Finite sample properties of pre-test estimators of spatial models

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July 1, 2013

Preliminary. Not to be quoted without permission of the authors.

Abstract

This paper examines the properties of pre-test strategies for a linear Cliff-Ord-type spatial model. We compare the performance of the ML estimator of the general model that allows for spatial spill-overs in the endogenous variables and disturbances, with three pre-test estimators based on Lagrange Multiplier tests and introduced in Florax *et al.* (2003), in the context of a Monte Carlo study. Our results show that the ML estimates based on the full model have better properties, and the size of hypothesis test is reasonably close to the nominal values. On the other hand we find that, even in a very simple setting, the bias of the estimates generated by a pre-testing strategy can be very large in some cases and the empirical size of tests can differ substantially from the nominal size.

1 Introduction

The recent literature on spatial econometrics has been concerned with model specification issues both in cross sectional as well as panel data analysis.[1] In the present paper we will focus on cross sectional models. Empirical work is often based on estimation strategies which entails estimating a linear model,

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[1]See e.g., Cliff & Ord (1972, 1973, 1981); Florax & Folmer (1992); Anselin *et al.* (1996); Anselin (1988a); Florax *et al.* (2003); Baltagi *et al.* (2007, 2003); Baltagi & Liu (2008); Baltagi *et al.* (2009, 2008); Burridge (1980), among many others.

followed by testing for dependences between spatial units, and re-estimation if spatial dependence cannot be rejected. It is well know that such pre-test strategies may potentially introduce bias for both the parameter estimates and corresponding standard errors; see, e.g., Leeb & Poetscher (2008). Of course, on the other hand, efficiency may be lost when estimating a more general model when it is not necessary. The purpose of this paper is to explore the implications of some common pre-test strategies.

In estimating Cliff-Ord-type models two forms of these dependences are usually considered in applied work corresponding to two different model specifications. The first form arises when the value of the dependent variable corresponding to each cross-sectional unit is jointly determined with the values at all other neighboring cross-sectional units. This is achieved through the inclusion of a weighted average of the dependent variable that is often described in the literature as a spatial lag. Consequently, the model that derives from it is referred to as a spatially autoregressive model (or, simply, spatial lag model, see, e.g., Anselin, 1988b). The second form of spatial dependence relates to the error term and specifies a spatial auto regressive process for the disturbances. Correspondingly, the model that derives from it is referred to as a spatial error model (see, e.g., Anselin, 1988b)

Burridge (1980) and Anselin *et al.* (1996) propose simple diagnostic tests, based on ordinary least squares (OLS) residuals, for spatial error autocorrelation or spatial lag dependence. Florax *et al.* (2003) suggest a simple selection criterion conditional upon the results of these specification tests. It should be noted that this testing strategy only leads to the estimation of either the spatial lag or the spatial error models and never of a model that contains both error and lag dependences (i.e. a full model). This is unfortunate because the spatial patterns implied by the full model are richer than those implied by either the spatial error or the spatial lag model (Kelejian & Prucha, 1998). As a consequence of this, such a pre-testing strategy may result in a situation where the asymptotic distribution of the estimator (derived without taking into account the pre-testing strategy) provides a bad approximation to the actual small sample distribution of the final stage estimator.

This paper examines the consequences of model misspecification in a linear Cliff-Ord-type spatial model. We compare the performance of the ML estimator of the general model that allows for spatial spill-overs in the endogenous variables and disturbances, with three pre-test estimators based on Lagrange Multiplier tests in the context of a Monte Carlo study. Our results show that ML estimates based on the full model are consistent, and the size of hypothesis tests is reasonably close to the nominal values. On the other hand, we find that even in simple settings the bias of the estimates generated by a pre-testing strategy can be very large and the empirical size of tests can differ substantially from the nominal size.

Section 2 briefly describes the models employed in the paper and presents the corresponding likelihoods over which our estimators are based. Section 3 introduces the LM tests while Section 4 derives the three pre-test estimators based on those LM tests. Section 5 describes the design of our Monte Carlo experiment and discusses the main evidence. Section 6 concludes.

2 The models

As we mentioned in the introduction, much of the empirical spatial econometrics literature has focused on the estimation of two alternative models relating to different forms of spatial dependence. In one case, spatial dependence is introduced via the disturbance process, where the disturbance term corresponding to one location is assumed to be jointly determined with those at other locations. In the other case, the dependent variable at one location is assumed to be jointly determined by its values at other locations. From an empirical perspective, each one of these two forms of dependence translates into two different Cliff-Ord-type spatial models. The model corresponding to the first case is known in the literature as spatial error model; while the model corresponding to the second one is known as the spatial lag model (Cliff & Ord, 1973; Anselin, 1988b). In what follows, we will briefly review these models and the corresponding likelihoods. Towards assessing the effects of misspecification we will furthermore consider the encompassing Cliff-Ord-type spatial model, which includes both forms of spatial effects (Anselin, 1988b). The estimates of this model remain consistent whether or not the true data generating process can be described by a spatial error or lag model, see, e.g., (Kelejian & Prucha, 1999, 2010). Of course, if the true data generating process corresponds to a spatial error or lag model, we expect some loss in efficiency when estimating the encompassing model. We will use the encompassing model as a benchmark for the pre-test estimators based on Lagrange Multiplier tests.

The approach frequently taken in empirical work is thus to start with the classical linear regression model

$$y = X\beta + \varepsilon \tag{1}$$

where y is an $n \times 1$ vector of observations on the dependent variable, X is an $n \times k$ matrix of observations on the non-stochastic explanatory variables, β a

$k \times 1$ vector of corresponding parameters, and ε an $n \times 1$ vector of innovations whose elements - for simplicity- are assumed to be i.i.d. $N(0, \sigma^2)$. Under regularity conditions the OLS estimator is also the ML estimator.

As an alternative to the above linear regression model (1), the error term can be specified as a spatial autoregressive process, leading to the spatial error model

$$\begin{aligned} y &= X\beta + u \\ u &= \rho W u + \varepsilon \end{aligned} \tag{2}$$

where u is the $n \times 1$ vector of disturbances, W is an $n \times n$ non-stochastic weighting matrix,[2] ρ is a scalar spatial autoregressive parameter with $|\rho| < 1$, and all other variables are defined as above. For efficiency we can estimate the model in (2) by ML (Ord, 1975), although OLS remains unbiased. The expression for the log likelihood function of the model under normal disturbances takes the form

$$L = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) + \ln|B| - \frac{1}{2\sigma^2}(y - X\beta)'B'B(y - X\beta) \tag{3}$$

where $B = I_N - \rho W$.[3] As an alternative to the maximum likelihood, a feasible GLS estimator, which utilizes a generalized method of moments estimator for ρ, has been suggested by Kelejian & Prucha (1999). However, in this paper we concentrate on the maximum likelihood estimator.

A further alternative to the linear regression model (1) which is often estimated in the empirical literature is the spatial autoregressive model

$$y = \lambda W y + X\beta + \varepsilon \tag{4}$$

where λ is a scalar spatial autoregressive parameter with $|\lambda| < 1$, and all other variables are defined as above. Because of the simultaneous nature of the spatial lag variable Wy is correlated with the disturbance term ε. Thus OLS is inconsistent, however the model can again be estimated efficiently by ML. The log likelihood takes the following form (Ord, 1975)

$$L = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) + \ln|A| - \frac{1}{2\sigma^2}(Ay - X\beta)'(Ay - X\beta) \tag{5}$$

where $A = I - \lambda W$. As an alternative to the maximum likelihood, the model could also be estimated by instrumental variables/generalized method of

[2]The assumptions made on the weights matrix are standard and we will not discuss them in this paper.

[3]For details on the maximum likelihood estimation see Anselin (1988b), Ch 12.

moments (Kelejian & Prucha, 1998), but again the present paper will focus on the maximum likelihood estimation.

Finally, consider the encompassing model which allows for spatial lags in the dependent variable, as well as in the disturbances, i.e.,

$$\begin{aligned} y =& \lambda W y + X\beta + u \\ u =& \rho W u + \varepsilon \end{aligned} \tag{6}$$

Consistent with the terminology introduced in the literature, we refer to this model as a spatial auto-regressive auto-regressive model (SARAR(1,1)).[4] Under normally distributed errors, the log likelihood is

$$\begin{aligned} L \quad = \quad & -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) + \frac{n}{2}\ln|B| + \frac{n}{2}\ln|A| \\ & -\frac{1}{2\sigma^2}(Ay - X\beta)'B'B(Ay - X\beta) \end{aligned} \tag{7}$$

We emphasize that contrarily to what is, e.g., stated in Florax & Folmer (1992) and Anselin (1988b), this does not constitute an obstacle to the identification of the parameters, as long as the matrix X contains at least one exogenous variable in addition to the constant term; see, e.g. Anselin *et al.* (1996, p. 81) and Kelejian & Prucha (1998, p. 105).

3 LM tests

In this section we define the LM tests for the absence of spatial dependence employed in the construction of our considered pretest estimators. Burridge (1980) derived the LM test statistic for the spatial error model. Anselin (1988b) derived LM tests more generally for the SARAR model. Requiring only the estimation of the restricted specification, LM tests have been considered particularly appealing in a spatial setting because of the computational difficulties related to the maximum likelihood estimation of the spatial models.[5]

[4]For a discussion on the assumption of the model see e.g., Kelejian & Prucha (1998, 1999, 2007, 2010); Lee (2002, 2003, 2004), among others.

For simplicity, and following the empirical literature, we assume that the spatial weighting matrices used to multiply the endogenous variables and vector of disturbances are the same.

[5]However, with the increase in power achieved by the modern computers, and the various methods to approximate the Jacobian term (LeSage & Pace, 2009), this problem has been somewhat mitigated.

With the first of these tests we wish to evaluate the hypothesis that the disturbances are independently normally distributed with constant variance (i.e. $\rho = 0$) against the alternative that they are generated by the first order spatial autoregression in (2). The LM-test statistics for this hypothesis is given by

$$LM_\rho = \frac{[e'We/(e'e/n)]^2}{tr[W'W + WW]} \tag{8}$$

where $e = y - X\hat{\beta}_{OLS}$ denotes the vector of OLS residuals, and tr is the trace operator.

The second LM-test statistics, which evaluates the null hypothesis that $\lambda = 0$ in (4) against the alternative of a spatial autoregressive process is given by

$$LM_\lambda = [e'Wy/(e'e/n)]^2/D \tag{9}$$

where $e = y - X\hat{\beta}_{OLS}$ denotes, as before, the vector of OLS residuals, $D = [(WX\hat{\beta}_{OLS})'M(WX\hat{\beta}_{OLS})/\hat{\sigma}^2_{OLS}] + tr(W'W + WW)$, $M = [I - X(X'X)^{-1}X']$, and $\hat{\beta}_{OLS}$ and $\hat{\sigma}^2_{OLS}$ are the OLS estimates of β and σ^2. Under the null hypothesis that the true model is (1) both LM_λ and LM_ρ are asymptotically distributed as $\chi^2(1)$.

The two test statistics presented above assume that the other form of dependence is not present. In other word, LM_ρ is derived under the null hypothesis that $\rho = 0$, but it assumes that also λ is zero. Using the general principles of specification testing with locally misspecified alternatives derived in Bera & Yoon (1993), Anselin *et al.* (1996) develop a set of diagnostics that are a robust version of (8) and (9). The expressions for the robust versions of the tests become, respectively,

$$LM^*_\rho = \frac{\{e'We/(e'e/n) - [tr(WW + W'W)/D]\, e'Wy/(e'e/n)\}^2}{tr(WW + W'W)/[1 - tr(WW + W'W)/D]} \tag{10}$$

and

$$LM^*_\lambda = \frac{[e'Wy/(e'e/n) - e'We/(e'e/n)]^2}{[D - tr(WW + W'W)]} \tag{11}$$

where D as well as the other symbols where defined before. Both the LM^*_λ and the LM^*_ρ statistics are asymptotically distributed as $\chi^2(1)$.

4 Pre-test estimators

The pre-tests estimators are based on the sequence of LM-tests presented in the previous section. We follow the algorithm illustrated in Florax *et al.*

(2003), which proposes three different "approaches" toward specification tests, each leading to a different pre-test estimator. Before reviewing these three approaches, we need to introduce some additional notation.

Let $\widehat{\beta}_{OLS}$ be the OLS estimator based on model (1), $\widehat{\rho}_{MLE}$ and $\widehat{\beta}_{MLE}$ the ML estimators corresponding to the model in (2) and, finally, $\widehat{\lambda}_{MLL}$ and $\widehat{\beta}_{MLL}$ the ML estimators corresponding to the model in (4). Also, let $\widehat{\lambda}_{ML}$, $\widehat{\rho}_{ML}$, and $\widehat{\beta}_{ML}$ denote the ML estimator for λ, ρ and β based on the full model in (6).

Thus, we define

$$\begin{aligned}
\widehat{\theta}_{OLS} &= (0, 0, \widehat{\beta}'_{OLS})', \\
\widehat{\theta}_{ML} &= (\widehat{\lambda}_{ML}, \widehat{\rho}_{ML}, \widehat{\beta}'_{ML})', \\
\widehat{\theta}_{MLL} &= (\widehat{\lambda}_{MLL}, 0, \widehat{\beta}'_{MLL})', \\
\widehat{\theta}_{MLE} &= (0, \widehat{\rho}_{MLE}, \widehat{\beta}'_{MLE})'.
\end{aligned}$$

The first approach in Florax *et al.* (2003) is based on the test statistics LM_λ and LM_ρ and can be summarized as follows:

1 Estimate the non-spatial model by OLS to obtain $\widehat{\beta}_{OLS}$ and the vector of OLS residuals $e = y - X\widehat{\beta}_{OLS}$.

2 Test the hypothesis of no spatial dependence due to an omitted spatial lag or to an omitted spatially autoregressive error using, respectively, LM_λ and LM_ρ.

3 If both tests statistics are not significant, then accept $H_0^A : \lambda = \rho = 0$. We refer to this estimator as $\widehat{\theta}_{OLS} = (0, 0, \widehat{\beta}'_{OLS})'$.

4 If LM_λ is significant and LM_ρ is not significant then accept H_0^B : $\lambda \neq 0; \rho = 0$ and estimate model (4) by maximum likelihood to get $\widehat{\theta}_{MLL} = (\widehat{\lambda}_{MLL}, 0, \widehat{\beta}'_{MLL})'$.

4 If LM_ρ is significant and LM_λ is not significant then accept H_0^C : $\lambda = 0; \rho \neq 0$ and estimate model (2) by maximum likelihood to get $\widehat{\theta}_{MLE} = (0, \widehat{\rho}_{MLE}, \widehat{\beta}'_{MLE})'$.

5 Finally, if both LM_λ and LM_ρ are significant, estimate the specification pointed by the more significant of the two tests.

Florax *et al.* (2003) refer to this approach as the "classic" approach because it is based on the test statistics LM_λ and LM_ρ.

Let $\widehat{\theta}_{PT1}$ denote the estimator for θ based on this approach, where "PT" stands for "pre-test". Then this estimator is formally given by

$$
\begin{aligned}
\widehat{\theta}_{PT1} \quad = \quad & \mathbf{1}\left(LM_{\lambda} < \chi_{.95}, LM_{\rho} < \chi_{.95}\right)\widehat{\theta}_{OLS} \\
& +\mathbf{1}\left(LM_{\lambda} \geq \chi_{.95}, LM_{\rho} < \chi_{.95}\right)\widehat{\theta}_{MLL} \\
& +\mathbf{1}\left(LM_{\lambda} < \chi_{.95}, LM_{\rho} \geq \chi_{.95}\right)\widehat{\theta}_{MLE} \\
& +\mathbf{1}\left(LM_{\lambda} \geq \chi_{.95}, LM_{\rho} \geq \chi_{.95}\right)\mathbf{1}\left(LM_{\lambda} \geq LM_{\rho}\right)\widehat{\theta}_{MLL} \\
& +\mathbf{1}\left(LM_{\lambda} \geq \chi_{.95}, LM_{\rho} \geq \chi_{.95}\right)\mathbf{1}\left(LM_{\lambda} < LM_{\rho}\right)\widehat{\theta}_{MLE}.
\end{aligned}
$$

where $\mathbf{1}(.)$ denotes the indicator function.

