n^*(\lambda)^{-1}V_{nt}^{**} \). Therefore, we are testing if the two types of spatial effects are of the same magnitude in the full specification (2). The score vector and information matrix under \( H_0^f \) and evaluated at the maximum likelihood estimators are

\[
\frac{\partial \ln L_{n,T}(\theta)}{\partial \theta} |_{H_0^f} = (\hat{z}_{\lambda,f} \quad \hat{z}_{\rho,f} \quad 0 \quad 0)^T, \quad \mathcal{I}_{|H_0^f}\end{align*}

Partition \( \mathcal{I}_{|H_0^f} \) so that

\[
\mathcal{I}_{|H_0^f} = \begin{pmatrix} \mathcal{I}_{11}^f & \mathcal{I}_{12}^f \\ \mathcal{I}_{21}^f & \mathcal{I}_{22}^f \end{pmatrix}, \quad \text{where} \quad \mathcal{I}_{11}^f |_{H_0^f} = \begin{pmatrix} \hat{b}_{1,f} + \hat{\nu}_f & * \\ \hat{b}_{2,f} & \hat{\lambda}_f^t \\ 0 & 0 \end{pmatrix}
\]

Then

\[
(\mathcal{I}_{11}^f - \mathcal{I}_{12}^f \mathcal{I}_{22}^{-1} \mathcal{I}_{21}^f) |_{H_0^f}^{-1} = \begin{pmatrix} \hat{b}_{1,f} \hat{\omega}_f \\ \hat{b}_{2,f} \hat{\lambda}_f \\ \hat{b}_{3,f} \hat{\lambda}_f \end{pmatrix}^{-1} = \frac{1}{b_{1,f}^* b_{2,f}^* - b_{2,f}^* b_{3,f}^* + b_{3,f}^* \hat{\omega}_f} \begin{pmatrix} \hat{b}_{3,f} - b_{2,f}^* \\ -b_{2,f}^* \end{pmatrix}
\]

and the LM test statistic is

\[
LM_f = \hat{\kappa}_{\lambda,f} \bar{z}^2_{\rho,f}
\]
Partition $I_{H_0'}$ so that

$$I_{H_0'} = \begin{pmatrix} I_{11}^f & I_{12}^f \\ I_{21}^f & I_{22}^f \end{pmatrix}, \quad \text{where } I_{11}^f|H_0' = \begin{pmatrix} \hat{b}_{1,f} + \hat{\nu}_f & \hat{b}_{2,f} \\ \tilde{b}_{2,f} & \hat{b}_{3,f} \end{pmatrix}$$

Then

$$(I_{11}^f - I_{12}^f \hat{I}^{-1}_{22} - I_{21}^f)^{-1} = \frac{1}{\tilde{b}_{1,f}^{\ast} \tilde{b}_{3,f}^{\ast} - \tilde{b}_{2,f}^{\ast}} \begin{pmatrix} \tilde{b}_{3,f}^{\ast} & -\hat{b}_{2,f}^{\ast} \\ -\tilde{b}_{2,f}^{\ast} & \tilde{b}_{1,f}^{\ast} + \hat{\omega}_f \end{pmatrix}$$

and the LM test is

$$LM_f = \hat{\kappa}_{3,f}^{\ast} \hat{z}_{\lambda,f}^{2} + \hat{\kappa}_{1,f}^{\ast} \hat{z}_{\rho,f}^{2} - 2\hat{\kappa}_{2,f} \hat{z}_{\lambda,f} \hat{z}_{\rho,f}$$

Under the null hypothesis $H_0'$, $LM_f \xrightarrow{d} \chi^2_1$.

