Testing for spatial dependence in a two-way fixed effects panel data model

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Abstract

In this paper, we derive a full set of Lagrange multiplier (LM) and likelihood ratio (LR) test statistics in a spatial panel data model with both individual and time effects, which is proposed by Lee and Yu (2010). Monte Carlo experiments are carried out to show their satisfactory finite sample performance. We apply our test statistics to the growth convergence example of 48 contiguous U.S. states, our empirical finding suggests positive spatial error autocorrelation and weakly positive spatial lag dependence.

Keywords: Spatial dependence, Lagrange multiplier, Likelihood ratio, Two-way fixed effects panel data model, Convergence

1. Introduction

Spatial econometric models have been rapidly developed during the last two decades after the pioneering work by Cliff and Ord (1973), Anselin (1988a). These models take into account the fact that different spatial units are related to each other, so that variables in different locations must be correlated. The standard approach is to use spatial weights matrices to capture the overall neighborhood effects. There are two popular forms of spatial correlation. The first is the spatial autoregressive process (SAR) which assumes correlation of dependent variables among neighbors. The second is the spatial error autocorrelation which allows the unobserved error terms to be spatially correlated. Spatial econometric models have been used in estimating the effects of geographical and social interaction. Recent applications include environmental Kuznets curve (Maddison, 2006), growth convergence (Ertur and Koch, 2007; Yu and Lee, 2012), spatial trade pattern and gravity model (Behrens et al., 2012), among others.

Hypothesis testing in the spatial econometric framework has been developing fast in the literature. For tests of spatial dependence in cross section models, see Anselin (1988a,b), Anselin et al. (1996), Anselin and Bera (1998), Anselin (2001), Yang (2010), Born and Breitung (2011), Lee and Yu (2011), Qu and Lee (2012). For tests of spatial dependence in panel data models, see Baltagi et al. (2003), Baltagi et al. (2007), Baltagi and Liu (2008),

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Baltagi et al. (2013), He and Lin (2011) for various LM and LR test statistics for spatial random effects panel data models. For spatial panel data fixed effects models, Lee and Yu (2010) propose two specifications, one is with only individual effects (one-way effects), and the other one is with both individual and time effects (two-way effects). Debarsy and Ertur (2010) derive the LM and LR test statistics in the one-way fixed effects model, while the generalized LM and LR tests for the two-way fixed effects model have not been developed yet. To test for spatial dependence in the two-way fixed effects model, one simple approach is to introduce time dummy variables and apply the formulae in Debarsy and Ertur (2010). However, the asymptotic properties of parameter estimators in Lee and Yu (2010) are for either finite or large T, and the number of time dummy variables will increase as T increases. Thus when T is comparable to or larger than n, there will be incidental parameter problem and the estimators for the time dummy variables will be inconsistent. Consequently, this problem will have an impact on the performance of the tests. In our Monte Carlo study, we find severe size distortion in this time dummy variable approach when T is comparable to or larger than n. Therefore, for the two-way fixed effects model, we suggest to derive the LM and LR test statistics directly from the transformed model based on Lee and Yu (2010).

The rest of the paper proceeds as follows: model specification and the estimation strategy in Lee and Yu (2010) are briefly reviewed in Section 2. In Section 3, we present formulae of the LM and LR test statistics corresponding to six different hypotheses, while the derivations are relegated to the Appendix. Monte Carlo experiment is conducted in Section 4 in order to evaluate the finite sample performance of these LM and LR tests. In Section 5, we provide an empirical example of growth convergence among 48 contiguous U.S. states to illustrate the use of these test statistics. Conclusion is made in Section 6.

2. Model and Hypotheses

2.1. The Model

The SAR panel data model with two-way fixed effects and SAR disturbances in Lee and Yu (2010) is

$$\begin{aligned} Y_{nt} &= \lambda_0 W_n Y_{nt} + X_{nt}\boldsymbol{\beta}_0 + \boldsymbol{c}_{n0} + \alpha_{t0} l_n + U_{nt} \\ U_{nt} &= \rho_0 M_n U_{nt} + V_{nt}, \qquad t = 1, 2, ..., T \end{aligned} \tag{1}$$

where $Y_{nt} = (y_{1t}, y_{2t}, ..., y_{nt})'$, $U_{nt} = (u_{1t}, u_{2t}, ..., u_{nt})'$ and $V_{nt} = (v_{1t}, v_{2t}, ..., v_{nt})'$ are $n \times 1$ vectors and v_{it} is i.i.d. across i and t with zero mean and variance σ_0^2. X_{nt} is a $n \times k$ matrix of non-stochastic time varying regressors, $\boldsymbol{\beta}_0 = (\beta_{10}, ..., \beta_{k0})'$ is the slope parameter vector. $\boldsymbol{c}_{n0}$ is a $n \times 1$ vector representing the individual effect, while α_{t0} represents the time effect, l_n is a $n \times 1$ vector of ones. W_n, M_n are two possibly different non-stochastic spatial weights matrices, with zero diagonals and normalized rows. λ_0, ρ_0 represent the magnitudes of spatial lag dependence and the spatial error autocorrelation, respectively. Lee and Yu (2010) show that the maximum likelihood method based on (1) does not consistently estimate σ_0^2. To solve this problem, they suggest the transformation approach. The transformed model can be written as

$$\begin{aligned} Y_{nt}^{**} &= \lambda_0 (F_{n,n-1}' W_n F_{n,n-1}) Y_{nt}^{**} + X_{nt}^{**}\boldsymbol{\beta}_0 + U_{nt}^{**} \\ U_{nt}^{**} &= \rho_0 (F_{n,n-1}' M_n F_{n,n-1}) U_{nt}^{**} + V_{nt}^{**}, \qquad t = 1, 2, ..., T-1 \end{aligned} \tag{2}$$

where $F_{T,T-1}, F_{n,n-1}$ are transformation matrices whose columns are the orthonormal eigenvectors corresponding to the eigenvalue ones of $J_T = I_T - \frac{1}{T}l_T l_T'$ and $J_n = I_n - \frac{1}{n}l_n l_n'$, respectively. I_T, I_n are identity matrices, and l_T is a $T \times 1$ vector of ones. $(Y_{n1}^{**'}, Y_{n2}^{**'}, ..., Y_{n(T-1)}^{**'})' = (F_{T,T-1}' \otimes F_{n,n-1}')(Y_{n1}', Y_{n2}', ..., Y_{nT}')$, and $X_{nt}^{**}, U_{nt}^{**}, V_{nt}^{**}$ are similarly defined, (see Lee and Yu (2010) for details of the transformation procedure and regularity assumptions). They show that the maximum likelihood estimator based on (2) is consistent. Let $\boldsymbol{\theta} = (\lambda, \rho, \boldsymbol{\beta}', \sigma)'$, then the log-likelihood function can be written as (see Lee and Yu, 2010)

$$\begin{aligned} \ln L_{n,T}(\boldsymbol{\theta}) = & -\frac{(n-1)(T-1)}{2}\ln(2\pi\sigma^2) + (T-1)[\ln|S_n(\lambda)| + \ln|R_n(\rho)|] \\ & - (T-1)[\ln(1-\lambda) + \ln(1-\rho)] - \frac{1}{2\sigma^2}\sum_{t=1}^{T-1} V_{nt}^{**'} V_{nt}^{**} \end{aligned} \quad (3)$$

where $V_{nt}^{**} = R_n^*(\rho)[S_n^*(\lambda)Y_{nt}^{**} - X_{nt}^{**}\boldsymbol{\beta}]$, $S_n(\lambda), R_n(\rho), S_n^*(\lambda)$ and $R_n^*(\rho)$ are defined in Appendix A.

2.2. Hypotheses

Six hypotheses in the following are considered:

(1) $H_0^a : \lambda = \rho = 0$, this is the joint hypothesis. Under the alternative, at least one spatial autoregressive parameter is not zero;

(2) $H_0^b : \lambda = 0$ $(\rho = 0)$, this is a marginal hypothesis, and there is only spatial lag dependence in the alternative model;

(3) $H_0^c : \rho = 0$ $(\lambda = 0)$, this is a marginal hypothesis, and there is only spatial error autocorrelation in the alternative model;

(4) $H_0^d : \lambda = 0$ $(\rho \neq 0)$, this is a conditional hypothesis. We are testing the existence of spatial lag dependence, allowing for the existence of spatial error autocorrelation;

(5) $H_0^e : \rho = 0$ $(\lambda \neq 0)$, this is a conditional hypothesis. We are testing the existence of spatial error autocorrelation, allowing for the existence of spatial lag dependence;

(6) $H_0^f : \lambda = \rho$, we are testing whether spatial lag dependence and spatial error autocorrelation are of the same magnitude.