The second approach, that they refer to as the robust approach, is almost identical to the previous one with the exception that it is performed with the robust versions of the LM tests. Let $\widehat{\theta}_{PT2}$ denote the estimator for θ based on this approach, then it is formally given by

$$
\begin{aligned}
\widehat{\theta}_{PT2} \quad = \quad & \mathbf{1}\left(LM^{*}_{\lambda} < \chi_{.95}, LM^{*}_{\rho} < \chi_{.95}\right)\widehat{\theta}_{OLS} \\
& +\mathbf{1}\left(LM^{*}_{\lambda} \geq \chi_{.95}, LM^{*}_{\rho} < \chi_{.95}\right)\widehat{\theta}_{MLL} \\
& +\mathbf{1}\left(LM^{*}_{\lambda} < \chi_{.95}, LM^{*}_{\rho} \geq \chi_{.95}\right)\widehat{\theta}_{MLE} \\
& +\mathbf{1}\left(LM^{*}_{\lambda} \geq \chi_{.95}, LM^{*}_{\rho} \geq \chi_{.95}\right)\mathbf{1}\left(LM^{*}_{\lambda} \geq LM^{*}_{\rho}\right)\widehat{\theta}_{MLL} \\
& +\mathbf{1}\left(LM^{*}_{\lambda} \geq \chi_{.95}, LM^{*}_{\rho} \geq \chi_{.95}\right)\mathbf{1}\left(LM^{*}_{\lambda} < LM^{*}_{\rho}\right)\widehat{\theta}_{MLE}.
\end{aligned}
$$

The third and last approach is an "hybrid" specification strategy in that it combines the use of both test statistics (classical and robust).

1 Estimate the non-spatial model by OLS to obtain $\widehat{\beta}_{OLS}$ and the vector of OLS residuals $e = y - X\widehat{\beta}_{OLS}$.

2 Test the hypothesis of no spatial dependence due to an omitted spatial lag or to an omitted spatially autoregressive error using, respectively, LM_{λ} and LM_{ρ}.

3 If both tests statistics are not significant, then accept $H_0^A : \lambda = \rho = 0$. We refer to this estimator as $\widehat{\theta}_{OLS} = (0, 0, \widehat{\beta}'_{OLS})'$.

4 If LM_{λ} is significant and LM_{ρ} is not significant then accept H_0^B : $\lambda \neq 0; \rho = 0$ and estimate model (4) by maximum likelihood to get $\widehat{\theta}_{MLL} = (\widehat{\lambda}_{MLL}, 0, \widehat{\beta}'_{MLL})'$.

4 If LM_ρ is significant and LM_λ is not significant then accept H_0^C : $\lambda = 0; \rho \neq 0$ and estimate model (2) by maximum likelihood to get $\widehat{\theta}_{MLE} = (0, \widehat{\rho}_{MLE}, \widehat{\beta}'_{MLE})'$.

5 Finally, if both LM_λ and LM_ρ are significant, estimate the specification pointed by the more significant of the two robust statistics LM_λ^* and LM_ρ^*.

Let $\widehat{\theta}_{PT3}$ denote the estimator for θ based on this approach. Then the estimator would be

$$\begin{aligned}
\widehat{\theta}_{PR3} \quad = \quad & \mathbf{1}\left(LM_\lambda < \chi_{.95}, LM_\rho < \chi_{.95}\right)\widehat{\theta}_{OLS} \\
& + \mathbf{1}\left(LM_\lambda \geq \chi_{.95}, LM_\rho < \chi_{.95}\right)\widehat{\theta}_{MLL} \\
& + \mathbf{1}\left(LM_\lambda < \chi_{.95}, LM_\rho \geq \chi_{.95}\right)\widehat{\theta}_{MLE} \\
& + \mathbf{1}\left(LM_\lambda \geq \chi_{.95}, LM_\rho \geq \chi_{.95}\right)\mathbf{1}\left(LM_\lambda^* \geq LM_\rho^*\right)\widehat{\theta}_{MLL} \\
& + \mathbf{1}\left(LM_\lambda \geq \chi_{.95}, LM_\rho \geq \chi_{.95}\right)\mathbf{1}\left(LM_\lambda^* < LM_\rho^*\right)\widehat{\theta}_{MLE}.
\end{aligned}$$

where the decision is based on LM_λ, LM_λ^*, LM_ρ, and LM_ρ^*.

As Florax *et al.* (2003) recognized, the performance of this last pre-test estimator is identical to the classical approach. This is because the order of the robust test is the same as the order of the classical test. In our Monte Carlo study we report the results for the third pre-test estimator just to confirm that they are indeed the same.

From the above definitions of the pre-test estimators it is obvious that they are highly nonlinear, and that they cannot be expected to be unbiased or consistent. Furthermore, it is obvious that the asymptotic distribution of those estimators will generally differ from those of $\widehat{\theta}_{OLS}$, $\widehat{\theta}_{MLL}$ and $\widehat{\theta}_{MLE}$. Of course, potential issues stemming from improper inference when using pre-test estimators are well known; see, e.g., Leeb & Poetscher (2008). The MC study is intended to shed some light on the importance of those issues in the estimation of spatial models.

5 Monte Carlo

In what follows, we report on a Monte Carlo study of the small sample properties of the three pre-test estimators defined above and of the ML estimator for the comprehensive model (6). The design of the Monte Carlo is, intentionally, very simple. Our findings suggest that even in a simple case, the properties of the estimates generated by the pre-testing strategy can be problematic.

Monte Carlo Design

In all of the experiments, the data are generated from the following model:

$$y = \lambda W y + x_1\beta_1 + x_2\beta_2 + u \qquad (12)$$
$$u = \rho W u + \varepsilon$$

The two regressors, x_1 and x_2, are normalized versions of income per capita and the proportion of housing units that are rental in 1980, in 760 counties in U.S. mid-western states.[6] We normalized the data by subtracting from each observation the corresponding sample average, and then dividing that result by the sample standard deviation. The first n values of these normalized variables were used in our Monte Carlo experiments. The regressors are held constant over all of the Monte Carlo trials. We consider one sample size, corresponding to a regular grid of dimension 23×23 and use the first n values of the regressors to generate the Monte Carlo samples. The regular grid leads to a sample size of $n = 529$. As for the spatial weighting matrix, we consider simply one definition of W based on the queen criterion (i.e. common borders and vertex). The values of β_1 and β_2 are set equal to one.[7] We consider the same set of values for both ρ and λ, namely -0.8,-0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8. Finally, we assume that the elements of the innovation vector are i.i.d. $N(0,1)$.[8]

Monte Carlo Results

The results of the Monte Carlo experiments are reported in tables 1-12. The first eight tables relate to the bias and MSE of, respectively, ρ, λ, β_1, and β_2. The first two columns of these tables contain the combinations of ρ and λ. The remaining columns report the maximum likelihood estimation of the full model, as well as each of the three pre-test estimators. Additionally, the last four tables report an empirical estimate of the frequency of rejections of the null hypothesis at the 5% level. They refer, respectively, to the maximum likelihood estimation of the full model and each of the three pre-test estimators.

Consider the results in table 1, which are based on 2,000 replications. The first thing to note is that the performance of this hybrid approach is

[6] These data were taken from Kelejian & Robinson (1995) and where also used by Arraiz *et al.* (2010).

[7] The target R^2 for the simulation was set to 0.6.

[8] All elaborations were performed using R statistical software (R Development Core Team, 2010) with the library **spdep** (Bivand *et al.* , 2010).

identical to the classical approach. As noted by Florax *et al.* (2003), this is because the order of the robust test is the same as the order of the classical test. Looking first at the averages, we note that the average bias of ρ_{ML} is almost fifty times lower than that of any of the pre-test estimators. Clearly, when one of the spatial parameters is zero the difference in terms of bias declines considerably. Furthermore, the highest bias seems to correspond to situations where there is substantial spatial dependence both of the error and of the lag type.

The figures in table 2 refer to the Monte Carlo estimates of the MSE of ρ calculated using the MLE of the full model and the three pre-test estimators. On average, the robust approach does not vary much compared to the other two approaches (0.0610 compared to 0.0602). Contrarily, the MSE of the ML estimator of the full model is considerably lower to that of the pre-test estimators (0.0038).

Table 3 and 4 display the results for λ. In particular, while table 3 reports the bias, figures in table 4 relate to the MSE. Consider first the results in table 3. Again looking at the averages, the results for λ confirm our findings for ρ_{ML} in that the bias for the ML estimator of the full model is considerably lower than that of any of the pre-test estimators. There are situations, however, in which the pre-test estimators perform very well. Not surprisingly these situations relate to at least one of the spatial parameters being zero. Results for the MSE in table 3 are also in line with the findings for ρ.

The results in table 5-8 pertain to the bias and MSE of the estimates of β_1 and β_2. Since the results for the two coefficient are almost identical, we will just focus on one of them (β_1). The bias of the pre-test estimators is way larger than that of the ML estimator of the full model. These results suggest that, even with a very simple design of the Monte Carlo analysis, the bias of the pre-test estimators can be very large. On the other hand, the ML estimates based on the full model are consistent for all the parameter values.

Let us move to the last four tables and discus the results on the size of the tests. Table 9 reports the frequency of rejection of the null hypothesis at the 5% level for the ML estimator of the full model. Looking first at column averages, the empirical size of the test is very close to the theoretical 5% level. There are, however, a few exceptions mostly related to extreme values of the spatial parameters. The size of the test for both β_1 and β_2 is extremely close to the nominal 5% value in all cases. Furthermore, we are considering a sample size of $n = 529$ observations and most of the spatial applications are based on datasets much larger than the one we are employing.

Tables 10-12 display the results for the pre-test estimators. Also in this case the results are very similar. Therefore, we just focus on the pre-test based on the classic approach. We only report the results for β_1 and β_2. On average, the size of the test is, respectively, 23% and 19%. In some experiments it can be as large as 90%. In particular, it reaches the value of 94% for β_1 (when $\rho = -0.8$ and $\lambda = 0$), and 82% for β_2 (when $\rho = 0.8$ and $\lambda = -0.8$).

6 Conclusions

This paper has examined some of the consequences of model misspecification in spatial econometrics. In particular, we have compared the performance of the ML estimator of the general model that allows for spatial spill-overs in the endogenous variables and disturbances, with three pre-test estimators based on Lagrange Multiplier tests in the context of a Monte Carlo study. Intentionally we have kept the Monte Carlo design very straightforward to stress that even in this simple case the bias of the estimates generated by the pre-testing strategy can be very large and the size of hypothesis test away from the nominal size. As opposed to this result, ML estimates based on the full model are consistent, and the size of hypothesis tests is reasonably close to the nominal values. A possible extension for future research would be to extend our Monte Carlo study to a panel framework.

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Table 1: Bias of ρ_{ML}, ρ_1, ρ_2, and ρ_3. Values based on 2,000 replications.

		ρ_{ML}	ρ_1	ρ_2	ρ_3
$\rho = -0.8$	$\lambda = -0.8$	0.0001	-0.2849	-0.2849	-0.2849
$\rho = -0.8$	$\lambda = -0.6$	-0.0015	-0.2728	-0.2712	-0.2728
$\rho = -0.8$	$\lambda = -0.4$	-0.0009	-0.1876	-0.0843	-0.1876
$\rho = -0.8$	$\lambda = -0.2$	-0.0017	-0.0300	0.1605	-0.0300
$\rho = -0.8$	$\lambda = 0$	-0.0038	-0.0016	-0.0026	-0.0016
$\rho = -0.8$	$\lambda = 0.2$	-0.0032	-0.2002	-0.1938	-0.2002
$\rho = -0.8$	$\lambda = 0.4$	-0.0020	-0.1562	-0.1562	-0.1562
$\rho = -0.8$	$\lambda = 0.6$	-0.0017	-0.1013	-0.1013	-0.1013
$\rho = -0.8$	$\lambda = 0.8$	-0.0013	-0.0449	-0.0449	-0.0449
$\rho = -0.6$	$\lambda = -0.8$	0.0011	-0.2085	-0.2085	-0.2085
$\rho = -0.6$	$\lambda = -0.6$	0.0011	-0.2122	-0.2101	-0.2122
$\rho = -0.6$	$\lambda = -0.4$	-0.0016	-0.1751	-0.0706	-0.1751
$\rho = -0.6$	$\lambda = -0.2$	-0.0013	-0.0249	0.1630	-0.0249
$\rho = -0.6$	$\lambda = 0$	-0.0025	-0.0089	-0.0016	-0.0089
$\rho = -0.6$	$\lambda = 0.2$	-0.0023	-0.1524	-0.1525	-0.1524
$\rho = -0.6$	$\lambda = 0.4$	-0.0032	-0.1258	-0.1258	-0.1258
$\rho = -0.6$	$\lambda = 0.6$	-0.0022	-0.0778	-0.0778	-0.0778
$\rho = -0.6$	$\lambda = 0.8$	-0.0011	-0.0340	-0.0340	-0.0340
$\rho = -0.4$	$\lambda = -0.8$	0.0022	-0.1395	-0.1395	-0.1395
$\rho = -0.4$	$\lambda = -0.6$	0.0037	-0.1415	-0.1399	-0.1415
$\rho = -0.4$	$\lambda = -0.4$	0.0025	-0.1250	-0.0286	-0.1250
$\rho = -0.4$	$\lambda = -0.2$	-0.0005	0.0860	0.1649	0.0860
$\rho = -0.4$	$\lambda = 0$	-0.0008	-0.0162	-0.0007	-0.0162
$\rho = -0.4$	$\lambda = 0.2$	-0.0004	-0.1173	-0.1187	-0.1173
$\rho = -0.4$	$\lambda = 0.4$	-0.0012	-0.1040	-0.1040	-0.1040
$\rho = -0.4$	$\lambda = 0.6$	-0.0022	-0.0605	-0.0605	-0.0605
$\rho = -0.4$	$\lambda = 0.8$	-0.0012	-0.0241	-0.0241	-0.0241
$\rho = -0.2$	$\lambda = -0.8$	0.0037	-0.0724	-0.0724	-0.0724
$\rho = -0.2$	$\lambda = -0.6$	0.0016	-0.0736	-0.0730	-0.0736
$\rho = -0.2$	$\lambda = -0.4$	-0.0004	-0.0698	0.0047	-0.0698
$\rho = -0.2$	$\lambda = -0.2$	-0.0009	0.1681	0.1772	0.1681
$\rho = -0.2$	$\lambda = 0$	-0.0021	-0.0091	0.0000	-0.0091
$\rho = -0.2$	$\lambda = 0.2$	-0.0010	-0.1031	-0.1037	-0.1031
$\rho = -0.2$	$\lambda = 0.4$	-0.0032	-0.1075	-0.1075	-0.1075
$\rho = -0.2$	$\lambda = 0.6$	-0.0016	-0.0592	-0.0592	-0.0592
$\rho = -0.2$	$\lambda = 0.8$	-0.0018	-0.0140	-0.0140	-0.0140
$\rho = 0$	$\lambda = -0.8$	0.0025	0.0001	0.0001	0.0001
$\rho = 0$	$\lambda = -0.6$	0.0027	0.0012	0.0020	0.0012
$\rho = 0$	$\lambda = -0.4$	-0.0017	0.0285	0.0672	0.0285
$\rho = 0$	$\lambda = -0.2$	-0.0003	0.1960	0.1872	0.1960
$\rho = 0$	$\lambda = 0$	-0.0037	0.0033	0.0019	0.0033
$\rho = 0$	$\lambda = 0.2$	-0.0038	-0.1111	-0.1115	-0.1111
$\rho = 0$	$\lambda = 0.4$	-0.0022	-0.1270	-0.1270	-0.1270
$\rho = 0$	$\lambda = 0.6$	-0.0026	-0.0781	-0.0781	-0.0781
$\rho = 0$	$\lambda = 0.8$	-0.0018	-0.0015	-0.0015	-0.0015
$\rho = 0.2$	$\lambda = -0.8$	0.0018	0.0855	0.0855	0.0855
$\rho = 0.2$	$\lambda = -0.6$	-0.0006	0.0839	0.0842	0.0839
$\rho = 0.2$	$\lambda = -0.4$	0.0046	0.2020	0.1479	0.2020
$\rho = 0.2$	$\lambda = -0.2$	-0.0003	0.1995	0.1955	0.1995
$\rho = 0.2$	$\lambda = 0$	0.0020	0.0088	0.0044	0.0088
$\rho = 0.2$	$\lambda = 0.2$	-0.0023	-0.1265	-0.1265	-0.1265
$\rho = 0.2$	$\lambda = 0.4$	-0.0031	-0.1731	-0.1731	-0.1731
$\rho = 0.2$	$\lambda = 0.6$	-0.0032	-0.1090	-0.1090	-0.1090
$\rho = 0.2$	$\lambda = 0.8$	-0.0027	0.0113	0.0113	0.0113
$\rho = 0.4$	$\lambda = -0.8$	0.0045	0.1973	0.1973	0.1973
$\rho = 0.4$	$\lambda = -0.6$	-0.0004	0.2067	0.1999	0.2067
$\rho = 0.4$	$\lambda = -0.4$	0.0017	0.3580	0.2890	0.3580
$\rho = 0.4$	$\lambda = -0.2$	0.0003	0.2002	0.1998	0.2002
$\rho = 0.4$	$\lambda = 0$	0.0011	0.0069	0.0055	0.0069
$\rho = 0.4$	$\lambda = 0.2$	-0.0014	-0.1504	-0.1504	-0.1504
$\rho = 0.4$	$\lambda = 0.4$	-0.0021	-0.2352	-0.2352	-0.2352
$\rho = 0.4$	$\lambda = 0.6$	-0.0045	-0.1630	-0.1630	-0.1630
$\rho = 0.4$	$\lambda = 0.8$	-0.0036	0.0175	0.0175	0.0175
$\rho = 0.6$	$\lambda = -0.8$	0.0054	0.3683	0.3680	0.3683
$\rho = 0.6$	$\lambda = -0.6$	0.0042	0.4655	0.4101	0.4655
$\rho = 0.6$	$\lambda = -0.4$	0.0041	0.3990	0.3921	0.3990
$\rho = 0.6$	$\lambda = -0.2$	0.0012	0.2003	0.2002	0.2003
$\rho = 0.6$	$\lambda = 0$	0.0022	0.0039	0.0039	0.0039
$\rho = 0.6$	$\lambda = 0.2$	-0.0017	-0.1717	-0.1717	-0.1717
$\rho = 0.6$	$\lambda = 0.4$	-0.0010	-0.2935	-0.2935	-0.2935
$\rho = 0.6$	$\lambda = 0.6$	-0.0052	-0.2748	-0.2748	-0.2748
$\rho = 0.6$	$\lambda = 0.8$	-0.0064	-0.0250	-0.0250	-0.0250
$\rho = 0.8$	$\lambda = -0.8$	0.0037	0.7423	0.7270	0.7423
$\rho = 0.8$	$\lambda = -0.6$	0.0058	0.5997	0.5984	0.5997
$\rho = 0.8$	$\lambda = -0.4$	0.0061	0.4000	0.4000	0.4000
$\rho = 0.8$	$\lambda = -0.2$	0.0076	0.2011	0.2011	0.2011
$\rho = 0.8$	$\lambda = 0$	0.0036	0.0031	0.0031	0.0031
$\rho = 0.8$	$\lambda = 0.2$	0.0035	-0.1879	-0.1879	-0.1879
$\rho = 0.8$	$\lambda = 0.4$	0.0016	-0.3394	-0.3394	-0.3394
$\rho = 0.8$	$\lambda = 0.6$	-0.0069	-0.3816	-0.3816	-0.3816
$\rho = 0.8$	$\lambda = 0.8$	-0.0120	-0.1534	-0.1534	-0.1534
	average	0.0026	0.1492	0.1462	0.1492