Reference


### Tables

#### Table 1: Performance of the Joint Tests, Sample Size $(49, 4)$, Normal Error

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$-0.8$</th>
<th>$-0.4$</th>
<th>$0$</th>
<th>$0.4$</th>
<th>$0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>$LM_a$</td>
<td>$LR_a$</td>
<td>$LM_a$</td>
<td>$LR_a$</td>
<td>$LM_a$</td>
</tr>
<tr>
<td>$-0.8$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.981</td>
</tr>
<tr>
<td>$-0.6$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.856</td>
</tr>
<tr>
<td>$-0.4$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.438</td>
</tr>
<tr>
<td>$-0.2$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.146</td>
</tr>
<tr>
<td>$0$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.999</td>
<td>0.999</td>
<td>0.049</td>
</tr>
<tr>
<td>$0.2$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.989</td>
<td>0.989</td>
<td>0.044</td>
</tr>
<tr>
<td>$0.4$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.980</td>
<td>0.980</td>
<td>0.044</td>
</tr>
<tr>
<td>$0.6$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.989</td>
<td>0.991</td>
<td>0.044</td>
</tr>
<tr>
<td>$0.8$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.044</td>
</tr>
</tbody>
</table>

#### Table 2: Performance of the Joint Tests, Sample Size $(16, 40)$, Normal Error

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$-0.8$</th>
<th>$-0.4$</th>
<th>$0$</th>
<th>$0.4$</th>
<th>$0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>$LM_a$</td>
<td>$LR_a$</td>
<td>$LM_a$</td>
<td>$LR_a$</td>
<td>$LM_a$</td>
</tr>
<tr>
<td>$-0.8$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$-0.6$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$-0.4$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.994</td>
</tr>
<tr>
<td>$-0.2$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.556</td>
</tr>
<tr>
<td>$0$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.045</td>
</tr>
<tr>
<td>$0.2$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.648</td>
</tr>
<tr>
<td>$0.4$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$0.6$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$0.8$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

#### Table 3: Performance of the Marginal Tests for $H_0^b$, Normal Error

<table>
<thead>
<tr>
<th>$\rho = 0$</th>
<th>(49, 4)</th>
<th>(16, 40)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$L_{M_b}$</td>
<td>$L_{M_b}^{DE}$</td>
</tr>
<tr>
<td>$-0.8$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$-0.6$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$-0.4$</td>
<td>0.997</td>
<td>1.000</td>
</tr>
<tr>
<td>$-0.2$</td>
<td>0.657</td>
<td>0.752</td>
</tr>
<tr>
<td>$0$</td>
<td>0.063</td>
<td>0.068</td>
</tr>
<tr>
<td>$0.2$</td>
<td>0.645</td>
<td>0.533</td>
</tr>
<tr>
<td>$0.4$</td>
<td>0.997</td>
<td>0.995</td>
</tr>
<tr>
<td>$0.6$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$0.8$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
### Table 4: Performance of the Marginal Tests for $H_0^c$, Normal Error

<table>
<thead>
<tr>
<th>$\lambda = 0$</th>
<th>(49, 4)</th>
<th>(16, 40)</th>
<th>$\lambda = -0.8$</th>
<th>$\lambda = -0.6$</th>
<th>$\lambda = -0.4$</th>
<th>$\lambda = -0.2$</th>
<th>$\lambda = 0$</th>
<th>$\lambda = 0.2$</th>
<th>$\lambda = 0.4$</th>
<th>$\lambda = 0.6$</th>
<th>$\lambda = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>$L_{Mc}$</td>
<td>$L_{MDE_c}$</td>
<td>$L_{Rc}$</td>
<td>$L_{RDE_c}$</td>
<td>$L_{Mc}$</td>
<td>$L_{MDE_c}$</td>
<td>$L_{Rc}$</td>
<td>$L_{RDE_c}$</td>
<td>$L_{Mc}$</td>
<td>$L_{MDE_c}$</td>
<td>$L_{Rc}$</td>
</tr>
<tr>
<td>0.0</td>
<td>0.997</td>
<td>0.999</td>
<td>0.998</td>
<td>0.950</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.2</td>
<td>0.929</td>
<td>0.965</td>
<td>0.950</td>
<td>0.787</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.4</td>
<td>0.576</td>
<td>0.750</td>
<td>0.663</td>
<td>0.437</td>
<td>0.997</td>
<td>1.000</td>
<td>0.998</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>0.6</td>
<td>0.182</td>
<td>0.331</td>
<td>0.241</td>
<td>0.133</td>
<td>0.671</td>
<td>1.000</td>
<td>0.694</td>
<td>0.978</td>
<td>0.978</td>
<td>0.978</td>
<td>0.978</td>
</tr>
</tbody>
</table>