3. LM and LR Test Statistics

3.1. LM and LR Tests for H_0^a

In practice, researchers first need to consider the joint null hypothesis H_0^a in order to decide whether it is necessary to include spatial effects in the model. Under H_0^a, both spatial lag dependence and spatial error autocorrelation are absent, the restricted model is the standard two-way fixed effects panel data model. We are testing the existence of both spatial lag dependence and spatial error autocorrelation parameters based on the alternative model (2). The LM test statistic is[1]

$$LM_a = \widehat{\kappa}_{3,a}\widehat{z}^2_{\lambda,a} + \widehat{\kappa}_{1,a}\widehat{z}^2_{\rho,a} - 2\widehat{\kappa}_{2,a}\widehat{z}_{\lambda,a}\widehat{z}_{\rho,a}$$

The advantage of LM test is that we only need to estimate the restricted model. For the LR test, both restricted and unrestricted models need to be estimated, and it is

$$LR_a = 2\left[\ln L_{n,T}(\widetilde{\boldsymbol{\theta}}_a) - \ln L_{n,T}(\widehat{\boldsymbol{\theta}}_a)\right]$$

Exploring the first order condition of the log-likelihood function with respect to σ^2 (see Appendix A), we can rewrite the LR test statistic as

$$LR_a = -(n-1)(T-1)\ln\left(\frac{\widetilde{\sigma}^2_a}{\widehat{\sigma}^2_a}\right) + 2(T-1)\ln\left(\frac{|\widetilde{S}_{n,a}\widetilde{R}_{n,a}|}{(1-\widetilde{\lambda}_a)(1-\widetilde{\rho}_a)}\right)$$

Under the joint null hypothesis H^a_0, LM_a and LR_a converge in distribution to χ^2_2.

3.2. LM and LR Tests for H^b_0

If the researcher has information that there is no spatial error autocorrelation, and wants to test if spatial lag dependence exists, then marginal tests corresponding to H^b_0 should be used. Under H^b_0, we assume there is no spatial lag dependence. We are testing the existence of spatial lag dependence based on the alternative model $S^*_n(\lambda)Y^{**}_{nt} = X^{**}_{nt}\boldsymbol{\beta} + V^{**}_{nt}$. The LM test statistic for this case is

$$LM_b = \frac{1}{\widehat{b}^*_{1,b} + \widehat{\omega}_b}\widehat{z}^2_{\lambda,b}$$

For the LR test, both restricted and unrestricted models need to be estimated, and it is

$$LR_b = 2\left[\ln L_{n,T}(\widetilde{\boldsymbol{\theta}}_b) - \ln L_{n,T}(\widehat{\boldsymbol{\theta}}_b)\right]$$

Similarly, by exploring the first order condition of the likelihood function with respect to σ^2, we can rewrite the LR test statistic as

$$LR_b = -(n-1)(T-1)\ln\left(\frac{\widetilde{\sigma}^2_b}{\widehat{\sigma}^2_b}\right) + 2(T-1)\ln\left(\frac{|\widetilde{S}_{n,b}|}{1-\widetilde{\lambda}_b}\right)$$

Under the marginal null hypothesis H^b_0, LM_b and LR_b are asymptotically distributed as χ^2_1.

[1]Throughout, we use $\widetilde{}$ to denote unrestricted estimator, and $\widehat{}$ to denote restricted estimator. In the main text, we do not present the formulae for relevant quantities in the expression of test statistics for the sake of compactness, and they can be found in Appendix A. The subscripts "a", "b", "c", "d", "e", "f" stand for relevant statistics and quantities for cases a, b, c, d, e, f, respectively.

3.3. LM and LR Tests for H_0^c

On the other hand, if the researcher has information that no spatial lag dependence exists, and wants to test whether $\rho = 0$, then the marginal tests corresponding to H_0^c should be used. Under H_0^c, we assume there is no spatial error autocorrelation. We are testing the existence of spatial error autocorrelation based on the alternative model $Y_{nt}^{**} = X_{nt}^{**}\boldsymbol{\beta} + R_n^*(\rho)^{-1}V_{nt}^{**}$. The LM test statistic is

$$LM_c = \frac{1}{\widehat{b}_{3,c}^*}\widehat{z}_{\rho,c}^2$$

For the LR test, it is

$$LR_c = 2\left[\ln L_{n,T}(\widetilde{\boldsymbol{\theta}}_c) - \ln L_{n,T}(\widehat{\boldsymbol{\theta}}_c)\right]$$

Similarly, by exploring the first order condition of the likelihood function with respect to σ^2, we can write the LR test statistic as

$$LR_c = -(n-1)(T-1)\ln\left(\frac{\widetilde{\sigma}_c^2}{\widehat{\sigma}_c^2}\right) + 2(T-1)\ln\left(\frac{|\widetilde{R}_{n,c}|}{1-\widetilde{\rho}_c}\right)$$

Under the marginal null hypothesis H_0^c, LM_c and LR_c are asymptotically distributed as χ_1^2.

3.4. LM and LR Tests for H_0^d

In practice, the information of $\lambda = 0$ or $\rho = 0$ is seldom available. In order to test the existence of one parameter, researcher must take into account the estimate of the other parameter. For instance, to test if $\lambda = 0$, the case that $\rho \neq 0$ must be allowed. Under the conditional null hypothesis H_0^d, we assume there is no spatial lag dependence. We are testing the existence of spatial lag dependence based on the alternative model (2). The LM test statistic is

$$LM_d = \widehat{\kappa}_{3,d}\widehat{z}_{\lambda,d}^2$$

For the LR test, it is

$$LR_d = 2\left[\ln L_{n,T}(\widetilde{\boldsymbol{\theta}}_d) - \ln L_{n,T}(\widehat{\boldsymbol{\theta}}_d)\right]$$

Similarly, by exploring the first order condition of the likelihood function with respect to σ^2, we can rewrite the LR test statistic as follows

$$LR_d = -(n-1)(T-1)\ln\left(\frac{\widetilde{\sigma}_d^2}{\widehat{\sigma}_d^2}\right) + 2(T-1)\left[\ln\left(\frac{|\widetilde{S}_{n,d}\widetilde{R}_{n,d}|}{(1-\widetilde{\lambda}_d)(1-\widetilde{\rho}_d)}\right) - \ln\left(\frac{|\widehat{R}_{n,d}|}{1-\widehat{\rho}_d}\right)\right]$$

Under the conditional null hypothesis H_0^d, LM_d and LR_d are asymptotically distributed as χ_1^2.

3.5. LM and LR Tests for H_0^e

Similarly, in order to test the existence of spatial error autocorrelation, the case that $\lambda \neq 0$ must be allowed. Under the conditional null hypothesis H_0^e, we assume there is no spatial error autocorrelation. We are testing the existence of spatial error autocorrelation based on the alternative model (2). The LM test statistic is

$$LM_e = \widehat{\kappa}_{1,e}\widehat{z}^2_{\rho,e}$$

For the LR test, it is

$$LR_e = 2\left[\ln L_{n,T}(\widetilde{\boldsymbol{\theta}}_e) - \ln L_{n,T}(\widehat{\boldsymbol{\theta}}_e)\right]$$

Similarly, by exploring the first order condition of the likelihood function with respect to σ^2, we can write the LR test statistic as

$$LR_e = -(n-1)(T-1)\ln\left(\frac{\widetilde{\sigma}_e^2}{\widehat{\sigma}_e^2}\right) + 2(T-1)\left[\ln\left(\frac{|\widetilde{S}_{n,e}\widetilde{R}_{n,e}|}{(1-\widetilde{\lambda}_e)(1-\widetilde{\rho}_e)}\right) - \ln\left(\frac{|\widehat{S}_{n,e}|}{1-\widehat{\lambda}_e}\right)\right]$$

Under the conditional null hypothesis H_0^e, LM_e and LR_e are asymptotically distributed as χ_1^2.

3.6. LM and LR Tests for $\lambda = \rho$

Another hypothesis of interest in empirical research is to test if the spatial lag dependence effect and spatial error autocorrelation effect are of the same magnitude. Specifically, we want to test the null hypothesis $H_0^f : \lambda = \rho$. The alternative model is the full specification in (2). The LM test statistic is

$$LM_f = \widehat{\kappa}_{3,f}\widehat{z}^2_{\lambda,f} + \widehat{\kappa}_{1,f}\widehat{z}^2_{\rho,f} - 2\widehat{\kappa}_{2,f}\widehat{z}_{\lambda,f}\widehat{z}_{\rho,f}$$

For the LR test, it is

$$LR_f = 2\left[\ln L_{n,T}(\widetilde{\boldsymbol{\theta}}_f) - \ln L_{n,T}(\widehat{\boldsymbol{\theta}}_f)\right]$$

Similarly, by exploring the first order condition of the likelihood function with respect to σ^2 and making use of the fact that $\widehat{\lambda}_f = \widehat{\rho}_f$, we can write the LR test statistic as follows

$$LR_f = -(n-1)(T-1)\ln\left(\frac{\widetilde{\sigma}_f^2}{\widehat{\sigma}_f^2}\right) + 2(T-1)\left[\ln\left(\frac{|\widetilde{S}_{n,f}\widetilde{R}_{n,f}|}{(1-\widetilde{\lambda}_f)(1-\widetilde{\rho}_f)}\right) - \ln\left(\frac{|\widehat{S}_{n,f}\widehat{R}_{n,f}|}{(1-\widehat{\lambda}_f)^2}\right)\right]$$

Under the conditional null hypothesis H_0^f, LM_f and LR_f are asymptotically distributed as χ_1^2.