Table 2: MSE of ρ_{ML}, ρ_1, ρ_2, and ρ_3. Values based on 2,000 replications.

		ρ_{ML}	ρ_1	ρ_2	ρ_3
$\rho = -0.8$	$\lambda = -0.8$	0.0046	0.0875	0.0875	0.0875
$\rho = -0.8$	$\lambda = -0.6$	0.0043	0.0959	0.0965	0.0959
$\rho = -0.8$	$\lambda = -0.4$	0.0039	0.0977	0.1128	0.0977
$\rho = -0.8$	$\lambda = -0.2$	0.0030	0.0592	0.0446	0.0592
$\rho = -0.8$	$\lambda = 0$	0.0024	0.0003	0.0004	0.0003
$\rho = -0.8$	$\lambda = 0.2$	0.0016	0.0418	0.0388	0.0418
$\rho = -0.8$	$\lambda = 0.4$	0.0010	0.0262	0.0262	0.0262
$\rho = -0.8$	$\lambda = 0.6$	0.0005	0.0111	0.0111	0.0111
$\rho = -0.8$	$\lambda = 0.8$	0.0001	0.0022	0.0022	0.0022
$\rho = -0.6$	$\lambda = -0.8$	0.0044	0.0467	0.0467	0.0467
$\rho = -0.6$	$\lambda = -0.6$	0.0042	0.0507	0.0518	0.0507
$\rho = -0.6$	$\lambda = -0.4$	0.0036	0.0547	0.0774	0.0547
$\rho = -0.6$	$\lambda = -0.2$	0.0033	0.0460	0.0418	0.0460
$\rho = -0.6$	$\lambda = 0$	0.0026	0.0009	0.0002	0.0009
$\rho = -0.6$	$\lambda = 0.2$	0.0018	0.0251	0.0250	0.0251
$\rho = -0.6$	$\lambda = 0.4$	0.0011	0.0201	0.0201	0.0201
$\rho = -0.6$	$\lambda = 0.6$	0.0005	0.0080	0.0080	0.0080
$\rho = -0.6$	$\lambda = 0.8$	0.0001	0.0013	0.0013	0.0013
$\rho = -0.4$	$\lambda = -0.8$	0.0047	0.0223	0.0223	0.0223
$\rho = -0.4$	$\lambda = -0.6$	0.0045	0.0237	0.0246	0.0237
$\rho = -0.4$	$\lambda = -0.4$	0.0040	0.0259	0.0546	0.0259
$\rho = -0.4$	$\lambda = -0.2$	0.0034	0.0385	0.0396	0.0385
$\rho = -0.4$	$\lambda = 0$	0.0026	0.0013	0.0001	0.0013
$\rho = -0.4$	$\lambda = 0.2$	0.0019	0.0169	0.0174	0.0169
$\rho = -0.4$	$\lambda = 0.4$	0.0012	0.0200	0.0200	0.0200
$\rho = -0.4$	$\lambda = 0.6$	0.0006	0.0088	0.0088	0.0088
$\rho = -0.4$	$\lambda = 0.8$	0.0002	0.0008	0.0008	0.0008
$\rho = -0.2$	$\lambda = -0.8$	0.0045	0.0082	0.0082	0.0082
$\rho = -0.2$	$\lambda = -0.6$	0.0044	0.0085	0.0088	0.0085
$\rho = -0.2$	$\lambda = -0.4$	0.0043	0.0104	0.0371	0.0104
$\rho = -0.2$	$\lambda = -0.2$	0.0037	0.0387	0.0385	0.0387
$\rho = -0.2$	$\lambda = 0$	0.0029	0.0006	0.0000	0.0006
$\rho = -0.2$	$\lambda = 0.2$	0.0019	0.0168	0.0171	0.0168
$\rho = -0.2$	$\lambda = 0.4$	0.0014	0.0319	0.0319	0.0319
$\rho = -0.2$	$\lambda = 0.6$	0.0007	0.0211	0.0211	0.0211
$\rho = -0.2$	$\lambda = 0.8$	0.0002	0.0004	0.0004	0.0004
$\rho = 0$	$\lambda = -0.8$	0.0047	0.0031	0.0031	0.0031
$\rho = 0$	$\lambda = -0.6$	0.0048	0.0031	0.0036	0.0031
$\rho = 0$	$\lambda = -0.4$	0.0047	0.0179	0.0334	0.0179
$\rho = 0$	$\lambda = -0.2$	0.0040	0.0397	0.0386	0.0397
$\rho = 0$	$\lambda = 0$	0.0035	0.0004	0.0002	0.0004
$\rho = 0$	$\lambda = 0.2$	0.0027	0.0232	0.0233	0.0232
$\rho = 0$	$\lambda = 0.4$	0.0017	0.0518	0.0518	0.0518
$\rho = 0$	$\lambda = 0.6$	0.0009	0.0465	0.0465	0.0465
$\rho = 0$	$\lambda = 0.8$	0.0003	0.0002	0.0002	0.0002
$\rho = 0.2$	$\lambda = -0.8$	0.0050	0.0108	0.0108	0.0108
$\rho = 0.2$	$\lambda = -0.6$	0.0054	0.0108	0.0110	0.0108
$\rho = 0.2$	$\lambda = -0.4$	0.0052	0.0725	0.0443	0.0725
$\rho = 0.2$	$\lambda = -0.2$	0.0051	0.0399	0.0393	0.0399
$\rho = 0.2$	$\lambda = 0$	0.0042	0.0010	0.0006	0.0010
$\rho = 0.2$	$\lambda = 0.2$	0.0032	0.0306	0.0306	0.0306
$\rho = 0.2$	$\lambda = 0.4$	0.0022	0.0812	0.0812	0.0812
$\rho = 0.2$	$\lambda = 0.6$	0.0012	0.0815	0.0815	0.0815
$\rho = 0.2$	$\lambda = 0.8$	0.0004	0.0024	0.0024	0.0024
$\rho = 0.4$	$\lambda = -0.8$	0.0054	0.0432	0.0432	0.0432
$\rho = 0.4$	$\lambda = -0.6$	0.0063	0.0518	0.0454	0.0518
$\rho = 0.4$	$\lambda = -0.4$	0.0065	0.1388	0.1000	0.1388
$\rho = 0.4$	$\lambda = -0.2$	0.0059	0.0401	0.0400	0.0401
$\rho = 0.4$	$\lambda = 0$	0.0056	0.0013	0.0011	0.0013
$\rho = 0.4$	$\lambda = 0.2$	0.0044	0.0377	0.0377	0.0377
$\rho = 0.4$	$\lambda = 0.4$	0.0033	0.1137	0.1137	0.1137
$\rho = 0.4$	$\lambda = 0.6$	0.0018	0.1328	0.1328	0.1328
$\rho = 0.4$	$\lambda = 0.8$	0.0007	0.0170	0.0170	0.0170
$\rho = 0.6$	$\lambda = -0.8$	0.0058	0.1413	0.1409	0.1413
$\rho = 0.6$	$\lambda = -0.6$	0.0070	0.2376	0.1828	0.2376
$\rho = 0.6$	$\lambda = -0.4$	0.0072	0.1594	0.1551	0.1594
$\rho = 0.6$	$\lambda = -0.2$	0.0079	0.0402	0.0401	0.0402
$\rho = 0.6$	$\lambda = 0$	0.0072	0.0013	0.0013	0.0013
$\rho = 0.6$	$\lambda = 0.2$	0.0066	0.0422	0.0422	0.0422
$\rho = 0.6$	$\lambda = 0.4$	0.0051	0.1404	0.1404	0.1404
$\rho = 0.6$	$\lambda = 0.6$	0.0032	0.2140	0.2140	0.2140
$\rho = 0.6$	$\lambda = 0.8$	0.0015	0.0797	0.0797	0.0797
$\rho = 0.8$	$\lambda = -0.8$	0.0058	0.5600	0.5377	0.5600
$\rho = 0.8$	$\lambda = -0.6$	0.0070	0.3597	0.3582	0.3597
$\rho = 0.8$	$\lambda = -0.4$	0.0083	0.1600	0.1600	0.1600
$\rho = 0.8$	$\lambda = -0.2$	0.0084	0.0409	0.0409	0.0409
$\rho = 0.8$	$\lambda = 0$	0.0089	0.0017	0.0017	0.0017
$\rho = 0.8$	$\lambda = 0.2$	0.0090	0.0433	0.0433	0.0433
$\rho = 0.8$	$\lambda = 0.4$	0.0081	0.1581	0.1581	0.1581
$\rho = 0.8$	$\lambda = 0.6$	0.0063	0.2859	0.2859	0.2859
$\rho = 0.8$	$\lambda = 0.8$	0.0039	0.2137	0.2137	0.2137
	average	0.0038	0.0610	0.0602	0.0610

Table 3: Bias λ_{ML}, λ_1, λ_2, and λ_3. Values based on 2,000 replications.