### Table 5: Performance of the Conditional Tests for $H_0^d$, Normal Error

<table>
<thead>
<tr>
<th>$\rho = -0.5$</th>
<th>(49, 4)</th>
<th>(16, 40)</th>
<th>$\rho = 0.5$</th>
<th>(49, 4)</th>
<th>(16, 40)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = -0.8$</td>
<td>$L_{Md}$</td>
<td>$L_{Rd}$</td>
<td>$L_{Md}$</td>
<td>$L_{Rd}$</td>
<td>$L_{Md}$</td>
</tr>
<tr>
<td>0.0</td>
<td>0.994</td>
<td>0.996</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.2</td>
<td>0.953</td>
<td>0.975</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.4</td>
<td>0.544</td>
<td>0.619</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>0.6</td>
<td>0.198</td>
<td>0.218</td>
<td>0.613</td>
<td>0.633</td>
<td>0.613</td>
</tr>
<tr>
<td>0.8</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

### Table 6: Performance of the Conditional Tests for $H_0^e$, Normal Error

<table>
<thead>
<tr>
<th>$\lambda = -0.5$</th>
<th>(49, 4)</th>
<th>(16, 40)</th>
<th>$\lambda = 0.5$</th>
<th>(49, 4)</th>
<th>(16, 40)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>$L_{Me}$</td>
<td>$L_{Re}$</td>
<td>$L_{Me}$</td>
<td>$L_{Re}$</td>
<td>$L_{Me}$</td>
</tr>
<tr>
<td>0.0</td>
<td>0.994</td>
<td>0.996</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.2</td>
<td>0.953</td>
<td>0.975</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.4</td>
<td>0.544</td>
<td>0.619</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>0.6</td>
<td>0.198</td>
<td>0.218</td>
<td>0.613</td>
<td>0.633</td>
<td>0.613</td>
</tr>
<tr>
<td>0.8</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Table 7: Performance of the Test $LM_f$, Sample Size $(49, 4)$, Normal Error

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$-0.8$</th>
<th>$-0.6$</th>
<th>$-0.4$</th>
<th>$-0.2$</th>
<th>$0$</th>
<th>$0.2$</th>
<th>$0.4$</th>
<th>$0.6$</th>
<th>$0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.8$</td>
<td>0.034</td>
<td>0.180</td>
<td>0.503</td>
<td>0.709</td>
<td>0.871</td>
<td>0.944</td>
<td>0.990</td>
<td>0.995</td>
<td>0.987</td>
</tr>
<tr>
<td>$-0.6$</td>
<td>0.156</td>
<td>0.031</td>
<td>0.159</td>
<td>0.401</td>
<td>0.645</td>
<td>0.823</td>
<td>0.938</td>
<td>0.972</td>
<td>0.968</td>
</tr>
<tr>
<td>$-0.4$</td>
<td>0.534</td>
<td>0.139</td>
<td>0.062</td>
<td>0.146</td>
<td>0.321</td>
<td>0.570</td>
<td>0.781</td>
<td>0.883</td>
<td>0.930</td>
</tr>
<tr>
<td>$-0.2$</td>
<td>0.834</td>
<td>0.434</td>
<td>0.126</td>
<td>0.051</td>
<td>0.137</td>
<td>0.286</td>
<td>0.526</td>
<td>0.723</td>
<td>0.826</td>
</tr>
<tr>
<td>$0$</td>
<td>0.982</td>
<td>0.785</td>
<td>0.404</td>
<td>0.135</td>
<td>0.047</td>
<td>0.099</td>
<td>0.288</td>
<td>0.491</td>
<td>0.642</td>
</tr>
<tr>
<td>$0.2$</td>
<td>0.988</td>
<td>0.957</td>
<td>0.740</td>
<td>0.373</td>
<td>0.106</td>
<td>0.045</td>
<td>0.124</td>
<td>0.242</td>
<td>0.445</td>
</tr>
<tr>
<td>$0.4$</td>
<td>1.000</td>
<td>0.997</td>
<td>0.931</td>
<td>0.698</td>
<td>0.357</td>
<td>0.144</td>
<td>0.054</td>
<td>0.105</td>
<td>0.280</td>
</tr>
<tr>
<td>$0.6$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.991</td>
<td>0.927</td>
<td>0.702</td>
<td>0.354</td>
<td>0.041</td>
<td>0.123</td>
<td>0.041</td>
</tr>
<tr>
<td>$0.8$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.999</td>
<td>0.990</td>
<td>0.933</td>
<td>0.738</td>
<td>0.371</td>
<td>0.123</td>
<td>0.041</td>
</tr>
</tbody>
</table>