4. Monte Carlo Simulation

In this section, we conduct a small Monte Carlo simulation to evaluate the size and power performance of the proposed test statistics. The data generating process is

$$Y_{nt} = \lambda_0 W_n Y_{nt} + X_{nt1}\beta_{10} + X_{nt2}\beta_{20} + \boldsymbol{c}_{n0} + \alpha_{t0} l_n + (I_n - \rho_0 M_n)^{-1} V_{nt},\ t = 1, ..., T$$

where $V_{nt} = (v_{1t}, v_{2t}, ..., v_{nt})'$, $v_{it} \sim$ i.i.n.$(0, \sigma_0^2)$, with $\sigma_0^2 = 5$. For the individual effects, each element of $\boldsymbol{c}_{n0}$ is generated independently from a uniform distribution on $[-5, 5]$. For the time effects, $\alpha_{(t+1)0} = 1.05\alpha_{t0}, t = 1, ..., T-1$, α_{10} is generated from a uniform distribution on $[0, 10]$. Each element of X_{nt1} is generated independently from $N(0, 16)$, while each element of X_{nt2} is generated independently from a uniform distribution on $[0, 10]$. For the spatial weights matrices, W_n is the first order rook contiguity matrix, and M_n is the first order queen contiguity matrix. For the population parameter values, we set $\beta_{10} = 0.5$, $\beta_{20} = 0.7$, both λ_0 and ρ_0 take values from -0.8 to 0.8, with increment 0.2. For each combination of parameter values, two sample sizes are chosen, namely, $n = 49, t = 4$ and $n = 16, t = 40$, with 1000 repetitions performed. The nominal size is set to be 0.05.

< − − − − − − − Table 1 and 2 Approximately Here − − − − − − >

The simulation results of the joint LM and LR test statistics are reported in Table 1 and Table 2, with the empirical size highlighted in boldface. For the sake of compactness, we only report the results for $\lambda = 0, \pm 0.4, \pm 0.8$, the rest of the results are similar. For the $(49, 4)$ sample, the empirical size of LM_a is 0.049 and that of LR_a is 0.044. For the $(16, 40)$ sample, the empirical size of LM_a is 0.045 and that of LR_a is 0.042, these empirical sizes are all within the 95% confidence interval of the frequency of rejection (FR).[2] Since we are testing the joint presence of spatial lag dependence parameter λ and spatial error autocorrelation parameter ρ, then LM_a and LR_a are supposed to deviate from χ_2^2 distribution asymptotically as either λ or ρ deviates from 0, and this is confirmed from our simulation results. FR increases rapidly when either λ or $\rho = 0$ deviates from 0. For example, when $\lambda = 0.4$, $\rho = -0.2$, FRs of LM_a, LR_a are 0.957, 0.970 for the $(49, 4)$ sample. FRs are almost uniformly 1 for the $(16, 40)$ sample, exhibiting very good power performance. In practice, researcher should first conduct the joint tests to determine if spatial effects actually exist. If the joint null hypothesis $\lambda = \rho = 0$ is rejected, then certain econometric specification with spatial effects should be used.

< − − − − − − − Table 3 and 4 Approximately Here − − − − − >

The experiment results of the marginal LM and LR test statistics are in Table 3 and Table 4. For these two marginal hypotheses, we also present results based on the time dummy variables approach using the test statistics in Debarsy and Ertur (2010), so LM_b^{DE}, LR_b^{DE}, LM_c^{DE}, LR_c^{DE} stand for the test statistics using their formulae. In Table 3, we evaluate the performance of test statistics corresponding to H_0^b. For the $(49, 4)$ sample, the empirical sizes of $LM_b, LM_b^{DE}, LR_b, LR_b^{DE}$ are 0.063, 0.068, 0.062, 0.069, respectively. Although LM_b^{DE}, LR_b^{DE} are a little oversized, they are still reasonable. The powers of the four tests in this case are generally comparable. For the $(16, 40)$ sample, however, in which case T is larger than n, the empirical sizes of LM_b, LR_b are $0.046, 0.045$, while those of LM_b^{DE}, LR_b^{DE} are $0.303, 0.317$. Thus test statistics based on the time dummy variables approach suffer from size distortion, and this distortion is even more severe for LM_c^{DE} and

[2]Using normal approximation to binomial distribution of the frequency of rejection (FR), a 95% confidence interval of FR is $\left(-1.96\sqrt{\frac{0.05\times 0.95}{1000}} + 0.05, 1.96\sqrt{\frac{0.05\times 0.95}{1000}} + 0.05\right) \approx (0.036, 0.064)$.

LR_c^{DE} as given in Table 4. For the $(16, 40)$ sample, the empirical sizes of LM_c^{DE}, LR_c^{DE} are $0.770, 0.476$, which could be very misleading. Also, when ρ deviates from 0 to 0.2, there is an unreasonable decrease in the powers of LM_c^{DE} and LR_c^{DE}. Therefore, in practice, we suggest researchers to use the test statistics derived from the transformation approach as presented in this paper, instead of taking the time dummy variables approach.

< − − − − − −Table 5 and 6 Approximately Here − − − −− >

Simulation results of the conditional LM and LR test statistics are summarized in Table 5 and Table 6. To save space, for each conditional hypothesis, we only report the results for two values of the conditional parameters. In Table 5, when $\rho = -0.5$, the empirical sizes of LM_d, LR_d are $0.046, 0.067$ for the $(49, 4)$ sample, while they are $0.056, 0.062$ for the $(16, 40)$ sample. The results for $\rho = 0.5$ are similar. In Table 6, when $\lambda = -0.5$, the empirical sizes of LM_e, LR_e are $0.038, 0.045$ for the $(49, 4)$ sample, although LM_e is a little under sized, but 0.038 is still within the FR's 95% confidence interval. Similarly, the empirical sizes of LM_e, LR_e are $0.047, 0.046$ for the $(16, 40)$ sample. The results for $\lambda = 0.5$ are similar. Conditional tests are important in practice since rejection of joint hypothesis does not give the researcher direction about which spatial effects actually exist, while marginal tests requires additional information about the data generating process. For example, marginal test LM_b or LR_b tend to over reject the null hypothesis $\lambda = 0$ when $\rho \neq 0$, and the model selection could be misleading. In this case, the conditional tests LM_d or LR_d should be used instead.

< − − − − − −Table 7 and 8 Approximately Here − − − −− >

The simulation results for the null hypothesis H_0^f are summarized in Table 7 and Table 8, only results for the $(49, 7)$ sample is reported for the sake of compactness. Under the null hypothesis $H_0^f : \lambda = \rho$, both LM_f and LR_f are asymptotically distributed as χ_1^2. As a result, the main diagonals of Table 7 and Table 8 demonstrate the empirical sizes of LM_f and LR_f, respectively. Most of the empirical sizes fall into the interval $(0.036, 0.064)$, with only a few exceptions. For example, when $\lambda = \rho = -0.8$, LM_f is a little undersized, with FR to be 0.034. On the other hand, when $\lambda = \rho = -0.4$, LR_f is a little oversized, with FR to be 0.071. When λ and ρ deviate from each other, the power function increases rapidly. In both Table 7 and Table 8, we can see the pattern that FR is larger the further away from the main diagonals. Moreover, the frequency of rejection of LM_f and LR_f are very close to each other, demonstrating their asymptotic equivalence. FR for the $(16, 40)$ sample is not provided here to save space, the performances of test statistics are generally better than that for the $(49, 7)$ sample.

For a robustness check, we use an alternative way to generate the innovation error terms. We generate v_{it} from a mixed distribution, with probability 0.5 from a log-normal distribution, and with probability 0.5 from a Weibull distribution. Specifically,

$$v_{it} = r_{it} - \mathrm{E}[r_{it}], \quad r_{it} = (1 - W_{it})Z_{1,it} + W_{it}Z_{2,it}$$

where W_{it} is a Bernoulli random variable with success probability $p = 0.5$, $\log Z_{1,it} \sim N(-2, 4)$, $Z_{2,it}$ is generated from a Weibull distribution with scale parameter 5 and shape parameter 1. $W_{it}, Z_{1,it}, Z_{2,it}$ are independent of each other. r_{it} is demeaned by the population mean so that $E[v_{it}] = 0$. To save space, we only summarize the results corresponding to H_0^d and H_0^e in Table 9 and Table 10. Generally, the performances of LM_d, LR_d, LM_e, LR_e are satisfactory even under non-normal distribution of the innovation error terms, although the power curves climb a little slower than those in the normal error case.

5. An Example: Growth Convergence Revisited

5.1. Conditional Growth Hypothesis

In this section, we illustrate the application of our developed LM and LR test statistics by revisiting the growth convergence hypothesis. In the neoclassical growth theory (Solow, 1956), absolute convergence refers to the hypothesis that economies with the same preferences and technology have the same steady state income per capita, and each economy will reach the steady state level of income per capita in the long run. Besides absolute convergence, the concept of conditional convergence has also been proposed to control for the difference in the steady state per capita income. The convergence or a negative relation between the initial level of income per capita and growth rate has been found in several studies (Barro, 1991; Barro and Sala-i-Martin, 1992; Mankiw et al., 1992; Islam, 1995). Islam (1995) argue that analysis of growth convergence should use panel data to control for unobserved country specific heterogeneity, and they found higher rate of convergence than that in Mankiw et al. (1992). Ertur and Koch (2007) build a spatial augmented Solow model, they estimate cross section convergence model and find spatial dependence. Yu and Lee (2012) estimate a spatial dynamic panel data model and find even higher rate of convergence than that in Islam (1995). Here for the illustration purpose, we estimate the following growth convergence model for the 48 contiguous U.S. states,

$$\begin{aligned} &\ln \boldsymbol{y}_t - \ln \boldsymbol{y}_{t-1} = \lambda_0 W_n(\ln \boldsymbol{y}_t - \ln \boldsymbol{y}_{t-1}) + \gamma_0 \ln \boldsymbol{y}_{t-1} + \boldsymbol{c}_{n0} + \alpha_{t0} l_n + U_{nt} \\ &U_{nt} = \rho_0 M_n U_{nt} + V_{nt}, \quad t = 1, ..., T \end{aligned}$$

where $\ln \boldsymbol{y}_t = (\ln y_{1t}, \ln y_{2t}, ..., \ln y_{nt})'$, $\ln \boldsymbol{y}_0$ is a $n \times 1$ vector of initial income per capita. $\boldsymbol{c}_{n0}$ represents the state specific effects, which incorporate the unobserved heterogeneous initial technology level, saving rate, growth rate of population and technology, capital depreciation rate, among others. α_{t0} represents the time effects. From the conditional growth theory, $\gamma_0 = -(1 - e^{-\phi\tau})$, where ϕ is the annual rate of convergence, and τ is the length of time interval in years.