		λ_{ML}	λ_1	λ_2	λ_3
$\rho = -0.8$	$\lambda = -0.8$	-0.0046	0.7956	0.7956	0.7956
$\rho = -0.8$	$\lambda = -0.6$	-0.0022	0.7691	0.7709	0.7691
$\rho = -0.8$	$\lambda = -0.4$	-0.0013	0.6304	0.7199	0.6304
$\rho = -0.8$	$\lambda = -0.2$	-0.0003	0.3054	0.7114	0.3054
$\rho = -0.8$	$\lambda = 0$	-0.0010	0.7582	0.7712	0.7582
$\rho = -0.8$	$\lambda = 0.2$	-0.0012	0.7828	0.7967	0.7828
$\rho = -0.8$	$\lambda = 0.4$	-0.0064	0.8004	0.8004	0.8004
$\rho = -0.8$	$\lambda = 0.6$	-0.0041	0.8007	0.8007	0.8007
$\rho = -0.8$	$\lambda = 0.8$	-0.0075	0.8000	0.8000	0.8000
$\rho = -0.6$	$\lambda = -0.8$	-0.0018	0.5994	0.5994	0.5994
$\rho = -0.6$	$\lambda = -0.6$	-0.0112	0.5946	0.5952	0.5946
$\rho = -0.6$	$\lambda = -0.4$	-0.0036	0.5393	0.5886	0.5393
$\rho = -0.6$	$\lambda = -0.2$	-0.0053	0.3873	0.5916	0.3873
$\rho = -0.6$	$\lambda = 0$	-0.0073	0.5700	0.5957	0.5700
$\rho = -0.6$	$\lambda = 0.2$	-0.0018	0.5715	0.5848	0.5715
$\rho = -0.6$	$\lambda = 0.4$	-0.0008	0.6095	0.6095	0.6095
$\rho = -0.6$	$\lambda = 0.6$	-0.0010	0.6035	0.6035	0.6035
$\rho = -0.6$	$\lambda = 0.8$	-0.0055	0.6000	0.6000	0.6000
$\rho = -0.4$	$\lambda = -0.8$	-0.0062	0.4000	0.4000	0.4000
$\rho = -0.4$	$\lambda = -0.6$	-0.0096	0.3984	0.3995	0.3984
$\rho = -0.4$	$\lambda = -0.4$	-0.0083	0.3781	0.3990	0.3781
$\rho = -0.4$	$\lambda = -0.2$	-0.0076	0.3496	0.3987	0.3496
$\rho = -0.4$	$\lambda = 0$	-0.0058	0.3000	0.3868	0.3000
$\rho = -0.4$	$\lambda = 0.2$	-0.0081	0.3776	0.3808	0.3776
$\rho = -0.4$	$\lambda = 0.4$	-0.0063	0.4308	0.4308	0.4308
$\rho = -0.4$	$\lambda = 0.6$	-0.0053	0.4116	0.4116	0.4116
$\rho = -0.4$	$\lambda = 0.8$	-0.0080	0.4000	0.4000	0.4000
$\rho = -0.2$	$\lambda = -0.8$	-0.0128	0.2000	0.2000	0.2000
$\rho = -0.2$	$\lambda = -0.6$	-0.0089	0.1996	0.1996	0.1996
$\rho = -0.2$	$\lambda = -0.4$	-0.0095	0.1973	0.2000	0.1973
$\rho = -0.2$	$\lambda = -0.2$	-0.0095	0.1925	0.1905	0.1925
$\rho = -0.2$	$\lambda = 0$	-0.0088	0.0869	0.1719	0.0869
$\rho = -0.2$	$\lambda = 0.2$	-0.0099	0.2256	0.2260	0.2256
$\rho = -0.2$	$\lambda = 0.4$	-0.0052	0.2856	0.2856	0.2856
$\rho = -0.2$	$\lambda = 0.6$	-0.0062	0.2429	0.2429	0.2429
$\rho = -0.2$	$\lambda = 0.8$	-0.0061	0.2000	0.2000	0.2000
$\rho = 0$	$\lambda = -0.8$	-0.0097	0.0000	0.0000	0.0000
$\rho = 0$	$\lambda = -0.6$	-0.0082	0.0000	0.0000	0.0000
$\rho = 0$	$\lambda = -0.4$	-0.0051	-0.0007	-0.0003	-0.0007
$\rho = 0$	$\lambda = -0.2$	-0.0098	-0.0123	-0.0238	-0.0123
$\rho = 0$	$\lambda = 0$	-0.0052	0.0019	0.0089	0.0019
$\rho = 0$	$\lambda = 0.2$	-0.0089	0.1357	0.1353	0.1357
$\rho = 0$	$\lambda = 0.4$	-0.0067	0.1791	0.1791	0.1791
$\rho = 0$	$\lambda = 0.6$	-0.0094	0.1032	0.1032	0.1032
$\rho = 0$	$\lambda = 0.8$	-0.0033	0.0000	0.0000	0.0000
$\rho = 0.2$	$\lambda = -0.8$	-0.0059	-0.2000	-0.2000	-0.2000
$\rho = 0.2$	$\lambda = -0.6$	-0.0083	-0.2000	-0.2000	-0.2000
$\rho = 0.2$	$\lambda = -0.4$	-0.0168	-0.2001	-0.2018	-0.2001
$\rho = 0.2$	$\lambda = -0.2$	-0.0048	-0.1672	-0.1659	-0.1672
$\rho = 0.2$	$\lambda = 0$	-0.0141	-0.0243	-0.0478	-0.0243
$\rho = 0.2$	$\lambda = 0.2$	-0.0094	0.0966	0.0965	0.0966
$\rho = 0.2$	$\lambda = 0.4$	-0.0070	0.1235	0.1235	0.1235
$\rho = 0.2$	$\lambda = 0.6$	-0.0101	-0.0129	-0.0129	-0.0129
$\rho = 0.2$	$\lambda = 0.8$	-0.0080	-0.1971	-0.1971	-0.1971
$\rho = 0.4$	$\lambda = -0.8$	-0.0080	-0.4000	-0.4000	-0.4000
$\rho = 0.4$	$\lambda = -0.6$	-0.0068	-0.4000	-0.3997	-0.4000
$\rho = 0.4$	$\lambda = -0.4$	-0.0082	-0.3751	-0.3357	-0.3751
$\rho = 0.4$	$\lambda = -0.2$	-0.0093	-0.1642	-0.1739	-0.1642
$\rho = 0.4$	$\lambda = 0$	-0.0095	-0.0172	-0.0264	-0.0172
$\rho = 0.4$	$\lambda = 0.2$	-0.0119	0.0842	0.0842	0.0842
$\rho = 0.4$	$\lambda = 0.4$	-0.0107	0.1041	0.1041	0.1041
$\rho = 0.4$	$\lambda = 0.6$	-0.0079	-0.0874	-0.0874	-0.0874
$\rho = 0.4$	$\lambda = 0.8$	-0.0074	-0.3769	-0.3769	-0.3769
$\rho = 0.6$	$\lambda = -0.8$	-0.0090	-0.5999	-0.5999	-0.5999
$\rho = 0.6$	$\lambda = -0.6$	-0.0074	-0.5643	-0.5276	-0.5643
$\rho = 0.6$	$\lambda = -0.4$	-0.0101	-0.2946	-0.2493	-0.2946
$\rho = 0.6$	$\lambda = -0.2$	-0.0109	-0.1202	-0.1251	-0.1202
$\rho = 0.6$	$\lambda = 0$	-0.0101	-0.0104	-0.0104	-0.0104
$\rho = 0.6$	$\lambda = 0.2$	-0.0089	0.0748	0.0748	0.0748
$\rho = 0.6$	$\lambda = 0.4$	-0.0128	0.0861	0.0861	0.0861
$\rho = 0.6$	$\lambda = 0.6$	-0.0093	-0.0797	-0.0797	-0.0797
$\rho = 0.6$	$\lambda = 0.8$	-0.0056	-0.4868	-0.4868	-0.4868
$\rho = 0.8$	$\lambda = -0.8$	-0.0061	-0.5748	-0.5220	-0.5748
$\rho = 0.8$	$\lambda = -0.6$	-0.0091	-0.2667	-0.2545	-0.2667
$\rho = 0.8$	$\lambda = -0.4$	-0.0072	-0.1570	-0.1572	-0.1570
$\rho = 0.8$	$\lambda = -0.2$	-0.0111	-0.0828	-0.0831	-0.0828
$\rho = 0.8$	$\lambda = 0$	-0.0112	-0.0104	-0.0104	-0.0104
$\rho = 0.8$	$\lambda = 0.2$	-0.0112	0.0485	0.0485	0.0485
$\rho = 0.8$	$\lambda = 0.4$	-0.0131	0.0487	0.0487	0.0487
$\rho = 0.8$	$\lambda = 0.6$	-0.0090	-0.0805	-0.0805	-0.0805
$\rho = 0.8$	$\lambda = 0.8$	-0.0066	-0.4905	-0.4905	-0.4905
	average	0.0075	0.3029	0.3145	0.3029

Table 4: MSE of λ_{ML}, λ_1, λ_2, and λ_3. Values based on 2,000 replications.

		λ_{ML}	λ_1	λ_2	λ_3
$\rho = -0.8$	$\lambda = -0.8$	0.0117	0.6394	0.6394	0.6394
$\rho = -0.8$	$\lambda = -0.6$	0.0113	0.6305	0.6312	0.6305
$\rho = -0.8$	$\lambda = -0.4$	0.0115	0.5640	0.6065	0.5640
$\rho = -0.8$	$\lambda = -0.2$	0.0106	0.3460	0.5911	0.3460
$\rho = -0.8$	$\lambda = 0$	0.0097	0.6106	0.6197	0.6106
$\rho = -0.8$	$\lambda = 0.2$	0.0092	0.6217	0.6362	0.6217
$\rho = -0.8$	$\lambda = 0.4$	0.0091	0.6408	0.6408	0.6408
$\rho = -0.8$	$\lambda = 0.6$	0.0090	0.6416	0.6416	0.6416
$\rho = -0.8$	$\lambda = 0.8$	0.0083	0.6400	0.6400	0.6400
$\rho = -0.6$	$\lambda = -0.8$	0.0125	0.3600	0.3600	0.3600
$\rho = -0.6$	$\lambda = -0.6$	0.0125	0.3600	0.3600	0.3600
$\rho = -0.6$	$\lambda = -0.4$	0.0124	0.3480	0.3582	0.3480
$\rho = -0.6$	$\lambda = -0.2$	0.0116	0.2816	0.3574	0.2816
$\rho = -0.6$	$\lambda = 0$	0.0112	0.3404	0.3576	0.3404
$\rho = -0.6$	$\lambda = 0.2$	0.0110	0.3354	0.3464	0.3354
$\rho = -0.6$	$\lambda = 0.4$	0.0097	0.3739	0.3739	0.3739
$\rho = -0.6$	$\lambda = 0.6$	0.0103	0.3667	0.3667	0.3667
$\rho = -0.6$	$\lambda = 0.8$	0.0090	0.3600	0.3600	0.3600
$\rho = -0.4$	$\lambda = -0.8$	0.0129	0.1600	0.1600	0.1600
$\rho = -0.4$	$\lambda = -0.6$	0.0129	0.1604	0.1602	0.1604
$\rho = -0.4$	$\lambda = -0.4$	0.0125	0.1604	0.1602	0.1604
$\rho = -0.4$	$\lambda = -0.2$	0.0122	0.1545	0.1597	0.1545
$\rho = -0.4$	$\lambda = 0$	0.0114	0.1182	0.1542	0.1182
$\rho = -0.4$	$\lambda = 0.2$	0.0113	0.1463	0.1481	0.1463
$\rho = -0.4$	$\lambda = 0.4$	0.0101	0.1962	0.1962	0.1962
$\rho = -0.4$	$\lambda = 0.6$	0.0098	0.1779	0.1779	0.1779
$\rho = -0.4$	$\lambda = 0.8$	0.0093	0.1600	0.1600	0.1600
$\rho = -0.2$	$\lambda = -0.8$	0.0119	0.0400	0.0400	0.0400
$\rho = -0.2$	$\lambda = -0.6$	0.0125	0.0402	0.0402	0.0402
$\rho = -0.2$	$\lambda = -0.4$	0.0117	0.0407	0.0400	0.0407
$\rho = -0.2$	$\lambda = -0.2$	0.0117	0.0404	0.0393	0.0404
$\rho = -0.2$	$\lambda = 0$	0.0107	0.0205	0.0345	0.0205
$\rho = -0.2$	$\lambda = 0.2$	0.0108	0.0549	0.0550	0.0549
$\rho = -0.2$	$\lambda = 0.4$	0.0100	0.1149	0.1149	0.1149
$\rho = -0.2$	$\lambda = 0.6$	0.0097	0.0906	0.0906	0.0906
$\rho = -0.2$	$\lambda = 0.8$	0.0090	0.0400	0.0400	0.0400
$\rho = 0$	$\lambda = -0.8$	0.0111	0.0000	0.0000	0.0000
$\rho = 0$	$\lambda = -0.6$	0.0110	0.0000	0.0000	0.0000
$\rho = 0$	$\lambda = -0.4$	0.0105	0.0003	0.0001	0.0003
$\rho = 0$	$\lambda = -0.2$	0.0110	0.0023	0.0043	0.0023
$\rho = 0$	$\lambda = 0$	0.0105	0.0050	0.0026	0.0050
$\rho = 0$	$\lambda = 0.2$	0.0103	0.0363	0.0363	0.0363
$\rho = 0$	$\lambda = 0.4$	0.0098	0.1018	0.1018	0.1018
$\rho = 0$	$\lambda = 0.6$	0.0097	0.0836	0.0836	0.0836
$\rho = 0$	$\lambda = 0.8$	0.0085	0.0000	0.0000	0.0000
$\rho = 0.2$	$\lambda = -0.8$	0.0092	0.0400	0.0400	0.0400
$\rho = 0.2$	$\lambda = -0.6$	0.0093	0.0400	0.0400	0.0400
$\rho = 0.2$	$\lambda = -0.4$	0.0095	0.0401	0.0412	0.0401
$\rho = 0.2$	$\lambda = -0.2$	0.0094	0.0324	0.0321	0.0324
$\rho = 0.2$	$\lambda = 0$	0.0089	0.0088	0.0136	0.0088
$\rho = 0.2$	$\lambda = 0.2$	0.0098	0.0466	0.0466	0.0466
$\rho = 0.2$	$\lambda = 0.4$	0.0090	0.1227	0.1227	0.1227
$\rho = 0.2$	$\lambda = 0.6$	0.0083	0.1227	0.1227	0.1227
$\rho = 0.2$	$\lambda = 0.8$	0.0074	0.0416	0.0416	0.0416
$\rho = 0.4$	$\lambda = -0.8$	0.0068	0.1600	0.1600	0.1600
$\rho = 0.4$	$\lambda = -0.6$	0.0074	0.1600	0.1598	0.1600
$\rho = 0.4$	$\lambda = -0.4$	0.0076	0.1456	0.1220	0.1456
$\rho = 0.4$	$\lambda = -0.2$	0.0080	0.0387	0.0440	0.0387
$\rho = 0.4$	$\lambda = 0$	0.0082	0.0104	0.0145	0.0104
$\rho = 0.4$	$\lambda = 0.2$	0.0079	0.0486	0.0486	0.0486
$\rho = 0.4$	$\lambda = 0.4$	0.0077	0.1309	0.1309	0.1309
$\rho = 0.4$	$\lambda = 0.6$	0.0074	0.1842	0.1842	0.1842
$\rho = 0.4$	$\lambda = 0.8$	0.0064	0.1637	0.1637	0.1637
$\rho = 0.6$	$\lambda = -0.8$	0.0044	0.3599	0.3599	0.3599
$\rho = 0.6$	$\lambda = -0.6$	0.0047	0.3298	0.2969	0.3298
$\rho = 0.6$	$\lambda = -0.4$	0.0049	0.1165	0.0787	0.1165
$\rho = 0.6$	$\lambda = -0.2$	0.0056	0.0193	0.0230	0.0193
$\rho = 0.6$	$\lambda = 0$	0.0059	0.0071	0.0071	0.0071
$\rho = 0.6$	$\lambda = 0.2$	0.0060	0.0364	0.0364	0.0364
$\rho = 0.6$	$\lambda = 0.4$	0.0063	0.1069	0.1069	0.1069
$\rho = 0.6$	$\lambda = 0.6$	0.0059	0.2137	0.2137	0.2137
$\rho = 0.6$	$\lambda = 0.8$	0.0054	0.3346	0.3346	0.3346
$\rho = 0.8$	$\lambda = -0.8$	0.0019	0.3940	0.3342	0.3940
$\rho = 0.8$	$\lambda = -0.6$	0.0021	0.0890	0.0759	0.0890
$\rho = 0.8$	$\lambda = -0.4$	0.0021	0.0275	0.0277	0.0275
$\rho = 0.8$	$\lambda = -0.2$	0.0025	0.0101	0.0104	0.0101
$\rho = 0.8$	$\lambda = 0$	0.0029	0.0049	0.0049	0.0049
$\rho = 0.8$	$\lambda = 0.2$	0.0031	0.0162	0.0162	0.0162
$\rho = 0.8$	$\lambda = 0.4$	0.0037	0.0644	0.0644	0.0644
$\rho = 0.8$	$\lambda = 0.6$	0.0039	0.1828	0.1828	0.1828
$\rho = 0.8$	$\lambda = 0.8$	0.0043	0.4513	0.4513	0.4513
	average	0.0087	0.1840	0.1881	0.1840

Table 5: Bias of $\beta_{1,ML}$, $\beta_{1,1}$, $\beta_{1,2}$, and $\beta_{1,3}$. Values based on 2,000 replications.