Table 8: Performance of the Test $LR_f$, Sample Size $(49, 4)$, Normal Error

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$-0.8$</th>
<th>$-0.6$</th>
<th>$-0.4$</th>
<th>$-0.2$</th>
<th>$0$</th>
<th>$0.2$</th>
<th>$0.4$</th>
<th>$0.6$</th>
<th>$0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.8$</td>
<td>0.042</td>
<td>0.183</td>
<td>0.514</td>
<td>0.742</td>
<td>0.899</td>
<td>0.960</td>
<td>0.995</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$-0.6$</td>
<td>0.164</td>
<td>0.038</td>
<td>0.170</td>
<td>0.423</td>
<td>0.676</td>
<td>0.851</td>
<td>0.961</td>
<td>0.990</td>
<td>0.994</td>
</tr>
<tr>
<td>$-0.4$</td>
<td>0.564</td>
<td>0.145</td>
<td>0.071</td>
<td>0.155</td>
<td>0.343</td>
<td>0.617</td>
<td>0.837</td>
<td>0.935</td>
<td>0.976</td>
</tr>
<tr>
<td>$-0.2$</td>
<td>0.857</td>
<td>0.453</td>
<td>0.127</td>
<td>0.057</td>
<td>0.157</td>
<td>0.315</td>
<td>0.572</td>
<td>0.786</td>
<td>0.912</td>
</tr>
<tr>
<td>$0$</td>
<td>0.988</td>
<td>0.963</td>
<td>0.403</td>
<td>0.126</td>
<td>0.047</td>
<td>0.047</td>
<td>0.047</td>
<td>0.047</td>
<td>0.047</td>
</tr>
<tr>
<td>$0.2$</td>
<td>0.999</td>
<td>0.964</td>
<td>0.750</td>
<td>0.380</td>
<td>0.109</td>
<td>0.046</td>
<td>0.143</td>
<td>0.285</td>
<td>0.523</td>
</tr>
<tr>
<td>$0.4$</td>
<td>1.000</td>
<td>0.997</td>
<td>0.735</td>
<td>0.306</td>
<td>0.099</td>
<td>0.050</td>
<td>0.115</td>
<td>0.312</td>
<td>0.523</td>
</tr>
<tr>
<td>$0.6$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.994</td>
<td>0.935</td>
<td>0.713</td>
<td>0.352</td>
<td>0.116</td>
<td>0.053</td>
<td>0.124</td>
</tr>
<tr>
<td>$0.8$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.999</td>
<td>0.999</td>
<td>0.944</td>
<td>0.744</td>
<td>0.381</td>
<td>0.116</td>
<td>0.038</td>
</tr>
</tbody>
</table>