5.2. Data

We obtain annual data of nominal state per capita income (SPI) since 1930 from the Bureau of Economic Analysis (BEA). We obtain the all urban consumers price index (CPI-U) from the Bureau of Labor Statistics (BLS), and deflate income per capita by CPI-U. We decided to use the data from 1950 to 2010 to avoid the impact of great recession and World War II. Following Islam (1995), we use the five-year span to calculate growth rate since a short time span is inappropriate for two reasons. First, by using a longer time span, the innovation term V_{nt} will be less influenced by business cycle fluctuations and is less likely to be serially correlated than in a yearly data setup. Second, spatial spillovers might take years to happen so that we may not have enough variation in a short time period. Thus $\tau = 5$ in our analysis, and $\ln \boldsymbol{y}_0 = \ln \boldsymbol{y}_{1950}, \ln \boldsymbol{y}_1 = \ln \boldsymbol{y}_{1955}, \cdots, \ln \boldsymbol{y}_{12} = \ln \boldsymbol{y}_{2010}$. For the spatial weights matrices, we set $W_n = M_n$ and first use the inverse of great circle distance (with different cutoff distances) between state capitals. We also use the $q-$nearest neighbors weights matrices for robustness check.

5.3. Results and Discussion

The calculated LM and LR test statistics are summarized in Table 11. The first row of Table 11 shows the values of test statistics corresponding to H_0^a. As can be seen, no matter which weights matrix we use, the joint null hypothesis $\lambda = \rho = 0$ is always rejected, with right tail probability to be 0. Thus, given that the general model is the spatial panel model with both individual and time effects, either spatial lag dependence or spatial error autocorrelation must exist. Without further information, however, we cannot conclude which type of spatial effects exist. For marginal tests corresponding to H_0^b, testing results from using different weights matrices are consistent. For instance, $LM_b = 184.33$, $LR_b = 116.65$ in the first two columns. If we believe that there is no spatial error autocorrelation, i.e. $\rho = 0$, then these are strong evidences for $\lambda \neq 0$. Nevertheless, we cannot rely only on these marginal tests since the assumption $\rho = 0$ may not be true, in which case we need to refer to the conditional tests for H_0^d. The results corresponding to H_0^d are somewhat mixed. When the cutoff distance is 800 or 1000 miles, the test statistics are significant at the 10% level. For example, $LM_d = 5.49$, $LR_d = 3.13$ when the cutoff distance is 800 miles, with $p-$values to be 0.019 and 0.077, respectively. But when the cutoff distance is 600 miles or $q = 6, 8$ when we use the $q-$nearest neighbors weights matrices, the test statistics are small, leading to the acceptance of no spatial lag dependence. On the other hand, for testing $\rho = 0$, LM_c and LR_c are statistically significant, suggesting that spatial error autocorrelation exists if $\lambda = 0$. Similarly, we need to refer to the conditional tests. The values of test statistics for H_0^e are large regardless of the choice of spatial weights matrices. For example, $LM_e = 33.22$, $LR_e = 21.62$ when the cutoff distance is 1000 miles, so we reject the null hypothesis that $\rho = 0$ ($\lambda \neq 0$). Finally, we are interested in testing whether the two types of spatial effects are of the same magnitude, i.e. $\lambda = \rho$, and the results are somewhat inconclusive. In all five cases, LM_f rejects H_0^f at the 5% significance level, while LR_f do not reject H_0^f at the 10% significance level in the cases that cutoff distances are 800 or 1000 miles. This inconsistency might be due to finite sample error or different finite sample behavior of LM and LR tests.

< − − − − −Table 11 Approximately Here − − − −− >

Considering all of these tests, we conclude that in the post 1950 economic growth of U.S. states, there exist spatial error autocorrelation and weak spatial lag dependence. We thus estimate the full model as well as the model with restriction $\lambda = 0$ (i.e. the spatial error model). Estimation results are summarized in Table 12 and Table 13 for five choices of spatial weights matrices. In Table 12 for the full model, results are similar regardless of the choice of spatial weights matrices. For instance, when cutoff distance is 800 miles, the estimated spatial lag coefficient is 0.295. This suggests that the growth rate of one state is positively related to the growth rate of its neighboring states, although it is not statistically significant. Also, the spatial error component is positively correlated, one explanation might be the technological spillovers. The estimated coefficient of initial income per capita is negative, with value -0.313 when the cutoff distance is 800 miles, and the implied annual convergence rate is 0.075. Our estimates of annual convergence rate are similar to that in Yu and Lee (2012), and are larger than that in Islam (1995). The results for spatial error model are similar to those of the full model. However, by comparing log likelihood values of these two models, the spatial error model is preferred.

< − − − − − −Table 12 and 13 Approximately Here − − − − − − >

6. Conclusion

In this paper, we consider hypothesis testing in a SAR panel data model with individual effects, time effects and SAR disturbances, which is proposed in Lee and Yu (2010). We argue that in this model framework, although test can be performed by including time dummy variables and use the formulae in Debarsy and Ertur (2010), there is incidental parameter problem when T becomes large or comparable to n. Since the estimates for the parameters of dummy variables are inconsistent, test statistics based on the one-way fixed effects model will perform poorly. We then employ the transformation approach in Lee and Yu (2010) to swipe out both individual and time effects. We first derive LM and LR tests for the joint null hypothesis that there is neither spatial lag dependence nor spatial error autocorrelation. Secondly, two marginal LM and LR tests for the SAR variable and SAR disturbances are derived, respectively. We next derive two conditional LM and LR tests for SAR variable and SAR disturbances. Finally, we derive LM and LR test to detect whether the two types of spatial autoregressive effects are of the same magnitude.

We conduct Monte Carlo experiments to evaluate finite sample performances of the suggested test statistics. These tests exhibit very good finite sample empirical size and power. An empirical example is provided to illustrate application of these LM and LR test statistics. We use this two-way fixed effects spatial panel data model to study the growth convergence for the 48 contiguous U.S. states. We find positive spatial error autocorrelation, possibly due to spatial technological spillovers. We also find weakly positive spatial lag dependence, suggesting that the growth rate of one state is positively correlated to that of its neighboring states. The implied annual rate of convergence in our estimation is between 0.075 and 0.086, which is similar to that in Yu and Lee (2012) and larger than that in Islam (1995).

Appendix A

A.1. Summary for Notations

Notation Set 1

$$S_n(\lambda) = I_n - \lambda W_n, R_n(\rho) = I_n - \rho M_n, G_n(\lambda) = W_n S_n(\lambda)^{-1}, H_n(\rho) = M_n R_n(\rho)^{-1}$$
$$W_n^* = F'_{n,n-1} W_n F_{n,n-1}, M_n^* = F'_{n,n-1} M_n F_{n,n-1}, S_n^*(\lambda) = I_{n-1} - \lambda W_n^*$$
$$R_n^*(\rho) = I_{n-1} - \rho M_n^*, G_n^*(\lambda) = W_n^* S_n^*(\lambda)^{-1}, H_n^*(\rho) = M_n^* R_n^*(\rho)^{-1}$$

Notation Set 2

$V_{nt}^{**} = R_n^*(\rho)[S_n^*(\lambda) Y_{nt}^{**} - X_{nt}^{**}\boldsymbol{\beta}]$, for restricted estimation, let $\widehat{V}_{nt}^{**} = \widehat{R}_n^*[\widehat{S}_n^* Y_{nt}^{**} - X_{nt}^{**}\widehat{\boldsymbol{\beta}}]$, $\widehat{\sigma}^2 = \frac{\left[\sum_{t=1}^{T-1} \widehat{V}_{nt}^{**\prime} \widehat{V}_{nt}^{**}\right]}{(n-1)(T-1)}$, where $\widehat{R}_n^* = R_n^*(\widehat{\rho})$, $\widehat{S}_n^* = S_n^*(\widehat{\lambda})$. Similarly, for the unrestricted estimation, let $\widetilde{V}_{nt}^{**} = \widetilde{R}_n^*[\widetilde{S}_n^* Y_{nt}^{**} - X_{nt}^{**}\widetilde{\boldsymbol{\beta}}]$, $\widetilde{\sigma}^2 = \frac{\left[\sum_{t=1}^{T-1} \widetilde{V}_{nt}^{**\prime} \widetilde{V}_{nt}^{**}\right]}{(n-1)(T-1)}$, where $\widetilde{R}_n^* = R_n^*(\widetilde{\rho})$, $\widetilde{S}_n^* = S_n^*(\widetilde{\lambda})$.