		β_{ML}	$\beta_{1,1}$	$\beta_{1,2}$	$\beta_{1,3}$
$\rho = -0.8$	$\lambda = -0.8$	-0.0008	0.0062	0.0062	0.0062
$\rho = -0.8$	$\lambda = -0.6$	-0.0002	0.0115	0.0084	0.0115
$\rho = -0.8$	$\lambda = -0.4$	-0.0001	-0.0043	-0.2252	-0.0043
$\rho = -0.8$	$\lambda = -0.2$	0.0000	-0.0585	-0.8226	-0.0585
$\rho = -0.8$	$\lambda = 0$	0.0014	-0.9281	-0.9349	-0.9281
$\rho = -0.8$	$\lambda = 0.2$	0.0016	0.0589	-0.1448	0.0589
$\rho = -0.8$	$\lambda = 0.4$	0.0013	0.0628	0.0628	0.0628
$\rho = -0.8$	$\lambda = 0.6$	0.0009	0.0621	0.0621	0.0621
$\rho = -0.8$	$\lambda = 0.8$	0.0032	0.0536	0.0536	0.0536
$\rho = -0.6$	$\lambda = -0.8$	-0.0009	0.0049	0.0049	0.0049
$\rho = -0.6$	$\lambda = -0.6$	0.0000	0.0132	0.0099	0.0132
$\rho = -0.6$	$\lambda = -0.4$	-0.0001	0.0088	-0.2092	0.0088
$\rho = -0.6$	$\lambda = -0.2$	0.0005	-0.2131	-0.8822	-0.2131
$\rho = -0.6$	$\lambda = 0$	0.0012	-0.8349	-0.9596	-0.8349
$\rho = -0.6$	$\lambda = 0.2$	-0.0006	0.0465	-0.0995	0.0465
$\rho = -0.6$	$\lambda = 0.4$	0.0039	0.0492	0.0492	0.0492
$\rho = -0.6$	$\lambda = 0.6$	0.0024	0.0468	0.0468	0.0468
$\rho = -0.6$	$\lambda = 0.8$	0.0012	0.0401	0.0401	0.0401
$\rho = -0.4$	$\lambda = -0.8$	0.0006	0.0051	0.0051	0.0051
$\rho = -0.4$	$\lambda = -0.6$	-0.0030	0.0065	0.0033	0.0065
$\rho = -0.4$	$\lambda = -0.4$	-0.0014	0.0086	-0.2012	0.0086
$\rho = -0.4$	$\lambda = -0.2$	-0.0001	-0.6126	-0.8804	-0.6126
$\rho = -0.4$	$\lambda = 0$	-0.0001	-0.4829	-0.9321	-0.4829
$\rho = -0.4$	$\lambda = 0.2$	-0.0003	0.0368	-0.0281	0.0368
$\rho = -0.4$	$\lambda = 0.4$	0.0013	0.0321	0.0321	0.0321
$\rho = -0.4$	$\lambda = 0.6$	0.0020	0.0311	0.0311	0.0311
$\rho = -0.4$	$\lambda = 0.8$	0.0029	0.0305	0.0305	0.0305
$\rho = -0.2$	$\lambda = -0.8$	-0.0003	0.0023	0.0023	0.0023
$\rho = -0.2$	$\lambda = -0.6$	0.0002	0.0054	0.0045	0.0054
$\rho = -0.2$	$\lambda = -0.4$	-0.0023	-0.0032	-0.1732	-0.0032
$\rho = -0.2$	$\lambda = -0.2$	0.0004	-0.8671	-0.8743	-0.8671
$\rho = -0.2$	$\lambda = 0$	-0.0002	-0.1518	-0.7917	-0.1518
$\rho = -0.2$	$\lambda = 0.2$	0.0005	0.0266	0.0077	0.0266
$\rho = -0.2$	$\lambda = 0.4$	-0.0004	0.0136	0.0136	0.0136
$\rho = -0.2$	$\lambda = 0.6$	0.0010	0.0122	0.0122	0.0122
$\rho = -0.2$	$\lambda = 0.8$	0.0024	0.0167	0.0167	0.0167
$\rho = 0$	$\lambda = -0.8$	-0.0014	-0.0009	-0.0009	-0.0009
$\rho = 0$	$\lambda = -0.6$	-0.0022	-0.0017	-0.0032	-0.0017
$\rho = 0$	$\lambda = -0.4$	-0.0025	-0.0954	-0.1885	-0.0954
$\rho = 0$	$\lambda = -0.2$	-0.0015	-0.8644	-0.7532	-0.8644
$\rho = 0$	$\lambda = 0$	0.0009	-0.0237	-0.5149	-0.0237
$\rho = 0$	$\lambda = 0.2$	0.0003	0.0124	0.0070	0.0124
$\rho = 0$	$\lambda = 0.4$	0.0009	-0.0047	-0.0047	-0.0047
$\rho = 0$	$\lambda = 0.6$	0.0002	-0.0102	-0.0102	-0.0102
$\rho = 0$	$\lambda = 0.8$	0.0034	0.0031	0.0031	0.0031
$\rho = 0.2$	$\lambda = -0.8$	-0.0009	-0.0030	-0.0030	-0.0030
$\rho = 0.2$	$\lambda = -0.6$	0.0009	-0.0055	-0.0059	-0.0055
$\rho = 0.2$	$\lambda = -0.4$	-0.0011	-0.4284	-0.1995	-0.4284
$\rho = 0.2$	$\lambda = -0.2$	-0.0021	-0.5047	-0.4634	-0.5047
$\rho = 0.2$	$\lambda = 0$	0.0007	-0.0017	-0.2019	-0.0017
$\rho = 0.2$	$\lambda = 0.2$	0.0000	-0.0038	-0.0043	-0.0038
$\rho = 0.2$	$\lambda = 0.4$	-0.0017	-0.0290	-0.0290	-0.0290
$\rho = 0.2$	$\lambda = 0.6$	-0.0009	-0.0364	-0.0364	-0.0364
$\rho = 0.2$	$\lambda = 0.8$	0.0010	-0.0179	-0.0179	-0.0179
$\rho = 0.4$	$\lambda = -0.8$	0.0011	-0.0048	-0.0048	-0.0048
$\rho = 0.4$	$\lambda = -0.6$	-0.0030	-0.0486	-0.0218	-0.0486
$\rho = 0.4$	$\lambda = -0.4$	0.0001	-0.7024	-0.1775	-0.7024
$\rho = 0.4$	$\lambda = -0.2$	-0.0008	-0.0967	-0.1424	-0.0967
$\rho = 0.4$	$\lambda = 0$	-0.0005	-0.0015	-0.0372	-0.0015
$\rho = 0.4$	$\lambda = 0.2$	0.0006	-0.0174	-0.0174	-0.0174
$\rho = 0.4$	$\lambda = 0.4$	-0.0010	-0.0491	-0.0491	-0.0491
$\rho = 0.4$	$\lambda = 0.6$	-0.0001	-0.0642	-0.0642	-0.0642
$\rho = 0.4$	$\lambda = 0.8$	0.0011	-0.0477	-0.0477	-0.0477
$\rho = 0.6$	$\lambda = -0.8$	-0.0005	-0.0136	-0.0126	-0.0136
$\rho = 0.6$	$\lambda = -0.6$	-0.0002	-0.4083	-0.0223	-0.4083
$\rho = 0.6$	$\lambda = -0.4$	-0.0004	-0.1784	-0.0135	-0.1784
$\rho = 0.6$	$\lambda = -0.2$	-0.0026	0.0132	0.0017	0.0132
$\rho = 0.6$	$\lambda = 0$	-0.0008	-0.0011	-0.0011	-0.0011
$\rho = 0.6$	$\lambda = 0.2$	-0.0013	-0.0303	-0.0303	-0.0303
$\rho = 0.6$	$\lambda = 0.4$	-0.0017	-0.0651	-0.0651	-0.0651
$\rho = 0.6$	$\lambda = 0.6$	-0.0031	-0.0967	-0.0967	-0.0967
$\rho = 0.6$	$\lambda = 0.8$	-0.0018	-0.0965	-0.0965	-0.0965
$\rho = 0.8$	$\lambda = -0.8$	0.0015	-0.1822	0.0630	-0.1822
$\rho = 0.8$	$\lambda = -0.6$	0.0007	0.0611	0.0990	0.0611
$\rho = 0.8$	$\lambda = -0.4$	-0.0010	0.0695	0.0691	0.0695
$\rho = 0.8$	$\lambda = -0.2$	0.0010	0.0350	0.0343	0.0350
$\rho = 0.8$	$\lambda = 0$	-0.0007	-0.0015	-0.0015	-0.0015
$\rho = 0.8$	$\lambda = 0.2$	-0.0011	-0.0373	-0.0373	-0.0373
$\rho = 0.8$	$\lambda = 0.4$	0.0006	-0.0730	-0.0730	-0.0730
$\rho = 0.8$	$\lambda = 0.6$	-0.0025	-0.1132	-0.1132	-0.1132
$\rho = 0.8$	$\lambda = 0.8$	-0.0035	-0.1526	-0.1526	-0.1526
	average	0.0012	0.1180	0.1562	0.1180

Table 6: MSE of $\beta_{1,ML}$, $\beta_{1,1}$, $\beta_{1,2}$, and $\beta_{1,3}$. Values based on 2,000 replications.

		β_{ML}	$\beta_{1,1}$	$\beta_{1,2}$	$\beta_{1,3}$
$\rho = -0.8$	$\lambda = -0.8$	0.0017	0.0022	0.0022	0.0022
$\rho = -0.8$	$\lambda = -0.6$	0.0019	0.0041	0.0074	0.0041
$\rho = -0.8$	$\lambda = -0.4$	0.0020	0.0084	0.2287	0.0084
$\rho = -0.8$	$\lambda = -0.2$	0.0020	0.0309	0.7954	0.0309
$\rho = -0.8$	$\lambda = 0$	0.0021	0.9126	0.9196	0.9126
$\rho = -0.8$	$\lambda = 0.2$	0.0023	0.0072	0.1935	0.0072
$\rho = -0.8$	$\lambda = 0.4$	0.0023	0.0070	0.0070	0.0070
$\rho = -0.8$	$\lambda = 0.6$	0.0022	0.0067	0.0067	0.0067
$\rho = -0.8$	$\lambda = 0.8$	0.0023	0.0058	0.0058	0.0058
$\rho = -0.6$	$\lambda = -0.8$	0.0018	0.0019	0.0019	0.0019
$\rho = -0.6$	$\lambda = -0.6$	0.0019	0.0025	0.0057	0.0025
$\rho = -0.6$	$\lambda = -0.4$	0.0019	0.0045	0.2174	0.0045
$\rho = -0.6$	$\lambda = -0.2$	0.0021	0.2044	0.8607	0.2044
$\rho = -0.6$	$\lambda = 0$	0.0022	0.8236	0.9438	0.8236
$\rho = -0.6$	$\lambda = 0.2$	0.0022	0.0048	0.1400	0.0048
$\rho = -0.6$	$\lambda = 0.4$	0.0022	0.0049	0.0049	0.0049
$\rho = -0.6$	$\lambda = 0.6$	0.0023	0.0047	0.0047	0.0047
$\rho = -0.6$	$\lambda = 0.8$	0.0023	0.0043	0.0043	0.0043
$\rho = -0.4$	$\lambda = -0.8$	0.0018	0.0019	0.0019	0.0019
$\rho = -0.4$	$\lambda = -0.6$	0.0019	0.0020	0.0052	0.0020
$\rho = -0.4$	$\lambda = -0.4$	0.0019	0.0029	0.2061	0.0029
$\rho = -0.4$	$\lambda = -0.2$	0.0020	0.5982	0.8594	0.5982
$\rho = -0.4$	$\lambda = 0$	0.0021	0.4824	0.9185	0.4824
$\rho = -0.4$	$\lambda = 0.2$	0.0024	0.0041	0.0652	0.0041
$\rho = -0.4$	$\lambda = 0.4$	0.0023	0.0034	0.0034	0.0034
$\rho = -0.4$	$\lambda = 0.6$	0.0022	0.0035	0.0035	0.0035
$\rho = -0.4$	$\lambda = 0.8$	0.0024	0.0035	0.0035	0.0035
$\rho = -0.2$	$\lambda = -0.8$	0.0018	0.0018	0.0018	0.0018
$\rho = -0.2$	$\lambda = -0.6$	0.0019	0.0020	0.0028	0.0020
$\rho = -0.2$	$\lambda = -0.4$	0.0020	0.0100	0.1731	0.0100
$\rho = -0.2$	$\lambda = -0.2$	0.0020	0.8440	0.8509	0.8440
$\rho = -0.2$	$\lambda = 0$	0.0022	0.1553	0.7796	0.1553
$\rho = -0.2$	$\lambda = 0.2$	0.0022	0.0032	0.0213	0.0032
$\rho = -0.2$	$\lambda = 0.4$	0.0024	0.0027	0.0027	0.0027
$\rho = -0.2$	$\lambda = 0.6$	0.0023	0.0028	0.0028	0.0028
$\rho = -0.2$	$\lambda = 0.8$	0.0023	0.0026	0.0026	0.0026
$\rho = 0$	$\lambda = -0.8$	0.0020	0.0020	0.0020	0.0020
$\rho = 0$	$\lambda = -0.6$	0.0020	0.0020	0.0034	0.0020
$\rho = 0$	$\lambda = -0.4$	0.0020	0.0919	0.1815	0.0919
$\rho = 0$	$\lambda = -0.2$	0.0021	0.8397	0.7286	0.8397
$\rho = 0$	$\lambda = 0$	0.0020	0.0261	0.5101	0.0261
$\rho = 0$	$\lambda = 0.2$	0.0022	0.0025	0.0081	0.0025
$\rho = 0$	$\lambda = 0.4$	0.0023	0.0024	0.0024	0.0024
$\rho = 0$	$\lambda = 0.6$	0.0023	0.0028	0.0028	0.0028
$\rho = 0$	$\lambda = 0.8$	0.0024	0.0023	0.0023	0.0023
$\rho = 0.2$	$\lambda = -0.8$	0.0022	0.0022	0.0022	0.0022
$\rho = 0.2$	$\lambda = -0.6$	0.0021	0.0030	0.0035	0.0030
$\rho = 0.2$	$\lambda = -0.4$	0.0022	0.4094	0.1871	0.4094
$\rho = 0.2$	$\lambda = -0.2$	0.0022	0.4853	0.4427	0.4853
$\rho = 0.2$	$\lambda = 0$	0.0022	0.0036	0.2024	0.0036
$\rho = 0.2$	$\lambda = 0.2$	0.0022	0.0024	0.0029	0.0024
$\rho = 0.2$	$\lambda = 0.4$	0.0023	0.0032	0.0032	0.0032
$\rho = 0.2$	$\lambda = 0.6$	0.0022	0.0040	0.0040	0.0040
$\rho = 0.2$	$\lambda = 0.8$	0.0023	0.0026	0.0026	0.0026
$\rho = 0.4$	$\lambda = -0.8$	0.0023	0.0026	0.0026	0.0026
$\rho = 0.4$	$\lambda = -0.6$	0.0024	0.0334	0.0082	0.0334
$\rho = 0.4$	$\lambda = -0.4$	0.0022	0.6741	0.1592	0.6741
$\rho = 0.4$	$\lambda = -0.2$	0.0023	0.0962	0.1405	0.0962
$\rho = 0.4$	$\lambda = 0$	0.0022	0.0024	0.0387	0.0024
$\rho = 0.4$	$\lambda = 0.2$	0.0023	0.0029	0.0029	0.0029
$\rho = 0.4$	$\lambda = 0.4$	0.0022	0.0047	0.0047	0.0047
$\rho = 0.4$	$\lambda = 0.6$	0.0023	0.0066	0.0066	0.0066
$\rho = 0.4$	$\lambda = 0.8$	0.0022	0.0048	0.0048	0.0048
$\rho = 0.6$	$\lambda = -0.8$	0.0026	0.0044	0.0035	0.0044
$\rho = 0.6$	$\lambda = -0.6$	0.0025	0.3827	0.0127	0.3827
$\rho = 0.6$	$\lambda = -0.4$	0.0025	0.2017	0.0414	0.2017
$\rho = 0.6$	$\lambda = -0.2$	0.0024	0.0051	0.0162	0.0051
$\rho = 0.6$	$\lambda = 0$	0.0024	0.0024	0.0024	0.0024
$\rho = 0.6$	$\lambda = 0.2$	0.0023	0.0033	0.0033	0.0033
$\rho = 0.6$	$\lambda = 0.4$	0.0024	0.0068	0.0068	0.0068
$\rho = 0.6$	$\lambda = 0.6$	0.0023	0.0118	0.0118	0.0118
$\rho = 0.6$	$\lambda = 0.8$	0.0023	0.0122	0.0122	0.0122
$\rho = 0.8$	$\lambda = -0.8$	0.0028	0.2379	0.0206	0.2379
$\rho = 0.8$	$\lambda = -0.6$	0.0027	0.0528	0.0177	0.0528
$\rho = 0.8$	$\lambda = -0.4$	0.0026	0.0074	0.0079	0.0074
$\rho = 0.8$	$\lambda = -0.2$	0.0024	0.0036	0.0046	0.0036
$\rho = 0.8$	$\lambda = 0$	0.0025	0.0023	0.0023	0.0023
$\rho = 0.8$	$\lambda = 0.2$	0.0025	0.0038	0.0038	0.0038
$\rho = 0.8$	$\lambda = 0.4$	0.0025	0.0079	0.0079	0.0079
$\rho = 0.8$	$\lambda = 0.6$	0.0023	0.0159	0.0159	0.0159
$\rho = 0.8$	$\lambda = 0.8$	0.0024	0.0270	0.0270	0.0270
	average	0.0022	0.0972	0.1374	0.0972

Table 7: Bias of $\beta_{2,ML}$, $\beta_{2,1}$, $\beta_{2,2}$, and $\beta_{2,3}$. Values based on 2,000 replications.