Table 9: Performance of the Conditional Tests for $H_0^d$, Mixed Error

<table>
<thead>
<tr>
<th>$\rho = -0.5$</th>
<th>$49, 4$</th>
<th>$16, 40$</th>
<th>$\rho = 0.5$</th>
<th>$49, 4$</th>
<th>$16, 40$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$LM_d$</td>
<td>$LR_d$</td>
<td>$LM_d$</td>
<td>$LR_d$</td>
<td>$LM_d$</td>
</tr>
<tr>
<td>$-0.8$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$-0.6$</td>
<td>0.999</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$-0.4$</td>
<td>0.932</td>
<td>0.958</td>
<td>1.000</td>
<td>1.000</td>
<td>0.918</td>
</tr>
<tr>
<td>$-0.2$</td>
<td>0.400</td>
<td>0.451</td>
<td>0.906</td>
<td>0.940</td>
<td>0.350</td>
</tr>
<tr>
<td>$0$</td>
<td>0.041</td>
<td>0.052</td>
<td>0.042</td>
<td>0.054</td>
<td>0.053</td>
</tr>
<tr>
<td>$0.2$</td>
<td>0.261</td>
<td>0.311</td>
<td>0.806</td>
<td>0.831</td>
<td>0.220</td>
</tr>
<tr>
<td>$0.4$</td>
<td>0.686</td>
<td>0.775</td>
<td>0.990</td>
<td>0.999</td>
<td>0.694</td>
</tr>
<tr>
<td>$0.6$</td>
<td>0.896</td>
<td>0.963</td>
<td>0.991</td>
<td>0.999</td>
<td>0.923</td>
</tr>
<tr>
<td>$0.8$</td>
<td>0.965</td>
<td>0.998</td>
<td>0.996</td>
<td>1.000</td>
<td>0.885</td>
</tr>
</tbody>
</table>

21
Table 10: Performance of the Conditional Tests for $H_0$, Mixed Error

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\rho$</th>
<th>$LM_e$</th>
<th>$LR_e$</th>
<th>$LM_e$</th>
<th>$LR_e$</th>
<th>$LM_e$</th>
<th>$LR_e$</th>
<th>$LM_e$</th>
<th>$LR_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.5$</td>
<td>$0.8$</td>
<td>$0.989$</td>
<td>$0.993$</td>
<td>$1.000$</td>
<td>$1.000$</td>
<td>$-0.8$</td>
<td>$0.884$</td>
<td>$0.923$</td>
<td>$0.997$</td>
</tr>
<tr>
<td>$-0.6$</td>
<td>$0.877$</td>
<td>$0.912$</td>
<td>$0.999$</td>
<td>$1.000$</td>
<td>$-0.6$</td>
<td>$0.677$</td>
<td>$0.725$</td>
<td>$0.978$</td>
<td>$0.990$</td>
</tr>
<tr>
<td>$-0.4$</td>
<td>$0.565$</td>
<td>$0.624$</td>
<td>$0.974$</td>
<td>$0.983$</td>
<td>$-0.4$</td>
<td>$0.343$</td>
<td>$0.370$</td>
<td>$0.891$</td>
<td>$0.911$</td>
</tr>
<tr>
<td>$-0.2$</td>
<td>$0.191$</td>
<td>$0.211$</td>
<td>$0.577$</td>
<td>$0.598$</td>
<td>$-0.2$</td>
<td>$0.119$</td>
<td>$0.128$</td>
<td>$0.378$</td>
<td>$0.380$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\rho$</th>
<th>$LM_e$</th>
<th>$LR_e$</th>
<th>$LM_e$</th>
<th>$LR_e$</th>
<th>$LM_e$</th>
<th>$LR_e$</th>
<th>$LM_e$</th>
<th>$LR_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.0$</td>
<td>$0.034$</td>
<td>$0.036$</td>
<td>$0.042$</td>
<td>$0.046$</td>
<td>$0.0$</td>
<td>$0.041$</td>
<td>$0.053$</td>
<td>$0.049$</td>
<td>$0.058$</td>
</tr>
<tr>
<td>$0.2$</td>
<td>$0.197$</td>
<td>$0.190$</td>
<td>$0.635$</td>
<td>$0.639$</td>
<td>$0.2$</td>
<td>$0.088$</td>
<td>$0.106$</td>
<td>$0.300$</td>
<td>$0.309$</td>
</tr>
<tr>
<td>$0.4$</td>
<td>$0.666$</td>
<td>$0.673$</td>
<td>$0.997$</td>
<td>$0.997$</td>
<td>$0.4$</td>
<td>$0.270$</td>
<td>$0.310$</td>
<td>$0.826$</td>
<td>$0.850$</td>
</tr>
<tr>
<td>$0.6$</td>
<td>$0.967$</td>
<td>$0.975$</td>
<td>$1.000$</td>
<td>$1.000$</td>
<td>$0.6$</td>
<td>$0.491$</td>
<td>$0.600$</td>
<td>$0.972$</td>
<td>$0.987$</td>
</tr>
<tr>
<td>$0.8$</td>
<td>$1.000$</td>
<td>$1.000$</td>
<td>$1.000$</td>
<td>$1.000$</td>
<td>$0.8$</td>
<td>$0.582$</td>
<td>$0.826$</td>
<td>$0.992$</td>
<td>$1.000$</td>
</tr>
</tbody>
</table>