Notation Set 3

$$z_\lambda = -(T-1)\mathrm{tr}[G_n^*(\lambda)] + \frac{1}{\sigma^2}\sum_{t=1}^{T-1}\left(V_{nt}^{**\prime} R_n^*(\rho) W_n^* Y_{nt}^{**}\right)$$

$$z_\rho = -(T-1)\mathrm{tr}[H_n^*(\rho)] + \frac{1}{\sigma^2}\sum_{t=1}^{t-1}\left(V_{nt}^{**\prime} M_n^* R_n^*(\rho)^{-1} V_{nt}^{**}\right)$$

$$\nu = \frac{1}{\sigma^2}\sum_{t=1}^{T-1}\left[R_n^*(\rho) G_n^*(\lambda) X_{nt}^{**}\boldsymbol{\beta}\right]' \left[R_n^*(\rho) G_n^*(\lambda) X_{nt}^{**}\boldsymbol{\beta}\right],\ \Lambda = \frac{1}{\sigma^2}\sum_{t=1}^{T-1}\left[R_n^*(\rho) G_n^*(\lambda) X_{nt}^{**}\boldsymbol{\beta}\right]' \left[R_n^*(\rho) X_{nt}^{**}\right]$$

$$\Delta = \frac{1}{\sigma^2}\sum_{t=1}^{T-1}[R_n^*(\rho) X_{nt}^{**}]'[R_n^*(\rho) X_{nt}^{**}],\ \omega = \nu - \Lambda\Delta^{-1}\Lambda'$$

and Let $\widehat{z}_\lambda, \widehat{z}_\rho, \widehat{\nu}, \widehat{\Lambda}, \widehat{\Delta}, \widehat{\omega}$ denote the restricted maximum likelihood estimators of corresponding quantities.

Notation Set 4

$$b_1 = (T-1)\mathrm{tr}\left[G_n^{*2}(\lambda)\right] + (T-1)\mathrm{tr}\left[(R_n^*(\rho) G_n^*(\lambda) R_n^*(\rho)^{-1})'(R_n^*(\rho) G_n^*(\lambda) R_n^*(\rho)^{-1})\right]$$
$$b_2 = (T-1)\mathrm{tr}[G_n^*(\lambda) H_n^*(\rho)] + (T-1)\mathrm{tr}\left[H_n^{*\prime}(\rho) R_n^*(\rho) G_n^*(\lambda) R_n^*(\rho)^{-1}\right]$$
$$b_3 = (T-1)\mathrm{tr}[H_n^*(\rho)(H_n^*(\rho) + H_n^{*\prime}(\rho))],\ b_1^* = b_1 - \frac{2(T-1)}{n-1}\mathrm{tr}^2[G_n^*(\lambda)]$$
$$b_2^* = b_2 - \frac{2(T-1)}{n-1}\mathrm{tr}[G_n^*(\lambda)]\mathrm{tr}[H_n^*(\rho)],\ b_3^* = b_3 - \frac{2(T-1)}{n-1}\mathrm{tr}^2[H_n^*(\rho)]$$
$$\kappa_1 = \frac{b_1^* + \omega}{b_1^* b_3^* - b_2^{*2} + b_3^*\omega},\ \kappa_2 = \frac{b_2^*}{b_1^* b_3^* - b_2^{*2} + b_3^*\omega},\ \kappa_3 = \frac{b_3^*}{b_1^* b_3^* - b_2^{*2} + b_3^*\omega}$$

and Let $\widehat{b}_1, \widehat{b}_2, \widehat{b}_3, \widehat{b}_1^*, \widehat{b}_2^*, \widehat{b}_3^*, \widehat{\kappa}_1, \widehat{\kappa}_2, \widehat{\kappa}_3$ denote the restricted maximum likelihood estimators of corresponding quantities.

A.2. Score Vector

Let the order be $\boldsymbol{\theta} = (\lambda, \rho, \boldsymbol{\beta'}, \sigma^2)'$, then

$$\frac{\partial \ln L_{n,T}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \left(\begin{array}{cccc} z_\lambda & z_\rho & \left[\frac{1}{\sigma^2}\sum_{t=1}^{T-1} X_{nt}^{**'} R_n^{*'}(\rho) V_{nt}^{**}\right] & \left[-\frac{(n-1)(T-1)}{2\sigma^2} + \frac{1}{2(\sigma^2)^2}\sum_{t=1}^{T-1} V_{nt}^{**'} V_{nt}^{**}\right] \end{array} \right)$$

A.3. Concentrated Log-likelihood Function

Let $\frac{\partial \ln L_{n,T}(\boldsymbol{\theta})}{\partial \boldsymbol{\beta}} = 0$, $\frac{\partial \ln L_{n,T}(\boldsymbol{\theta})}{\partial \sigma^2} = 0$, we get

$$\widehat{\boldsymbol{\beta}} = \left(\sum_{t=1}^{T-1} X_{nt}^{**'} \widehat{R}_n^{*'} \widehat{R}_n^* X_{nt}^{**}\right)^{-1} \left(\sum_{t=1}^{T-1} X_{nt}^{**'} \widehat{R}_n^{*'} \widehat{R}_n^* \widehat{S}_n^* Y_{nt}^{**}\right), \; \widehat{\sigma}^2 = \frac{\sum_{t=1}^{T-1} \widehat{V}_{nt}^{**'} \widehat{V}_{nt}^{**}}{(n-1)(T-1)}$$

plug it into the log-likelihood function, we obtain the concentrated log-likelihood function

$$\begin{aligned} \ln L_{n,T}^c(\boldsymbol{\theta}) = & -\frac{(n-1)(T-1)}{2}[\ln(2\pi)+1] + (T-1)[\ln|S_n(\lambda)| + \ln|R_n(\rho)|] \\ & -(T-1)[\ln(1-\lambda) + \ln(1-\rho)] - \frac{(n-1)(T-1)}{2}\ln(\widehat{\sigma}^2) \end{aligned}$$

A.4 Information Matrix

The information matrix is

$$\mathcal{I} = \begin{pmatrix} \mathcal{I}_{\lambda\lambda} & * & * & * \\ \mathcal{I}_{\rho\lambda} & \mathcal{I}_{\rho\rho} & * & * \\ \mathcal{I}_{\boldsymbol{\beta}\lambda} & \mathcal{I}_{\boldsymbol{\beta}\rho} & \mathcal{I}_{\boldsymbol{\beta}\boldsymbol{\beta}'} & * \\ \mathcal{I}_{\sigma^2\lambda} & \mathcal{I}_{\sigma^2\rho} & \mathcal{I}_{\sigma^2\boldsymbol{\beta}} & \mathcal{I}_{\sigma^2\sigma^2} \end{pmatrix} = \begin{pmatrix} b_1+\nu & * & * & * \\ b_2 & b_3 & * & * \\ \Lambda' & 0 & \Delta & * \\ \frac{T-1}{\sigma^2}\mathrm{tr}[G_n^*(\lambda)] & \frac{T-1}{\sigma^2}\mathrm{tr}[H_n^*(\rho)] & 0 & \frac{(n-1)(T-1)}{2(\sigma^2)^2} \end{pmatrix}$$

Appendix B

B.1 The Joint Test Statistic $\boldsymbol{LM_a}$

The joint null hypothesis we consider is $H_0^a : \lambda = \rho = 0$, and the restricted model is $Y_{nt}^{**} = X_{nt}^{**}\boldsymbol{\beta} + V_{nt}^{**}$. The restricted estimator is the standard OLS estimator, i.e., $\widehat{\boldsymbol{\beta}}_a = \left[\sum_{t=1}^{T-1} X_{nt}^{**'} X_{nt}^{**}\right]^{-1} \left[\sum_{t=1}^{T-1} X_{nt}^{**'} Y_{nt}^{**}\right]$. We are testing the existence of both spatial lag dependence and spatial error autocorrelation parameters in the full model (2). Under the row normalization assumption of W_n, M_n, we have $\mathrm{tr}(J_n W_n) = \mathrm{tr}(J_n M_n) = -1$. Using this fact, the score vector and information matrix under H_0^a and evaluated at the OLS estimator are

$$\frac{\partial \ln L_{n,T}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}|_{H_0^a} = \left(\begin{array}{cccc} \widehat{z}_{\lambda,a} & \widehat{z}_{\rho,a} & 0 & 0 \end{array} \right), \; \mathcal{I}_{|H_0^a} = \begin{pmatrix} \widehat{b}_{1,a}+\widehat{\nu}_a & * & * & * \\ \widehat{b}_{2,a} & \widehat{b}_{3,a} & * & * \\ \widehat{\Lambda}_a' & 0 & \widehat{\Delta}_a & * \\ \frac{1-T}{\widehat{\sigma}_a^2} & \frac{1-T}{\widehat{\sigma}_a^2} & 0 & \frac{(n-1)(T-1)}{2(\widehat{\sigma}_a^2)^2} \end{pmatrix}$$

Partition $\mathcal{I}_{|H_0^a}$ so that

$$\mathcal{I}_{|H_0^a} = \begin{pmatrix} \mathcal{I}_{11}^a & \mathcal{I}_{12}^a \\ \mathcal{I}_{21}^a & \mathcal{I}_{22}^a \end{pmatrix}_{|H_0^a} \text{ where } \mathcal{I}_{11|H_0^a}^a = \begin{pmatrix} \widehat{b}_{1,a} + \widehat{\nu}_a & \widehat{b}_{2,a} \\ \widehat{b}_{2,a} & \widehat{b}_{3,a} \end{pmatrix}$$

Then

$$(\mathcal{I}_{11}^a - \mathcal{I}_{12}^a \mathcal{I}_{22}^{a-1} \mathcal{I}_{21}^a)_{|H_0^a}^{-1} = \begin{pmatrix} \widehat{b}_{1,a}^* + \widehat{\omega}_a & \widehat{b}_{2,a}^* \\ \widehat{b}_{2,a}^* & \widehat{b}_{3,a}^* \end{pmatrix}^{-1} = \frac{1}{\widehat{b}_{1,a}^* \widehat{b}_{3,a}^* - \widehat{b}_{2,a}^{*2} + \widehat{b}_{3,a}^* \widehat{\omega}_a} \begin{pmatrix} \widehat{b}_{3,a}^* & -\widehat{b}_{2,a}^* \\ -\widehat{b}_{2,a}^* & \widehat{b}_{1,a}^* + \widehat{\omega}_a \end{pmatrix}$$

and the joint LM test statistic is

$$LM_a = \widehat{\kappa}_{3,a} \widehat{z}_{\lambda,a}^2 + \widehat{\kappa}_{1,a} \widehat{z}_{\rho,a}^2 - 2\widehat{\kappa}_{2,a} \widehat{z}_{\lambda,a} \widehat{z}_{\rho,a}$$

Under the joint null hypothesis H_0^a, $LM_a \xrightarrow{d} \chi_2^2$.