		$\beta_{2,ML}$	$\beta_{2,1}$	$\beta_{2,2}$	$\beta_{3,1}$
$\rho = -0.8$	$\lambda = -0.8$	-0.0012	-0.0230	-0.0230	-0.0230
$\rho = -0.8$	$\lambda = -0.6$	0.0016	-0.0147	-0.0138	-0.0147
$\rho = -0.8$	$\lambda = -0.4$	0.0010	-0.0172	0.0453	-0.0172
$\rho = -0.8$	$\lambda = -0.2$	0.0004	-0.0170	0.1912	-0.0170
$\rho = -0.8$	$\lambda = 0$	0.0012	0.2273	0.2304	0.2273
$\rho = -0.8$	$\lambda = 0.2$	0.0011	0.0165	0.0639	0.0165
$\rho = -0.8$	$\lambda = 0.4$	-0.0001	0.0201	0.0201	0.0201
$\rho = -0.8$	$\lambda = 0.6$	0.0015	0.0231	0.0231	0.0231
$\rho = -0.8$	$\lambda = 0.8$	0.0017	0.0196	0.0196	0.0196
$\rho = -0.6$	$\lambda = -0.8$	0.0008	-0.0143	-0.0143	-0.0143
$\rho = -0.6$	$\lambda = -0.6$	-0.0017	-0.0121	-0.0112	-0.0121
$\rho = -0.6$	$\lambda = -0.4$	-0.0020	-0.0100	0.0469	-0.0100
$\rho = -0.6$	$\lambda = -0.2$	-0.0018	0.0380	0.2072	0.0380
$\rho = -0.6$	$\lambda = 0$	0.0000	0.2043	0.2345	0.2043
$\rho = -0.6$	$\lambda = 0.2$	0.0006	0.0129	0.0471	0.0129
$\rho = -0.6$	$\lambda = 0.4$	-0.0004	0.0154	0.0154	0.0154
$\rho = -0.6$	$\lambda = 0.6$	0.0001	0.0161	0.0161	0.0161
$\rho = -0.6$	$\lambda = 0.8$	0.0011	0.0152	0.0152	0.0152
$\rho = -0.4$	$\lambda = -0.8$	-0.0003	-0.0106	-0.0106	-0.0106
$\rho = -0.4$	$\lambda = -0.6$	-0.0013	-0.0079	-0.0070	-0.0079
$\rho = -0.4$	$\lambda = -0.4$	0.0002	-0.0036	0.0482	-0.0036
$\rho = -0.4$	$\lambda = -0.2$	0.0007	0.1444	0.2097	0.1444
$\rho = -0.4$	$\lambda = 0$	0.0002	0.1183	0.2270	0.1183
$\rho = -0.4$	$\lambda = 0.2$	0.0002	0.0100	0.0254	0.0100
$\rho = -0.4$	$\lambda = 0.4$	-0.0007	0.0094	0.0094	0.0094
$\rho = -0.4$	$\lambda = 0.6$	0.0009	0.0113	0.0113	0.0113
$\rho = -0.4$	$\lambda = 0.8$	-0.0003	0.0092	0.0092	0.0092
$\rho = -0.2$	$\lambda = -0.8$	-0.0008	-0.0062	-0.0062	-0.0062
$\rho = -0.2$	$\lambda = -0.6$	-0.0004	-0.0037	-0.0035	-0.0037
$\rho = -0.2$	$\lambda = -0.4$	0.0014	0.0023	0.0427	0.0023
$\rho = -0.2$	$\lambda = -0.2$	0.0004	0.2070	0.2074	0.2070
$\rho = -0.2$	$\lambda = 0$	-0.0003	0.0366	0.1925	0.0366
$\rho = -0.2$	$\lambda = 0.2$	-0.0007	0.0054	0.0099	0.0054
$\rho = -0.2$	$\lambda = 0.4$	0.0001	0.0033	0.0033	0.0033
$\rho = -0.2$	$\lambda = 0.6$	-0.0005	0.0028	0.0028	0.0028
$\rho = -0.2$	$\lambda = 0.8$	0.0018	0.0072	0.0072	0.0072
$\rho = 0$	$\lambda = -0.8$	-0.0020	-0.0019	-0.0019	-0.0019
$\rho = 0$	$\lambda = -0.6$	0.0000	0.0002	0.0005	0.0002
$\rho = 0$	$\lambda = -0.4$	-0.0012	0.0217	0.0436	0.0217
$\rho = 0$	$\lambda = -0.2$	-0.0016	0.2024	0.1729	0.2024
$\rho = 0$	$\lambda = 0$	-0.0007	0.0053	0.1253	0.0053
$\rho = 0$	$\lambda = 0.2$	0.0000	0.0010	0.0024	0.0010
$\rho = 0$	$\lambda = 0.4$	-0.0014	-0.0076	-0.0076	-0.0076
$\rho = 0$	$\lambda = 0.6$	0.0014	-0.0045	-0.0045	-0.0045
$\rho = 0$	$\lambda = 0.8$	0.0014	0.0013	0.0013	0.0013
$\rho = 0.2$	$\lambda = -0.8$	0.0003	0.0066	0.0066	0.0066
$\rho = 0.2$	$\lambda = -0.6$	-0.0012	0.0032	0.0033	0.0032
$\rho = 0.2$	$\lambda = -0.4$	0.0012	0.1056	0.0480	0.1056
$\rho = 0.2$	$\lambda = -0.2$	-0.0003	0.1186	0.1089	0.1186
$\rho = 0.2$	$\lambda = 0$	-0.0013	0.0001	0.0498	0.0001
$\rho = 0.2$	$\lambda = 0.2$	0.0011	-0.0059	-0.0058	-0.0059
$\rho = 0.2$	$\lambda = 0.4$	0.0008	-0.0175	-0.0175	-0.0175
$\rho = 0.2$	$\lambda = 0.6$	0.0005	-0.0173	-0.0173	-0.0173
$\rho = 0.2$	$\lambda = 0.8$	-0.0009	-0.0080	-0.0080	-0.0080
$\rho = 0.4$	$\lambda = -0.8$	-0.0001	0.0138	0.0138	0.0138
$\rho = 0.4$	$\lambda = -0.6$	-0.0010	0.0161	0.0093	0.0161
$\rho = 0.4$	$\lambda = -0.4$	-0.0007	0.1775	0.0530	0.1775
$\rho = 0.4$	$\lambda = -0.2$	0.0008	0.0369	0.0487	0.0369
$\rho = 0.4$	$\lambda = 0$	0.0008	0.0021	0.0111	0.0021
$\rho = 0.4$	$\lambda = 0.2$	-0.0015	-0.0175	-0.0175	-0.0175
$\rho = 0.4$	$\lambda = 0.4$	-0.0005	-0.0332	-0.0332	-0.0332
$\rho = 0.4$	$\lambda = 0.6$	-0.0009	-0.0341	-0.0341	-0.0341
$\rho = 0.4$	$\lambda = 0.8$	0.0022	-0.0154	-0.0154	-0.0154
$\rho = 0.6$	$\lambda = -0.8$	-0.0001	0.0270	0.0268	0.0270
$\rho = 0.6$	$\lambda = -0.6$	0.0002	0.1250	0.0345	0.1250
$\rho = 0.6$	$\lambda = -0.4$	-0.0023	0.0890	0.0555	0.0890
$\rho = 0.6$	$\lambda = -0.2$	-0.0012	0.0262	0.0291	0.0262
$\rho = 0.6$	$\lambda = 0$	0.0010	0.0017	0.0017	0.0017
$\rho = 0.6$	$\lambda = 0.2$	0.0000	-0.0248	-0.0248	-0.0248
$\rho = 0.6$	$\lambda = 0.4$	0.0004	-0.0465	-0.0465	-0.0465
$\rho = 0.6$	$\lambda = 0.6$	-0.0010	-0.0552	-0.0552	-0.0552
$\rho = 0.6$	$\lambda = 0.8$	0.0009	-0.0378	-0.0378	-0.0378
$\rho = 0.8$	$\lambda = -0.8$	-0.0018	0.1645	0.1154	0.1645
$\rho = 0.8$	$\lambda = -0.6$	0.0007	0.1235	0.1164	0.1235
$\rho = 0.8$	$\lambda = -0.4$	-0.0002	0.0745	0.0746	0.0745
$\rho = 0.8$	$\lambda = -0.2$	-0.0008	0.0345	0.0347	0.0345
$\rho = 0.8$	$\lambda = 0$	0.0019	0.0022	0.0022	0.0022
$\rho = 0.8$	$\lambda = 0.2$	-0.0019	-0.0345	-0.0345	-0.0345
$\rho = 0.8$	$\lambda = 0.4$	-0.0013	-0.0604	-0.0604	-0.0604
$\rho = 0.8$	$\lambda = 0.6$	-0.0017	-0.0757	-0.0757	-0.0757
$\rho = 0.8$	$\lambda = 0.8$	-0.0002	-0.0669	-0.0669	-0.0669
	average	0.0009	0.0403	0.0497	0.0403

Table 8: MSE of $\beta_{2,ML}$, $\beta_{2,1}$, $\beta_{2,2}$, and $\beta_{2,3}$. Values based on 2,000 replications.

		$\beta_{2,ML}$	$\lambda_{2,1}$	$\lambda_{2,2}$	$\lambda_{2,3}$
$\rho = -0.8$	$\lambda = -0.8$	0.0018	0.0027	0.0027	0.0027
$\rho = -0.8$	$\lambda = -0.6$	0.0020	0.0030	0.0030	0.0030
$\rho = -0.8$	$\lambda = -0.4$	0.0018	0.0041	0.0124	0.0041
$\rho = -0.8$	$\lambda = -0.2$	0.0020	0.0053	0.0469	0.0053
$\rho = -0.8$	$\lambda = 0$	0.0020	0.0572	0.0585	0.0572
$\rho = -0.8$	$\lambda = 0.2$	0.0019	0.0027	0.0190	0.0027
$\rho = -0.8$	$\lambda = 0.4$	0.0021	0.0031	0.0031	0.0031
$\rho = -0.8$	$\lambda = 0.6$	0.0021	0.0032	0.0032	0.0032
$\rho = -0.8$	$\lambda = 0.8$	0.0020	0.0030	0.0030	0.0030
$\rho = -0.6$	$\lambda = -0.8$	0.0019	0.0023	0.0023	0.0023
$\rho = -0.6$	$\lambda = -0.6$	0.0019	0.0023	0.0024	0.0023
$\rho = -0.6$	$\lambda = -0.4$	0.0020	0.0028	0.0107	0.0028
$\rho = -0.6$	$\lambda = -0.2$	0.0020	0.0123	0.0482	0.0123
$\rho = -0.6$	$\lambda = 0$	0.0020	0.0517	0.0587	0.0517
$\rho = -0.6$	$\lambda = 0.2$	0.0020	0.0024	0.0132	0.0024
$\rho = -0.6$	$\lambda = 0.4$	0.0019	0.0026	0.0026	0.0026
$\rho = -0.6$	$\lambda = 0.6$	0.0021	0.0027	0.0027	0.0027
$\rho = -0.6$	$\lambda = 0.8$	0.0020	0.0026	0.0026	0.0026
$\rho = -0.4$	$\lambda = -0.8$	0.0019	0.0021	0.0021	0.0021
$\rho = -0.4$	$\lambda = -0.6$	0.0020	0.0021	0.0021	0.0021
$\rho = -0.4$	$\lambda = -0.4$	0.0020	0.0023	0.0109	0.0023
$\rho = -0.4$	$\lambda = -0.2$	0.0020	0.0353	0.0493	0.0353
$\rho = -0.4$	$\lambda = 0$	0.0020	0.0313	0.0565	0.0313
$\rho = -0.4$	$\lambda = 0.2$	0.0020	0.0022	0.0068	0.0022
$\rho = -0.4$	$\lambda = 0.4$	0.0021	0.0023	0.0023	0.0023
$\rho = -0.4$	$\lambda = 0.6$	0.0022	0.0024	0.0024	0.0024
$\rho = -0.4$	$\lambda = 0.8$	0.0020	0.0023	0.0023	0.0023
$\rho = -0.2$	$\lambda = -0.8$	0.0021	0.0021	0.0021	0.0021
$\rho = -0.2$	$\lambda = -0.6$	0.0021	0.0022	0.0022	0.0022
$\rho = -0.2$	$\lambda = -0.4$	0.0021	0.0025	0.0102	0.0025
$\rho = -0.2$	$\lambda = -0.2$	0.0021	0.0497	0.0495	0.0497
$\rho = -0.2$	$\lambda = 0$	0.0021	0.0114	0.0476	0.0114
$\rho = -0.2$	$\lambda = 0.2$	0.0020	0.0021	0.0034	0.0021
$\rho = -0.2$	$\lambda = 0.4$	0.0021	0.0021	0.0021	0.0021
$\rho = -0.2$	$\lambda = 0.6$	0.0022	0.0023	0.0023	0.0023
$\rho = -0.2$	$\lambda = 0.8$	0.0021	0.0022	0.0022	0.0022
$\rho = 0$	$\lambda = -0.8$	0.0021	0.0021	0.0021	0.0021
$\rho = 0$	$\lambda = -0.6$	0.0022	0.0021	0.0022	0.0021
$\rho = 0$	$\lambda = -0.4$	0.0021	0.0068	0.0108	0.0068
$\rho = 0$	$\lambda = -0.2$	0.0021	0.0487	0.0419	0.0487
$\rho = 0$	$\lambda = 0$	0.0020	0.0034	0.0314	0.0034
$\rho = 0$	$\lambda = 0.2$	0.0022	0.0022	0.0026	0.0022
$\rho = 0$	$\lambda = 0.4$	0.0022	0.0023	0.0023	0.0023
$\rho = 0$	$\lambda = 0.6$	0.0023	0.0025	0.0025	0.0025
$\rho = 0$	$\lambda = 0.8$	0.0022	0.0022	0.0022	0.0022
$\rho = 0.2$	$\lambda = -0.8$	0.0022	0.0022	0.0022	0.0022
$\rho = 0.2$	$\lambda = -0.6$	0.0021	0.0022	0.0022	0.0022
$\rho = 0.2$	$\lambda = -0.4$	0.0022	0.0259	0.0124	0.0259
$\rho = 0.2$	$\lambda = -0.2$	0.0023	0.0294	0.0271	0.0294
$\rho = 0.2$	$\lambda = 0$	0.0022	0.0023	0.0140	0.0023
$\rho = 0.2$	$\lambda = 0.2$	0.0021	0.0022	0.0022	0.0022
$\rho = 0.2$	$\lambda = 0.4$	0.0021	0.0026	0.0026	0.0026
$\rho = 0.2$	$\lambda = 0.6$	0.0023	0.0032	0.0032	0.0032
$\rho = 0.2$	$\lambda = 0.8$	0.0022	0.0023	0.0023	0.0023
$\rho = 0.4$	$\lambda = -0.8$	0.0024	0.0028	0.0028	0.0028
$\rho = 0.4$	$\lambda = -0.6$	0.0023	0.0045	0.0028	0.0045
$\rho = 0.4$	$\lambda = -0.4$	0.0023	0.0436	0.0129	0.0436
$\rho = 0.4$	$\lambda = -0.2$	0.0023	0.0078	0.0111	0.0078
$\rho = 0.4$	$\lambda = 0$	0.0022	0.0022	0.0043	0.0022
$\rho = 0.4$	$\lambda = 0.2$	0.0021	0.0024	0.0024	0.0024
$\rho = 0.4$	$\lambda = 0.4$	0.0022	0.0037	0.0037	0.0037
$\rho = 0.4$	$\lambda = 0.6$	0.0021	0.0041	0.0041	0.0041
$\rho = 0.4$	$\lambda = 0.8$	0.0022	0.0026	0.0026	0.0026
$\rho = 0.6$	$\lambda = -0.8$	0.0025	0.0037	0.0036	0.0037
$\rho = 0.6$	$\lambda = -0.6$	0.0022	0.0312	0.0050	0.0312
$\rho = 0.6$	$\lambda = -0.4$	0.0024	0.0166	0.0074	0.0166
$\rho = 0.6$	$\lambda = -0.2$	0.0022	0.0030	0.0037	0.0030
$\rho = 0.6$	$\lambda = 0$	0.0023	0.0022	0.0022	0.0022
$\rho = 0.6$	$\lambda = 0.2$	0.0021	0.0026	0.0026	0.0026
$\rho = 0.6$	$\lambda = 0.4$	0.0021	0.0046	0.0046	0.0046
$\rho = 0.6$	$\lambda = 0.6$	0.0020	0.0059	0.0059	0.0059
$\rho = 0.6$	$\lambda = 0.8$	0.0022	0.0044	0.0044	0.0044
$\rho = 0.8$	$\lambda = -0.8$	0.0024	0.0386	0.0198	0.0386
$\rho = 0.8$	$\lambda = -0.6$	0.0023	0.0187	0.0164	0.0187
$\rho = 0.8$	$\lambda = -0.4$	0.0023	0.0078	0.0079	0.0078
$\rho = 0.8$	$\lambda = -0.2$	0.0023	0.0032	0.0033	0.0032
$\rho = 0.8$	$\lambda = 0$	0.0023	0.0020	0.0020	0.0020
$\rho = 0.8$	$\lambda = 0.2$	0.0023	0.0032	0.0032	0.0032
$\rho = 0.8$	$\lambda = 0.4$	0.0022	0.0057	0.0057	0.0057
$\rho = 0.8$	$\lambda = 0.6$	0.0023	0.0083	0.0083	0.0083
$\rho = 0.8$	$\lambda = 0.8$	0.0021	0.0076	0.0076	0.0076
	average	0.0021	0.0088	0.0108	0.0088