Table 11: Testing for Spatial Effects (Five-Year Span, 1950–2010)

<table>
<thead>
<tr>
<th>$H_a$</th>
<th>$q = 6$</th>
<th>$q = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$600$</td>
<td>$138.41$</td>
<td>$234.92$</td>
</tr>
<tr>
<td>$800$</td>
<td>$219.32$</td>
<td>$216.00$</td>
</tr>
<tr>
<td>$1000$</td>
<td>$211.61$</td>
<td>$216.00$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$H_b$</th>
<th>$q = 6$</th>
<th>$q = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$600$</td>
<td>$184.33$</td>
<td>$169.81$</td>
</tr>
<tr>
<td>$800$</td>
<td>$158.38$</td>
<td>$159.44$</td>
</tr>
<tr>
<td>$1000$</td>
<td>$148.22$</td>
<td>$149.74$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$H_c$</th>
<th>$q = 6$</th>
<th>$q = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$600$</td>
<td>$249.38$</td>
<td>$233.74$</td>
</tr>
<tr>
<td>$800$</td>
<td>$219.04$</td>
<td>$216.00$</td>
</tr>
<tr>
<td>$1000$</td>
<td>$211.18$</td>
<td>$214.33$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$H_d$</th>
<th>$q = 6$</th>
<th>$q = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$600$</td>
<td>$0.17$</td>
<td>$0.02$</td>
</tr>
<tr>
<td>$800$</td>
<td>$5.49$</td>
<td>$4.42$</td>
</tr>
<tr>
<td>$1000$</td>
<td>$3.13$</td>
<td>$3.02$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$H_e$</th>
<th>$q = 6$</th>
<th>$q = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$600$</td>
<td>$16.40$</td>
<td>$12.54$</td>
</tr>
<tr>
<td>$800$</td>
<td>$21.76$</td>
<td>$12.54$</td>
</tr>
<tr>
<td>$1000$</td>
<td>$19.76$</td>
<td>$12.54$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$H_f$</th>
<th>$q = 6$</th>
<th>$q = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$600$</td>
<td>$6.10$</td>
<td>$7.20$</td>
</tr>
<tr>
<td>$800$</td>
<td>$5.62$</td>
<td>$7.20$</td>
</tr>
<tr>
<td>$1000$</td>
<td>$4.23$</td>
<td>$7.20$</td>
</tr>
</tbody>
</table>

Table 12: Unrestricted Estimation for Growth Convergence among U.S. States, 1950–2010

<table>
<thead>
<tr>
<th>Estimates</th>
<th>$600$ miles</th>
<th>$800$ miles</th>
<th>$1000$ miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$0.060$</td>
<td>$0.295$</td>
<td>$0.363$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$0.724$***</td>
<td>$0.669$***</td>
<td>$0.724$***</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$-0.320$***</td>
<td>$-0.313$***</td>
<td>$-0.313$***</td>
</tr>
</tbody>
</table>

| Implied $\phi$| $0.077$       | $0.075$       | $0.076$       | $0.086$       | $0.084$       |

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$
<table>
<thead>
<tr>
<th>Estimates</th>
<th>600 miles</th>
<th>800 miles</th>
<th>1000 miles</th>
<th>$q = 6$</th>
<th>$q = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.752***</td>
<td>0.803***</td>
<td>0.873***</td>
<td>0.676***</td>
<td>0.737***</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.054)</td>
<td>(0.060)</td>
<td>(0.044)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$-0.320^{***}$</td>
<td>$-0.319^{***}$</td>
<td>$-0.322^{***}$</td>
<td>$-0.349^{***}$</td>
<td>$-0.349^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.029)</td>
<td>(0.030)</td>
<td>(0.030)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Implied $\phi$</td>
<td>0.077</td>
<td>0.077</td>
<td>0.078</td>
<td>0.086</td>
<td>0.085</td>
</tr>
</tbody>
</table>

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$