B.2 The Marginal Test Statistic LM_b

The first marginal hypothesis is $H_0^b : \lambda = 0$ $(\rho = 0)$, and the restricted model is $Y_{nt}^{**} = X_{nt}^{**}\boldsymbol{\beta} + V_{nt}^{**}$. Therefore, we are testing the existence of the spatial lag dependence parameter in the alternative model $S_n^*(\lambda) Y_{nt}^{**} = X_{nt}^{**}\boldsymbol{\beta} + V_{nt}^{**}$. The score vector and information matrix under H_0^b and evaluated at the OLS estimator are

$$\left.\frac{\partial \ln L_{n,T}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right|_{H_0^b} = \begin{pmatrix} \widehat{z}_{\lambda,b} & 0 & 0 \end{pmatrix}, \; \mathcal{I}_{|H_0^b}^b = \begin{pmatrix} \widehat{b}_{1,b} + \widehat{\nu}_b & * & * \\ \widehat{\Lambda}_b' & \widehat{\Delta}_b & * \\ \frac{1-T}{\widehat{\sigma}_b^2} & 0 & \frac{(n-1)(T-1)}{2(\widehat{\sigma}_b^2)^2} \end{pmatrix}$$

Partition $\mathcal{I}_{|H_0^b}^b$ so that

$$\mathcal{I}_{|H_0^b}^b = \begin{pmatrix} \mathcal{I}_{11}^b & \mathcal{I}_{12}^b \\ \mathcal{I}_{21}^b & \mathcal{I}_{22}^b \end{pmatrix}_{|H_0^b}, \quad \text{where } \mathcal{I}_{11|H_0^b}^b = \widehat{b}_{1,b} + \widehat{\nu}_b$$

Then $(\mathcal{I}_{11}^b - \mathcal{I}_{12}^b \mathcal{I}_{22}^{b-1} \mathcal{I}_{21}^b)_{|H_0^b}^{-1} = \frac{1}{\widehat{b}_{1,b}^* + \widehat{\omega}_b}$, and the LM test statistic is

$$LM_b = \frac{1}{\widehat{b}_{1,b}^* + \widehat{\omega}_b} \widehat{z}_{\lambda,b}^2$$

Under the marginal null hypothesis H_0^b, $LM_b \xrightarrow{d} \chi_1^2$.

B.3 The Marginal Test Statistic LM_c

The second marginal hypothesis is $H_0^c: \rho = 0$ ($\lambda = 0$), and the restricted model is $Y_{nt}^{**} = X_{nt}^{**}\boldsymbol{\beta} + V_{nt}^{**}$. Therefore, we are testing the existence of the spatial error autocorrelation parameter in the alternative model $Y_{nt}^{**} = X_{nt}^{**}\boldsymbol{\beta} + R_n^*(\rho)^{-1}V_{nt}^{**}$. The score vector and information matrix under H_0^c and evaluated at the OLS estimator are

$$\frac{\partial \ln L_{n,T}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}_{|H_0^c} = \begin{pmatrix} \widehat{z}_{\rho,c} & 0 & 0 \end{pmatrix}, \ \mathcal{I}_{|H_0^c}^c = \begin{pmatrix} \widehat{b}_{3,c} & * & * \\ 0 & \widehat{\Delta}_c & * \\ \frac{1-T}{\widehat{\sigma}_c^2} & 0 & \frac{(n-1)(T-1)}{2(\widehat{\sigma}_c^2)^2} \end{pmatrix}$$

Partition $\mathcal{I}_{|H_0^c}^c$ so that

$$\mathcal{I}_{|H_0^c}^c = \begin{pmatrix} \mathcal{I}_{11}^c & \mathcal{I}_{12}^c \\ \mathcal{I}_{21}^c & \mathcal{I}_{22}^c \end{pmatrix}_{|H_0^c}, \text{ where } \mathcal{I}_{11|H_0^c}^c = \widehat{b}_{3,c}$$

Then $(\mathcal{I}_{11}^c - \mathcal{I}_{12}^c \mathcal{I}_{22}^{c-1} \mathcal{I}_{21}^c)_{|H_0^c}^{-1} = \frac{1}{\widehat{b}_{3,c}^*}$, and the LM test statistic is

$$LM_c = \frac{1}{\widehat{b}_{3,c}^*} \widehat{z}_{\rho,c}^2$$

B.4 The Conditional Test Statistic LM_d

The first conditional hypothesis is $H_0^d: \lambda = 0$, allowing for the possiblity that $\rho \neq 0$. The restricted model is $Y_{nt}^{**} = X_{nt}^{**}\boldsymbol{\beta} + R_n^*(\rho)^{-1}V_{nt}^{**}$. Therefore, we are testing the existence of the spatial lag dependence parameter in the full model (2). The score vector and information matrix under H_0^d and evaluated at the maximum likelihood estimator are

$$\frac{\partial \ln L_{n,T}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}_{|H_0^d} = \begin{pmatrix} \widehat{z}_{\lambda,d} & 0 & 0 & 0 \end{pmatrix}', \ \mathcal{I}_{|H_0^d} = \begin{pmatrix} \widehat{b}_{1,d} + \widehat{\nu}_d & * & * & * \\ \widehat{b}_{2,d} & \widehat{b}_{3,d} & * & * \\ \widehat{\Lambda}_d' & 0 & \widehat{\Delta}_d & * \\ \frac{1-T}{\widehat{\sigma}_d^2} & \frac{T-1}{\widehat{\sigma}_d^2}\mathrm{tr}(\widehat{H}_{n,d}^*) & 0 & \frac{(n-1)(T-1)}{2(\widehat{\sigma}_d^2)^2} \end{pmatrix}$$

Partition $\mathcal{I}_{|H_0^d}$ so that

$$\mathcal{I}_{|H_0^d} = \begin{pmatrix} \mathcal{I}_{11}^d & \mathcal{I}_{12}^d \\ \mathcal{I}_{21}^d & \mathcal{I}_{22}^d \end{pmatrix}_{|H_0^d}, \quad \text{where } \mathcal{I}_{11|H_0^d}^d = \begin{pmatrix} \widehat{b}_{1,d} + \widehat{\nu}_d & * \\ \widehat{b}_{2,d} & \widehat{b}_{3,d} \end{pmatrix}$$

Then

$$(\mathcal{I}_{11}^d - \mathcal{I}_{12}^d \mathcal{I}_{22}^{d-1} \mathcal{I}_{21}^d)_{|H_0^d}^{-1} = \begin{pmatrix} \widehat{b}_{1,d}^* + \widehat{\omega}_d & \widehat{b}_{2,d}^* \\ \widehat{b}_{2,d}^* & \widehat{b}_{3,d}^* \end{pmatrix}^{-1} = \frac{1}{\widehat{b}_{1,d}^*\widehat{b}_{3,d}^* - \widehat{b}_{2,d}^{*2} + \widehat{b}_{3,d}^*\widehat{\omega}_d} \begin{pmatrix} \widehat{b}_{3,d}^* & -\widehat{b}_{2,d}^* \\ -\widehat{b}_{2,d}^* & \widehat{b}_{1,d}^* + \widehat{\omega}_d \end{pmatrix}$$

and the LM test statistic is

$$LM_d = \widehat{\kappa}_{3,d} \widehat{z}_{\lambda,d}^2$$

Under the conditional null hypothesis H_0^d, $LM_d \xrightarrow{d} \chi_1^2$.