Table 9: Size of test for the full model. Values based on 2,000 replications.

		ρ	λ	β_1	β_2
$\rho = -0.8$	$\lambda = -0.8$	0.0495	0.0180	0.0435	0.0535
$\rho = -0.8$	$\lambda = -0.6$	0.0570	0.0360	0.0475	0.0565
$\rho = -0.8$	$\lambda = -0.4$	0.0565	0.0530	0.0560	0.0505
$\rho = -0.8$	$\lambda = -0.2$	0.0475	0.0595	0.0455	0.0585
$\rho = -0.8$	$\lambda = 0$	0.0595	0.0550	0.0455	0.0540
$\rho = -0.8$	$\lambda = 0.2$	0.0535	0.0495	0.0545	0.0465
$\rho = -0.8$	$\lambda = 0.4$	0.0545	0.0495	0.0530	0.0580
$\rho = -0.8$	$\lambda = 0.6$	0.0435	0.0485	0.0485	0.0510
$\rho = -0.8$	$\lambda = 0.8$	0.0580	0.0570	0.0565	0.0590
$\rho = -0.6$	$\lambda = -0.8$	0.0500	0.0510	0.0480	0.0520
$\rho = -0.6$	$\lambda = -0.6$	0.0510	0.0555	0.0495	0.0515
$\rho = -0.6$	$\lambda = -0.4$	0.0410	0.0535	0.0505	0.0580
$\rho = -0.6$	$\lambda = -0.2$	0.0620	0.0580	0.0555	0.0500
$\rho = -0.6$	$\lambda = 0$	0.0520	0.0575	0.0555	0.0555
$\rho = -0.6$	$\lambda = 0.2$	0.0590	0.0585	0.0510	0.0475
$\rho = -0.6$	$\lambda = 0.4$	0.0565	0.0490	0.0530	0.0465
$\rho = -0.6$	$\lambda = 0.6$	0.0570	0.0680	0.0515	0.0570
$\rho = -0.6$	$\lambda = 0.8$	0.0460	0.0540	0.0545	0.0525
$\rho = -0.4$	$\lambda = -0.8$	0.0595	0.0515	0.0470	0.0465
$\rho = -0.4$	$\lambda = -0.6$	0.0615	0.0605	0.0540	0.0515
$\rho = -0.4$	$\lambda = -0.4$	0.0575	0.0525	0.0540	0.0500
$\rho = -0.4$	$\lambda = -0.2$	0.0530	0.0520	0.0455	0.0505
$\rho = -0.4$	$\lambda = 0$	0.0485	0.0510	0.0480	0.0485
$\rho = -0.4$	$\lambda = 0.2$	0.0545	0.0500	0.0620	0.0460
$\rho = -0.4$	$\lambda = 0.4$	0.0490	0.0480	0.0520	0.0560
$\rho = -0.4$	$\lambda = 0.6$	0.0585	0.0575	0.0460	0.0565
$\rho = -0.4$	$\lambda = 0.8$	0.0510	0.0530	0.0600	0.0490
$\rho = -0.2$	$\lambda = -0.8$	0.0560	0.0560	0.0485	0.0515
$\rho = -0.2$	$\lambda = -0.6$	0.0485	0.0545	0.0515	0.0605
$\rho = -0.2$	$\lambda = -0.4$	0.0595	0.0495	0.0555	0.0575
$\rho = -0.2$	$\lambda = -0.2$	0.0465	0.0530	0.0480	0.0565
$\rho = -0.2$	$\lambda = 0$	0.0555	0.0445	0.0510	0.0575
$\rho = -0.2$	$\lambda = 0.2$	0.0405	0.0550	0.0475	0.0500
$\rho = -0.2$	$\lambda = 0.4$	0.0545	0.0510	0.0565	0.0500
$\rho = -0.2$	$\lambda = 0.6$	0.0465	0.0535	0.0490	0.0625
$\rho = -0.2$	$\lambda = 0.8$	0.0495	0.0530	0.0515	0.0540
$\rho = 0$	$\lambda = -0.8$	0.0565	0.0580	0.0495	0.0520
$\rho = 0$	$\lambda = -0.6$	0.0545	0.0530	0.0560	0.0575
$\rho = 0$	$\lambda = -0.4$	0.0595	0.0530	0.0470	0.0610
$\rho = 0$	$\lambda = -0.2$	0.0520	0.0570	0.0500	0.0495
$\rho = 0$	$\lambda = 0$	0.0590	0.0615	0.0455	0.0470
$\rho = 0$	$\lambda = 0.2$	0.0495	0.0610	0.0535	0.0575
$\rho = 0$	$\lambda = 0.4$	0.0575	0.0510	0.0535	0.0490
$\rho = 0$	$\lambda = 0.6$	0.0445	0.0580	0.0430	0.0630
$\rho = 0$	$\lambda = 0.8$	0.0505	0.0645	0.0480	0.0615
$\rho = 0.2$	$\lambda = -0.8$	0.0545	0.0555	0.0525	0.0525
$\rho = 0.2$	$\lambda = -0.6$	0.0580	0.0575	0.0490	0.0440
$\rho = 0.2$	$\lambda = -0.4$	0.0510	0.0545	0.0495	0.0500
$\rho = 0.2$	$\lambda = -0.2$	0.0590	0.0530	0.0570	0.0540
$\rho = 0.2$	$\lambda = 0$	0.0590	0.0415	0.0545	0.0500
$\rho = 0.2$	$\lambda = 0.2$	0.0530	0.0635	0.0550	0.0545
$\rho = 0.2$	$\lambda = 0.4$	0.0460	0.0520	0.0530	0.0450
$\rho = 0.2$	$\lambda = 0.6$	0.0560	0.0505	0.0440	0.0605
$\rho = 0.2$	$\lambda = 0.8$	0.0510	0.0540	0.0505	0.0540
$\rho = 0.4$	$\lambda = -0.8$	0.0570	0.0550	0.0515	0.0555
$\rho = 0.4$	$\lambda = -0.6$	0.0580	0.0595	0.0570	0.0510
$\rho = 0.4$	$\lambda = -0.4$	0.0645	0.0680	0.0490	0.0535
$\rho = 0.4$	$\lambda = -0.2$	0.0535	0.0620	0.0520	0.0560
$\rho = 0.4$	$\lambda = 0$	0.0665	0.0640	0.0560	0.0585
$\rho = 0.4$	$\lambda = 0.2$	0.0595	0.0585	0.0550	0.0515
$\rho = 0.4$	$\lambda = 0.4$	0.0605	0.0585	0.0470	0.0580
$\rho = 0.4$	$\lambda = 0.6$	0.0490	0.0580	0.0520	0.0575
$\rho = 0.4$	$\lambda = 0.8$	0.0580	0.0515	0.0435	0.0590
$\rho = 0.6$	$\lambda = -0.8$	0.0555	0.0640	0.0540	0.0545
$\rho = 0.6$	$\lambda = -0.6$	0.0660	0.0490	0.0520	0.0410
$\rho = 0.6$	$\lambda = -0.4$	0.0505	0.0455	0.0585	0.0520
$\rho = 0.6$	$\lambda = -0.2$	0.0695	0.0645	0.0545	0.0435
$\rho = 0.6$	$\lambda = 0$	0.0635	0.0655	0.0655	0.0505
$\rho = 0.6$	$\lambda = 0.2$	0.0575	0.0665	0.0480	0.0515
$\rho = 0.6$	$\lambda = 0.4$	0.0670	0.0595	0.0555	0.0595
$\rho = 0.6$	$\lambda = 0.6$	0.0570	0.0545	0.0560	0.0425
$\rho = 0.6$	$\lambda = 0.8$	0.0555	0.0485	0.0480	0.0560
$\rho = 0.8$	$\lambda = -0.8$	0.0535	0.0635	0.0565	0.0535
$\rho = 0.8$	$\lambda = -0.6$	0.0580	0.0545	0.0510	0.0420
$\rho = 0.8$	$\lambda = -0.4$	0.0700	0.0515	0.0485	0.0490
$\rho = 0.8$	$\lambda = -0.2$	0.0625	0.0500	0.0475	0.0500
$\rho = 0.8$	$\lambda = 0$	0.0670	0.0680	0.0525	0.0505
$\rho = 0.8$	$\lambda = 0.2$	0.0735	0.0595	0.0475	0.0540
$\rho = 0.8$	$\lambda = 0.4$	0.0790	0.0715	0.0555	0.0505
$\rho = 0.8$	$\lambda = 0.6$	0.0845	0.0835	0.0505	0.0595
$\rho = 0.8$	$\lambda = 0.8$	0.0870	0.0930	0.0585	0.0615
	average	0.0565	0.0558	0.0516	0.0530

Table 10: Size of test for the pre-test estimator based of the classic approach. Values based on 2,000 replications.

		β_1	β_2
$\rho = -0.8$	$\lambda = -0.8$	0.0390	0.0680
$\rho = -0.8$	$\lambda = -0.6$	0.0760	0.0710
$\rho = -0.8$	$\lambda = -0.4$	0.2200	0.1595
$\rho = -0.8$	$\lambda = -0.2$	0.5290	0.2485
$\rho = -0.8$	$\lambda = 0$	0.9480	0.9370
$\rho = -0.8$	$\lambda = 0.2$	0.2365	0.0580
$\rho = -0.8$	$\lambda = 0.4$	0.2390	0.0815
$\rho = -0.8$	$\lambda = 0.6$	0.2215	0.0800
$\rho = -0.8$	$\lambda = 0.8$	0.1745	0.0685
$\rho = -0.6$	$\lambda = -0.8$	0.0420	0.0575
$\rho = -0.6$	$\lambda = -0.6$	0.0540	0.0515
$\rho = -0.6$	$\lambda = -0.4$	0.1270	0.0920
$\rho = -0.6$	$\lambda = -0.2$	0.4825	0.2925
$\rho = -0.6$	$\lambda = 0$	0.8650	0.8605
$\rho = -0.6$	$\lambda = 0.2$	0.1900	0.0540
$\rho = -0.6$	$\lambda = 0.4$	0.1620	0.0550
$\rho = -0.6$	$\lambda = 0.6$	0.1575	0.0645
$\rho = -0.6$	$\lambda = 0.8$	0.1320	0.0545
$\rho = -0.4$	$\lambda = -0.8$	0.0415	0.0465
$\rho = -0.4$	$\lambda = -0.6$	0.0530	0.0445
$\rho = -0.4$	$\lambda = -0.4$	0.0800	0.0565
$\rho = -0.4$	$\lambda = -0.2$	0.7070	0.6530
$\rho = -0.4$	$\lambda = 0$	0.5215	0.5190
$\rho = -0.4$	$\lambda = 0.2$	0.1585	0.0515
$\rho = -0.4$	$\lambda = 0.4$	0.1150	0.0570
$\rho = -0.4$	$\lambda = 0.6$	0.1065	0.0615
$\rho = -0.4$	$\lambda = 0.8$	0.1025	0.0485
$\rho = -0.2$	$\lambda = -0.8$	0.0365	0.0520
$\rho = -0.2$	$\lambda = -0.6$	0.0460	0.0605
$\rho = -0.2$	$\lambda = -0.4$	0.0670	0.0615
$\rho = -0.2$	$\lambda = -0.2$	0.9050	0.8895
$\rho = -0.2$	$\lambda = 0$	0.2100	0.2090
$\rho = -0.2$	$\lambda = 0.2$	0.1120	0.0495
$\rho = -0.2$	$\lambda = 0.4$	0.0760	0.0460
$\rho = -0.2$	$\lambda = 0.6$	0.0780	0.0625
$\rho = -0.2$	$\lambda = 0.8$	0.0610	0.0550
$\rho = 0$	$\lambda = -0.8$	0.0490	0.0495
$\rho = 0$	$\lambda = -0.6$	0.0550	0.0525
$\rho = 0$	$\lambda = -0.4$	0.1375	0.1455
$\rho = 0$	$\lambda = -0.2$	0.8930	0.8805
$\rho = 0$	$\lambda = 0$	0.0730	0.0700
$\rho = 0$	$\lambda = 0.2$	0.0705	0.0585
$\rho = 0$	$\lambda = 0.4$	0.0575	0.0590
$\rho = 0$	$\lambda = 0.6$	0.0775	0.0790
$\rho = 0$	$\lambda = 0.8$	0.0460	0.0635
$\rho = 0.2$	$\lambda = -0.8$	0.0570	0.0575
$\rho = 0.2$	$\lambda = -0.6$	0.0555	0.0485
$\rho = 0.2$	$\lambda = -0.4$	0.4720	0.4695
$\rho = 0.2$	$\lambda = -0.2$	0.5480	0.5310
$\rho = 0.2$	$\lambda = 0$	0.0610	0.0515
$\rho = 0.2$	$\lambda = 0.2$	0.0675	0.0510
$\rho = 0.2$	$\lambda = 0.4$	0.1000	0.0860
$\rho = 0.2$	$\lambda = 0.6$	0.1450	0.1215
$\rho = 0.2$	$\lambda = 0.8$	0.0690	0.0585
$\rho = 0.4$	$\lambda = -0.8$	0.0715	0.0710
$\rho = 0.4$	$\lambda = -0.6$	0.1095	0.0980
$\rho = 0.4$	$\lambda = -0.4$	0.7515	0.7455
$\rho = 0.4$	$\lambda = -0.2$	0.1470	0.1580
$\rho = 0.4$	$\lambda = 0$	0.0630	0.0605
$\rho = 0.4$	$\lambda = 0.2$	0.0860	0.0775
$\rho = 0.4$	$\lambda = 0.4$	0.1880	0.1445
$\rho = 0.4$	$\lambda = 0.6$	0.2840	0.1730
$\rho = 0.4$	$\lambda = 0.8$	0.1720	0.0735
$\rho = 0.6$	$\lambda = -0.8$	0.1010	0.0955
$\rho = 0.6$	$\lambda = -0.6$	0.5040	0.4870
$\rho = 0.6$	$\lambda = -0.4$	0.3000	0.3570
$\rho = 0.6$	$\lambda = -0.2$	0.0705	0.0870
$\rho = 0.6$	$\lambda = 0$	0.0685	0.0520
$\rho = 0.6$	$\lambda = 0.2$	0.1130	0.0900
$\rho = 0.6$	$\lambda = 0.4$	0.3025	0.2125
$\rho = 0.6$	$\lambda = 0.6$	0.5290	0.2945
$\rho = 0.6$	$\lambda = 0.8$	0.4885	0.1505
$\rho = 0.8$	$\lambda = -0.8$	0.6445	0.6860
$\rho = 0.8$	$\lambda = -0.6$	0.5515	0.6450
$\rho = 0.8$	$\lambda = -0.4$	0.2790	0.3410
$\rho = 0.8$	$\lambda = -0.2$	0.1165	0.1115
$\rho = 0.8$	$\lambda = 0$	0.0500	0.0510
$\rho = 0.8$	$\lambda = 0.2$	0.1330	0.1215
$\rho = 0.8$	$\lambda = 0.4$	0.3280	0.2900
$\rho = 0.8$	$\lambda = 0.6$	0.6265	0.4005
$\rho = 0.8$	$\lambda = 0.8$	0.7850	0.2975
	average	0.2379	0.1936

Table 11: Size of test for the pre-test estimator based of the robust approach. Values based on 2,000 replications.

		β_1	β_2
$\rho = -0.8$	$\lambda = -0.8$	0.0390	0.0680
$\rho = -0.8$	$\lambda = -0.6$	0.0780	0.0720
$\rho = -0.8$	$\lambda = -0.4$	0.3690	0.3035
$\rho = -0.8$	$\lambda = -0.2$	0.9305	0.8710
$\rho = -0.8$	$\lambda = 0$	0.9530	0.9485
$\rho = -0.8$	$\lambda = 0.2$	0.3560	0.2225
$\rho = -0.8$	$\lambda = 0.4$	0.2390	0.0815
$\rho = -0.8$	$\lambda = 0.6$	0.2215	0.0800
$\rho = -0.8$	$\lambda = 0.8$	0.1745	0.0685
$\rho = -0.6$	$\lambda = -0.8$	0.0420	0.0575
$\rho = -0.6$	$\lambda = -0.6$	0.0570	0.0535
$\rho = -0.6$	$\lambda = -0.4$	0.2975	0.2510
$\rho = -0.6$	$\lambda = -0.2$	0.9210	0.9025
$\rho = -0.6$	$\lambda = 0$	0.9790	0.9760
$\rho = -0.6$	$\lambda = 0.2$	0.2795	0.1770
$\rho = -0.6$	$\lambda = 0.4$	0.1620	0.0550
$\rho = -0.6$	$\lambda = 0.6$	0.1575	0.0645
$\rho = -0.6$	$\lambda = 0.8$	0.1320	0.0545
$\rho = -0.4$	$\lambda = -0.8$	0.0415	0.0465
$\rho = -0.4$	$\lambda = -0.6$	0.0555	0.0465
$\rho = -0.4$	$\lambda = -0.4$	0.2620	0.2425
$\rho = -0.4$	$\lambda = -0.2$	0.9140	0.9055
$\rho = -0.4$	$\lambda = 0$	0.9495	0.9480
$\rho = -0.4$	$\lambda = 0.2$	0.2045	0.1080
$\rho = -0.4$	$\lambda = 0.4$	0.1150	0.0570
$\rho = -0.4$	$\lambda = 0.6$	0.1065	0.0615
$\rho = -0.4$	$\lambda = 0.8$	0.1025	0.0485
$\rho = -0.2$	$\lambda = -0.8$	0.0365	0.0520
$\rho = -0.2$	$\lambda = -0.6$	0.0470	0.0615
$\rho = -0.2$	$\lambda = -0.4$	0.2250	0.2135
$\rho = -0.2$	$\lambda = -0.2$	0.9075	0.8975
$\rho = -0.2$	$\lambda = 0$	0.8200	0.8140
$\rho = -0.2$	$\lambda = 0.2$	0.1275	0.0670
$\rho = -0.2$	$\lambda = 0.4$	0.0760	0.0460
$\rho = -0.2$	$\lambda = 0.6$	0.0780	0.0625
$\rho = -0.2$	$\lambda = 0.8$	0.0610	0.0550
$\rho = 0$	$\lambda = -0.8$	0.0490	0.0495
$\rho = 0$	$\lambda = -0.6$	0.0565	0.0540
$\rho = 0$	$\lambda = -0.4$	0.2285	0.2315
$\rho = 0$	$\lambda = -0.2$	0.7905	0.7680
$\rho = 0$	$\lambda = 0$	0.5505	0.5405
$\rho = 0$	$\lambda = 0.2$	0.0760	0.0635
$\rho = 0$	$\lambda = 0.4$	0.0575	0.0590
$\rho = 0$	$\lambda = 0.6$	0.0775	0.0790
$\rho = 0$	$\lambda = 0.8$	0.0460	0.0635
$\rho = 0.2$	$\lambda = -0.8$	0.0570	0.0575
$\rho = 0.2$	$\lambda = -0.6$	0.0560	0.0490
$\rho = 0.2$	$\lambda = -0.4$	0.2540	0.2425
$\rho = 0.2$	$\lambda = -0.2$	0.5155	0.4935
$\rho = 0.2$	$\lambda = 0$	0.2525	0.2435
$\rho = 0.2$	$\lambda = 0.2$	0.0680	0.0515
$\rho = 0.2$	$\lambda = 0.4$	0.1000	0.0860
$\rho = 0.2$	$\lambda = 0.6$	0.1450	0.1215
$\rho = 0.2$	$\lambda = 0.8$	0.0690	0.0585
$\rho = 0.4$	$\lambda = -0.8$	0.0715	0.0710
$\rho = 0.4$	$\lambda = -0.6$	0.0840	0.0705
$\rho = 0.4$	$\lambda = -0.4$	0.2340	0.2380
$\rho = 0.4$	$\lambda = -0.2$	0.1905	0.1960
$\rho = 0.4$	$\lambda = 0$	0.0935	0.0950
$\rho = 0.4$	$\lambda = 0.2$	0.0860	0.0775
$\rho = 0.4$	$\lambda = 0.4$	0.1880	0.1445
$\rho = 0.4$	$\lambda = 0.6$	0.2840	0.1730
$\rho = 0.4$	$\lambda = 0.8$	0.1720	0.0735
$\rho = 0.6$	$\lambda = -0.8$	0.1000	0.0950
$\rho = 0.6$	$\lambda = -0.6$	0.1350	0.1405
$\rho = 0.6$	$\lambda = -0.4$	0.1590	0.2150
$\rho = 0.6$	$\lambda = -0.2$	0.0800	0.0975
$\rho = 0.6$	$\lambda = 0$	0.0685	0.0520
$\rho = 0.6$	$\lambda = 0.2$	0.1130	0.0900
$\rho = 0.6$	$\lambda = 0.4$	0.3025	0.2125
$\rho = 0.6$	$\lambda = 0.6$	0.5290	0.2945
$\rho = 0.6$	$\lambda = 0.8$	0.4885	0.1505
$\rho = 0.8$	$\lambda = -0.8$	0.4975	0.5475
$\rho = 0.8$	$\lambda = -0.6$	0.5340	0.6220
$\rho = 0.8$	$\lambda = -0.4$	0.2795	0.3415
$\rho = 0.8$	$\lambda = -0.2$	0.1170	0.1120
$\rho = 0.8$	$\lambda = 0$	0.0500	0.0510
$\rho = 0.8$	$\lambda = 0.2$	0.1330	0.1215
$\rho = 0.8$	$\lambda = 0.4$	0.3280	0.2900
$\rho = 0.8$	$\lambda = 0.6$	0.6265	0.4005
$\rho = 0.8$	$\lambda = 0.8$	0.7850	0.2975
	average	0.2679	0.2299

Table 12: Size of test for the pre-test estimator based of the hybrid approach. Values based on 2,000 replications.

		β_1	β_2
$\rho = -0.8$	$\lambda = -0.8$	0.0390	0.0680
$\rho = -0.8$	$\lambda = -0.6$	0.0760	0.0710
$\rho = -0.8$	$\lambda = -0.4$	0.2200	0.1595
$\rho = -0.8$	$\lambda = -0.2$	0.5290	0.2485
$\rho = -0.8$	$\lambda = 0$	0.9480	0.9370
$\rho = -0.8$	$\lambda = 0.2$	0.2365	0.0580
$\rho = -0.8$	$\lambda = 0.4$	0.2390	0.0815
$\rho = -0.8$	$\lambda = 0.6$	0.2215	0.0800
$\rho = -0.8$	$\lambda = 0.8$	0.1745	0.0685
$\rho = -0.6$	$\lambda = -0.8$	0.0420	0.0575
$\rho = -0.6$	$\lambda = -0.6$	0.0540	0.0515
$\rho = -0.6$	$\lambda = -0.4$	0.1270	0.0920
$\rho = -0.6$	$\lambda = -0.2$	0.4825	0.2925
$\rho = -0.6$	$\lambda = 0$	0.8650	0.8605
$\rho = -0.6$	$\lambda = 0.2$	0.1900	0.0540
$\rho = -0.6$	$\lambda = 0.4$	0.1620	0.0550
$\rho = -0.6$	$\lambda = 0.6$	0.1575	0.0645
$\rho = -0.6$	$\lambda = 0.8$	0.1320	0.0545
$\rho = -0.4$	$\lambda = -0.8$	0.0415	0.0465
$\rho = -0.4$	$\lambda = -0.6$	0.0530	0.0445
$\rho = -0.4$	$\lambda = -0.4$	0.0800	0.0565
$\rho = -0.4$	$\lambda = -0.2$	0.7070	0.6530
$\rho = -0.4$	$\lambda = 0$	0.5215	0.5190
$\rho = -0.4$	$\lambda = 0.2$	0.1585	0.0515
$\rho = -0.4$	$\lambda = 0.4$	0.1150	0.0570
$\rho = -0.4$	$\lambda = 0.6$	0.1065	0.0615
$\rho = -0.4$	$\lambda = 0.8$	0.1025	0.0485
$\rho = -0.2$	$\lambda = -0.8$	0.0365	0.0520
$\rho = -0.2$	$\lambda = -0.6$	0.0460	0.0605
$\rho = -0.2$	$\lambda = -0.4$	0.0670	0.0615
$\rho = -0.2$	$\lambda = -0.2$	0.9050	0.8895
$\rho = -0.2$	$\lambda = 0$	0.2100	0.2090
$\rho = -0.2$	$\lambda = 0.2$	0.1120	0.0495
$\rho = -0.2$	$\lambda = 0.4$	0.0760	0.0460
$\rho = -0.2$	$\lambda = 0.6$	0.0780	0.0625
$\rho = -0.2$	$\lambda = 0.8$	0.0610	0.0550
$\rho = 0$	$\lambda = -0.8$	0.0490	0.0495
$\rho = 0$	$\lambda = -0.6$	0.0550	0.0525
$\rho = 0$	$\lambda = -0.4$	0.1375	0.1455
$\rho = 0$	$\lambda = -0.2$	0.8930	0.8805
$\rho = 0$	$\lambda = 0$	0.0730	0.0700
$\rho = 0$	$\lambda = 0.2$	0.0705	0.0585
$\rho = 0$	$\lambda = 0.4$	0.0575	0.0590
$\rho = 0$	$\lambda = 0.6$	0.0775	0.0790
$\rho = 0$	$\lambda = 0.8$	0.0460	0.0635
$\rho = 0.2$	$\lambda = -0.8$	0.0570	0.0575
$\rho = 0.2$	$\lambda = -0.6$	0.0555	0.0485
$\rho = 0.2$	$\lambda = -0.4$	0.4720	0.4695
$\rho = 0.2$	$\lambda = -0.2$	0.5480	0.5310
$\rho = 0.2$	$\lambda = 0$	0.0610	0.0515
$\rho = 0.2$	$\lambda = 0.2$	0.0675	0.0510
$\rho = 0.2$	$\lambda = 0.4$	0.1000	0.0860
$\rho = 0.2$	$\lambda = 0.6$	0.1450	0.1215
$\rho = 0.2$	$\lambda = 0.8$	0.0690	0.0585
$\rho = 0.4$	$\lambda = -0.8$	0.0715	0.0710
$\rho = 0.4$	$\lambda = -0.6$	0.1095	0.0980
$\rho = 0.4$	$\lambda = -0.4$	0.7515	0.7455
$\rho = 0.4$	$\lambda = -0.2$	0.1470	0.1580
$\rho = 0.4$	$\lambda = 0$	0.0630	0.0605
$\rho = 0.4$	$\lambda = 0.2$	0.0860	0.0775
$\rho = 0.4$	$\lambda = 0.4$	0.1880	0.1445
$\rho = 0.4$	$\lambda = 0.6$	0.2840	0.1730
$\rho = 0.4$	$\lambda = 0.8$	0.1720	0.0735
$\rho = 0.6$	$\lambda = -0.8$	0.1010	0.0955
$\rho = 0.6$	$\lambda = -0.6$	0.5040	0.4870
$\rho = 0.6$	$\lambda = -0.4$	0.3000	0.3570
$\rho = 0.6$	$\lambda = -0.2$	0.0705	0.0870
$\rho = 0.6$	$\lambda = 0$	0.0685	0.0520
$\rho = 0.6$	$\lambda = 0.2$	0.1130	0.0900
$\rho = 0.6$	$\lambda = 0.4$	0.3025	0.2125
$\rho = 0.6$	$\lambda = 0.6$	0.5290	0.2945
$\rho = 0.6$	$\lambda = 0.8$	0.4885	0.1505
$\rho = 0.8$	$\lambda = -0.8$	0.6445	0.6860
$\rho = 0.8$	$\lambda = -0.6$	0.5515	0.6450
$\rho = 0.8$	$\lambda = -0.4$	0.2790	0.3410
$\rho = 0.8$	$\lambda = -0.2$	0.1165	0.1115
$\rho = 0.8$	$\lambda = 0$	0.0500	0.0510
$\rho = 0.8$	$\lambda = 0.2$	0.1330	0.1215
$\rho = 0.8$	$\lambda = 0.4$	0.3280	0.2900
$\rho = 0.8$	$\lambda = 0.6$	0.6265	0.4005
$\rho = 0.8$	$\lambda = 0.8$	0.7850	0.2975
	average	0.2379	0.1936