B.5 The Conditional Test Statistic $\boldsymbol{LM_e}$

The second conditional hypothesis is $H_0^e : \rho = 0$, allowing for the possibility that $\lambda \neq 0$. The restricted model is $S_n^*(\lambda)Y_{nt}^{**} = X_{nt}^{**}\boldsymbol{\beta} + V_{nt}^{**}$. Therefore, we are testing the existence of the spatial error autocorrelation parameter in the full model (2). The score vector and information matrix under H_0^e and evaluated at the maximum likelihood estimator are

$$\frac{\partial \ln L_{n,T}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}_{|H_0^e} = \begin{pmatrix} 0 & \widehat{z}_{\rho,e} & 0 & 0 \end{pmatrix}', \ \mathcal{I}_{|H_0^e} = \begin{pmatrix} \widehat{b}_{1,e} + \widehat{\nu}_e & * & * & * \\ \widehat{b}_{2,e} & \widehat{b}_{3,e} & * & * \\ \widehat{\Lambda}_e' & 0 & \widehat{\Delta}_e & * \\ \frac{T-1}{\widehat{\sigma}_e^2}\mathrm{tr}(\widehat{G}_{n,e}^*) & \frac{1-T}{\widehat{\sigma}_e^2} & 0 & \frac{(n-1)(T-1)}{2(\widehat{\sigma}_e^2)^2} \end{pmatrix}$$

Partition $\mathcal{I}_{|H_0^e}$ so that

$$\mathcal{I}_{|H_0^e} = \begin{pmatrix} \mathcal{I}_{11}^e & \mathcal{I}_{12}^e \\ \mathcal{I}_{21}^e & \mathcal{I}_{22}^e \end{pmatrix}_{|H_0^e}, \quad \text{where } \mathcal{I}_{11|H_0^b}^e = \begin{pmatrix} \widehat{b}_{1,e} + \widehat{\nu}_e & * \\ \widehat{b}_{2,e} & \widehat{b}_{3,e} \end{pmatrix}$$

Then

$$(\mathcal{I}_{11}^e - \mathcal{I}_{12}^e \mathcal{I}_{22}^{e-1} \mathcal{I}_{21}^e)_{|H_0^e}^{-1} = \begin{pmatrix} \widehat{b}_{1,e}^* + \widehat{\omega}_e & \widehat{b}_{2,e}^* \\ \widehat{b}_{2,e}^* & \widehat{b}_{3,e}^* \end{pmatrix}^{-1} = \frac{1}{\widehat{b}_{1,e}^*\widehat{b}_{3,e}^* - \widehat{b}_{2,e}^{*2} + \widehat{b}_{3,e}^*\widehat{\omega}_e} \begin{pmatrix} \widehat{b}_{3,e}^* & -\widehat{b}_{2,e}^* \\ -\widehat{b}_{2,e}^* & \widehat{b}_{1,e}^* + \widehat{\omega}_e \end{pmatrix}$$

and the LM test statistic is

$$LM_e = \widehat{\kappa}_{1,e}\widehat{z}_{\rho,e}^2$$

Under the conditional null hypothesis H_0^e, $LM_e \xrightarrow{d} \chi_1^2$.

B.6 The Equality Test $\boldsymbol{LM_f}$

The null hypothesis is $H_0^f : \lambda = \rho$. Using λ to denote the common value of λ and ρ, then the restricted model is $S_n^*(\lambda)Y_{nt}^{**} = X_{nt}^{**}\boldsymbol{\beta} + R_n^*(\lambda)^{-1}V_{nt}^{**}$. Therefore, we are testing if the two types of spatial effects are of the same magnitude in the full specification (2). The score vector and information matrix under H_0^f and evaluated at the maximum likelihood estimators are

$$\frac{\partial \ln L_{n,T}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}_{|H_0^f} = \begin{pmatrix} \widehat{z}_{\lambda,f} & \widehat{z}_{\rho,f} & 0 & 0 \end{pmatrix}, \ \mathcal{I}_{|H_0^f} = \begin{pmatrix} \widehat{b}_{1,f} + \widehat{\nu}_f & * & * & * \\ \widehat{b}_{2,f} & \widehat{b}_{3,f} & * & * \\ \widehat{\Lambda}_f' & 0 & \widehat{\Delta}_f & * \\ \frac{T-1}{\widehat{\sigma}_f^2}\mathrm{tr}(\widehat{G}_{n,f}^*) & \frac{T-1}{\widehat{\sigma}_f^2}\mathrm{tr}(\widehat{H}_{n,f}^*) & 0 & \frac{(n-1)(T-1)}{2(\widehat{\sigma}_f^2)^2} \end{pmatrix}$$

Partition $\mathcal{I}_{|H_0^f}$ so that

$$\mathcal{I}_{|H_0^f} = \begin{pmatrix} \mathcal{I}_{11}^f & \mathcal{I}_{12}^f \\ \mathcal{I}_{21}^f & \mathcal{I}_{22}^f \end{pmatrix}_{|H_0^f}, \text{ where } \mathcal{I}_{11|H_0^f}^f = \begin{pmatrix} \widehat{b}_{1,f} + \widehat{\nu}_f & \widehat{b}_{2,f} \\ \widehat{b}_{2,f} & \widehat{b}_{3,f} \end{pmatrix}$$

Then

$$(\mathcal{I}_{11}^f - \mathcal{I}_{12}^f \mathcal{I}_{22}^{f-1} \mathcal{I}_{21}^f)^{-1}_{|H_0^f} = \begin{pmatrix} \widehat{b}_{1,f}^* + \widehat{\omega}_f & \widehat{b}_{2,f}^* \\ \widehat{b}_{2,f}^* & \widehat{b}_{3,f}^* \end{pmatrix}^{-1} = \frac{1}{\widehat{b}_{1,f}^* \widehat{b}_{3,f}^* - \widehat{b}_{2,f}^{*2} + \widehat{b}_{3,f}^* \widehat{\omega}_f} \begin{pmatrix} \widehat{b}_{3,f}^* & -\widehat{b}_{2,f}^* \\ -\widehat{b}_{2,f}^* & \widehat{b}_{1,f}^* + \widehat{\omega}_f \end{pmatrix}$$

and the LM test is

$$LM_f = \widehat{\kappa}_{3,f} \widehat{z}_{\lambda,f}^2 + \widehat{\kappa}_{1,f} \widehat{z}_{\rho,f}^2 - 2\widehat{\kappa}_{2,f} \widehat{z}_{\lambda,f} \widehat{z}_{\rho,f}$$

Under the null hypothesis H_0^f, $LM_f \xrightarrow{d} \chi_1^2$.

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Tables

Table 1: Performance of the Joint Tests, Sample Size $(49, 4)$, Normal Error

λ	-0.8		-0.4		0		0.4		0.8	
ρ	LM_a	LR_a	LM_a	LR_a	LM_a	LR_a	LM_a	LR_a	LM_a	LR_a
-0.8	1.000	1.000	1.000	1.000	0.981	0.994	0.978	0.989	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	0.856	0.923	0.942	0.966	1.000	1.000
-0.4	1.000	1.000	1.000	1.000	0.438	0.545	0.924	0.953	1.000	1.000
-0.2	1.000	1.000	1.000	1.000	0.146	0.211	0.957	0.970	1.000	1.000
0	1.000	1.000	0.999	0.999	**0.049**	**0.044**	0.996	0.996	1.000	1.000
0.2	1.000	1.000	0.989	0.989	0.269	0.235	0.999	0.999	1.000	1.000
0.4	1.000	1.000	0.980	0.980	0.789	0.752	1.000	1.000	1.000	1.000
0.6	1.000	1.000	0.989	0.991	0.991	0.989	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 2: Performance of the Joint Tests, Sample Size $(16, 40)$, Normal Error

ρ	-0.8		-0.4		0		0.4		0.8	
λ	LM_a	LR_a	LM_a	LR_a	LM_a	LR_a	LM_a	LR_a	LM_a	LR_a
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.4	1.000	1.000	1.000	1.000	0.994	0.995	1.000	1.000	1.000	1.000
-0.2	1.000	1.000	1.000	1.000	0.556	0.592	1.000	1.000	1.000	1.000
0	1.000	1.000	1.000	1.000	**0.045**	**0.042**	1.000	1.000	1.000	1.000
0.2	1.000	1.000	1.000	1.000	0.648	0.617	1.000	1.000	1.000	1.000
0.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 3: Performance of the Marginal Tests for H_0^b, Normal Error

$\rho = 0$	$(49, 4)$				$(16, 40)$			
λ	LM_b	LM_b^{DE}	LR_b	LR_b^{DE}	LM_b	LM_b^{DE}	LR_b	LR_b^{DE}
-0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-0.4	0.997	1.000	0.998	1.000	1.000	1.000	1.000	1.000
-0.2	0.657	0.752	0.658	0.765	0.999	1.000	0.999	1.000
0.0	**0.063**	**0.068**	**0.062**	**0.069**	**0.046**	**0.303**	**0.045**	**0.317**
0.2	0.645	0.533	0.656	0.555	1.000	0.931	1.000	0.941
0.4	0.997	0.995	0.997	0.995	1.000	1.000	1.000	1.000
0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 4: Performance of the Marginal Tests for H_0^c, Normal Error

$\lambda = 0$	$(49,4)$				$(16,40)$			
ρ	LM_c	LM_c^{DE}	LR_c	LR_c^{DE}	LM_c	LM_c^{DE}	LR_c	LR_c^{DE}
−0.8	0.997	0.999	0.998	0.950	1.000	1.000	1.000	1.000
−0.6	0.929	0.965	0.950	0.787	1.000	1.000	1.000	1.000
−0.4	0.576	0.750	0.663	0.437	0.997	1.000	0.998	0.999
−0.2	0.182	0.331	0.241	0.133	0.671	1.000	0.694	0.978
0.0	**0.053**	**0.051**	**0.044**	**0.025**	**0.046**	**0.770**	**0.049**	**0.476**
0.2	0.322	0.172	0.288	0.073	0.752	0.057	0.736	0.021
0.4	0.810	0.715	0.796	0.525	0.999	0.733	0.999	0.567
0.6	0.987	0.974	0.987	0.923	1.000	1.000	1.000	0.996
0.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5: Performance of the Conditional Tests for H_0^d, Normal Error

$\rho = -0.5$	$(49,4)$		$(16,40)$		$\rho = 0.5$	$(49,4)$		$(16,40)$	
λ	LM_d	LR_d	LM_d	LR_d	λ	LM_d	LR_d	LM_d	LR_d
−0.8	1.000	1.000	1.000	1.000	−0.8	1.000	1.000	1.000	1.000
−0.6	1.000	1.000	1.000	1.000	−0.6	1.000	1.000	1.000	1.000
−0.4	0.995	0.996	1.000	1.000	−0.4	0.992	0.989	1.000	1.000
−0.2	0.595	0.656	0.996	0.997	−0.2	0.591	0.555	0.993	0.994
0.0	**0.046**	**0.067**	**0.056**	**0.062**	0.0	**0.064**	**0.069**	**0.051**	**0.062**
0.2	0.476	0.515	0.989	0.990	0.2	0.430	0.504	0.982	0.989
0.4	0.953	0.975	1.000	1.000	0.4	0.943	0.969	1.000	1.000
0.6	1.000	1.000	1.000	1.000	0.6	0.993	1.000	1.000	1.000
0.8	1.000	1.000	1.000	1.000	0.8	0.999	1.000	1.000	1.000

Table 6: Performance of the Conditional Tests for H_0^e, Normal Error

$\lambda = -0.5$	$(49,4)$		$(16,40)$		$\lambda = 0.5$	$(49,4)$		$(16,40)$	
ρ	LM_e	LR_e	LM_e	LR_e	ρ	LM_e	LR_e	LM_e	LR_e
−0.8	0.994	0.996	1.000	1.000	−0.8	0.956	0.975	1.000	1.000
−0.6	0.905	0.934	1.000	1.000	−0.6	0.798	0.837	1.000	1.000
−0.4	0.544	0.619	0.999	0.999	−0.4	0.438	0.471	0.975	0.980
−0.2	0.198	0.218	0.613	0.633	−0.2	0.145	0.172	0.496	0.522
0.0	**0.038**	**0.045**	**0.047**	**0.046**	0.0	**0.045**	**0.051**	**0.057**	**0.054**
0.2	0.257	0.243	0.666	0.656	0.2	0.172	0.156	0.511	0.504
0.4	0.728	0.713	0.997	0.998	0.4	0.496	0.512	0.983	0.983
0.6	0.968	0.968	1.000	1.000	0.6	0.783	0.819	1.000	1.000
0.8	1.000	1.000	1.000	1.000	0.8	0.923	0.976	1.000	1.000

Table 7: Performance of the Test LM_f, Sample Size $(49, 4)$, Normal Error

λ	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
ρ					LM_f				
-0.8	**0.034**	0.180	0.503	0.709	0.871	0.944	0.990	0.995	0.987
-0.6	0.156	**0.031**	0.159	0.401	0.645	0.823	0.938	0.972	0.968
-0.4	0.534	0.139	**0.062**	0.146	0.321	0.570	0.781	0.883	0.930
-0.2	0.834	0.434	0.126	**0.051**	0.137	0.286	0.526	0.723	0.826
0	0.982	0.785	0.404	0.135	**0.047**	0.099	0.288	0.491	0.642
0.2	0.998	0.957	0.740	0.373	0.106	**0.045**	0.124	0.242	0.445
0.4	1.000	0.997	0.931	0.698	0.357	0.114	**0.054**	0.105	0.280
0.6	1.000	1.000	0.991	0.927	0.702	0.354	0.112	**0.041**	0.101
0.8	1.000	1.000	0.999	0.990	0.933	0.738	0.371	0.123	**0.041**

Table 8: Performance of the Test LR_f, Sample Size $(49, 4)$, Normal Error

λ	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
ρ					LR_f				
-0.8	**0.042**	0.183	0.514	0.742	0.899	0.960	0.995	1.000	1.000
-0.6	0.164	**0.038**	0.170	0.423	0.676	0.851	0.961	0.990	0.994
-0.4	0.564	0.145	**0.071**	0.155	0.343	0.617	0.837	0.935	0.976
-0.2	0.857	0.453	0.127	**0.057**	0.157	0.315	0.572	0.786	0.912
0	0.988	0.803	0.403	0.126	**0.047**	0.113	0.320	0.545	0.763
0.2	0.999	0.964	0.750	0.380	0.109	**0.046**	0.143	0.285	0.523
0.4	1.000	0.997	0.935	0.710	0.356	0.099	**0.050**	0.115	0.312
0.6	1.000	1.000	0.994	0.935	0.713	0.352	0.116	**0.053**	0.124
0.8	1.000	1.000	0.999	0.994	0.944	0.744	0.381	0.116	**0.038**

Table 9: Performance of the Conditional Tests for H_0^d, Mixed Error

$\rho = -0.5$	$(49, 4)$		$(16, 40)$		$\rho = 0.5$	$(49, 4)$		$(16, 40)$	
λ	LM_d	LR_d	LM_d	LR_d	λ	LM_d	LR_d	LM_d	LR_d
-0.8	1.000	1.000	1.000	1.000	-0.8	1.000	1.000	1.000	1.000
-0.6	0.999	1.000	1.000	1.000	-0.6	0.998	1.000	1.000	1.000
-0.4	0.932	0.958	1.000	1.000	-0.4	0.918	0.916	1.000	1.000
-0.2	0.400	0.451	0.906	0.940	-0.2	0.350	0.317	0.903	0.904
0.0	**0.041**	**0.052**	**0.042**	**0.054**	0.0	**0.053**	**0.063**	**0.053**	**0.067**
0.2	0.261	0.311	0.806	0.831	0.2	0.220	0.325	0.755	0.829
0.4	0.686	0.775	0.990	0.999	0.4	0.694	0.843	0.987	0.994
0.6	0.896	0.963	0.991	0.999	0.6	0.923	0.989	0.998	1.000
0.8	0.965	0.998	0.996	1.000	0.8	0.885	0.993	1.000	1.000

Table 10: Performance of the Conditional Tests for H_0^e, Mixed Error

$\lambda=-0.5$	(49, 4)		(16, 40)		$\lambda=0.5$	(49, 4)		(16, 40)	
ρ	LM_e	LR_e	LM_e	LR_e	ρ	LM_e	LR_e	LM_e	LR_e
-0.8	0.989	0.993	1.000	1.000	-0.8	0.884	0.923	0.997	0.999
-0.6	0.877	0.912	0.999	1.000	-0.6	0.677	0.725	0.978	0.990
-0.4	0.565	0.624	0.974	0.983	-0.4	0.343	0.370	0.891	0.911
-0.2	0.191	0.211	0.577	0.598	-0.2	0.119	0.128	0.378	0.380
0.0	**0.034**	**0.036**	**0.042**	**0.046**	0.0	**0.041**	**0.053**	**0.049**	**0.058**
0.2	0.197	0.190	0.635	0.639	0.2	0.088	0.106	0.300	0.309
0.4	0.666	0.673	0.997	0.997	0.4	0.270	0.310	0.826	0.850
0.6	0.967	0.975	1.000	1.000	0.6	0.491	0.600	0.972	0.987
0.8	1.000	1.000	1.000	1.000	0.8	0.582	0.826	0.992	1.000

Table 11: Testing for Spatial Effects (Five−Year Span, 1950 – 2010)

	600 miles		800 miles		1000 miles		$q=6$		$q=8$	
	LM	LR	LM	LR	LM	LR	LM	LR	LM	LR
H_0^a	249.69	138.41	219.32	115.29	211.61	107.96	216.00	138.85	234.92	132.76
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_0^b	184.33	116.65	158.38	95.54	148.22	86.35	159.44	109.52	169.81	103.72
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_0^c	249.38	138.27	219.04	112.16	211.18	103.54	214.33	138.83	233.74	132.26
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_0^d	0.17	0.14	5.49	3.13	8.68	4.42	0.02	0.02	0.81	0.50
	(0.679)	(0.713)	(0.019)	(0.077)	(0.003)	(0.036)	(0.884)	(0.902)	(0.369)	(0.478)
H_0^e	16.40	21.76	26.20	19.76	33.22	21.62	27.28	29.32	35.79	29.04
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_0^f	6.10	5.62	4.23	2.02	4.88	1.88	12.02	8.80	12.54	7.20
	(0.014)	(0.018)	(0.040)	(0.155)	(0.027)	(0.171)	(0.001)	(0.003)	(0.000)	(0.007)

Table 12: Unrestricted Estimation for Growth Convergence among U.S. States, 1950 – 2010

Estimates	600 miles	800 miles	1000 miles	$q=6$	$q=8$
λ	0.060	0.295	0.363	0.016	0.099
	(0.185)	(0.234)	(0.260)	(0.160)	(0.179)
ρ	0.724***	0.669***	0.724***	0.668***	0.695***
	(0.101)	(0.168)	(0.190)	(0.090)	(0.104)
γ	−0.320***	−0.313***	−0.315***	−0.349***	−0.342***
	(0.029)	(0.030)	(0.030)	(0.030)	(0.031)
Log-likelihood	9.685×10^2	9.570×10^2	9.533×10^2	9.688×10^2	9.657×10^2
Implied ϕ	0.077	0.075	0.076	0.086	0.084

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 13: Restricted Estimation ($\lambda = 0$) for Growth Convergence among U.S. States, 1950 − 2010

Estimates	600 miles	800 miles	1000 miles	$q = 6$	$q = 8$
ρ	0.752***	0.803***	0.873***	0.676***	0.737***
	(0.048)	(0.054)	(0.060)	(0.044)	(0.047)
γ	−0.320***	−0.319***	−0.322***	−0.349***	−0.349***
	(0.029)	(0.029)	(0.030)	(0.030)	(0.031)
Log-likelihood	9.685×10^2	9.554×10^2	9.511×10^2	9.688×10^2	9.655×10^2
Implied ϕ	0.077	0.077	0.078	0.086	0.085

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$