

Spatial Econometric Origin-Destination Modeling of U.S. – Canadian Trade Flows

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Abstract

We explore international trade flows from 10 Canadian provinces to the lower 48 U.S. states using spatial econometric origin-destination techniques based on a recently introduced spatial autoregressive variant of the traditional gravity model put forth by LeSage and Pace (2008). The gravity model in international trade has seen wide application, but very little work has been done that explores spatial dependence in trade flows. The spatial variants of the gravity model used here include three spatial autoregressive parameters to explicitly account for origin based, destination based, and origin-to-destination based neighborhood influences. Maximum likelihood estimated spatial lag and spatial error gravity models are applied and compared to the traditional non-spatial least squares estimation. Also included is a check on the robustness of the standard great circle distance measurement when compared to transportation network distance.

Keywords: gravity model, international trade, spatial autoregressive regression model

Introduction

The gravity model has been widely used in international trade for the greater half of a century. At its most basic formulation, the gravity model states that the value of trade between an origin-destination (OD) pair is proportional to their economic masses and inversely proportional to the distance between them. Various specifications of the gravity model have been developed from this simple equation and fit the data surprising well; with an R^2 averaging 0.7 across published literature (Baldwin & Taglioni, 2006) leading Chaney (2011) to call the gravity model one of the most stable and robust empirical models in economics.

Implicit in all specifications of the gravity model is the importance of spatial relationships in international trade. While several authors have used the gravity model to investigate these relationships, the large base of empirical research using the gravity model in international trade has all but ignored Anselin and Griffith (1988) in their clarification on the ways in which standard econometric models fail to remain applicable for spatial data.

In its standard least squares estimation the gravity model, also known as the spatial interaction model in the regional science literature, assumes independence among flows, an assumption that seems inappropriate for many spatial interactions. Known as the first law of geography, Tobler (1970, p. 236) suggests “everything is related to everything else, but near things are more related than distant things.” This deceptively simple statement is operationalized through spatial autocorrelation and if least squares

estimation is used in its presence the resulting parameter estimates can suffer from bias and inefficiency (Anselin, 1988). While the assumption of independent flows has seen almost no debate in the international trade literature, it has long been questioned in the regional science community, where the gravity model has been widely applied to areas such as migration and transportation flows. Griffith (2007) provides an historical review of the regional science literature on this issue and credits Curry (1972) for being the first to hypothesize the presence of spatial dependence in spatial interaction flows; while later work, such as that by Griffith and Jones (1980), reiterates and further refines the idea that distance effect estimates are confounded by unacknowledged spatial autocorrelation.

Since international trade flows are inherently spatial interactions there is no reason for the a priori assumption that trade is an aberration of Tobler's first law and that each observation is independent of neighboring observations. However, this is the assumption that underlies the estimation of the gravity model in international trade and this theoretical misalignment also manifests itself empirically, generating concern not only from the theoretical perspective, but operationally, as well.

In order to account for spatial dependence in regression analysis, one must look to models originating in the spatial econometrics literature, such as the widely used spatial lag and spatial error models described by Anselin (1988). These models use spatial weights matrices and special estimation techniques to properly account for spatial dependence. These spatial weights matrices, however, are constructed to model dependence among n regions, resulting in a square n by n matrix. As a consequence, using these models in an origin-destination setting such as international trade, where all n

regions act as origins each having as many as $n - 1$ destinations, remained a stumbling block; essentially preventing the gravity model literature from taking advantage of these useful methods. That is until LeSage and Pace (2008) bridged this gap and developed methodology to extend the traditional spatial weights matrix and corresponding spatial lag and spatial error models to account for spatial effects in origin-destination flows.

We use the methodology put forth by LeSage and Pace (2008) to investigate the differences in model performance when controlling for spatial dependence in international trade flows. Both a spatial lag and spatial error type gravity model were ran on Canadian provincial exports to the lower 48 U.S. states and compared to the traditional, non-spatial, least squares gravity model estimation.

Gravity Model Background

The inspiration for the gravity model comes from Newtonian physics and the law of universal gravity (Zhang & Kristensen, 1995), in which the attraction of two masses is directly proportional to the product of their masses and inversely proportional to the distance between them. Functionally:

$$Force\ of\ Gravity = G \frac{M_1 M_2}{(dist_{12})^2}$$

In trade, force of gravity is replaced with the value of trade, the gravitational constant G is analogous to the intercept constant, M_1 and M_2 are each trading partners' specific economic masses represented by individual economic characteristics, such as gross domestic product (GDP), population, GDP per capita, as well as other measures.

The standard approach to applying the gravity model in international trade is by using its log-normal functional form in a least squares estimation resulting in a model in matrix notation as shown below

$$\mathbf{y} = \alpha \mathbf{1}_n + \mathbf{X}_o \boldsymbol{\beta}_o + \mathbf{X}_d \boldsymbol{\beta}_d + \gamma \mathbf{d} + \boldsymbol{\varepsilon}$$

Equation 1

where \mathbf{y} is an n by 1 vector of logged trade flows, $\mathbf{1}_n$ is an n by 1 vector of ones with α denoting its constant parameter term, \mathbf{X}_o and \mathbf{X}_d are n by k matrices of k explanatory variables for each the origin and destination, respectively, logged unless otherwise specified, with $\boldsymbol{\beta}_o$ and $\boldsymbol{\beta}_d$ being their associated k by 1 parameter vectors, \mathbf{d} is an n by 1 vector of the logged distance between each OD pair with γ being its associated scalar parameter. The disturbances are represented by the n by 1 vector $\boldsymbol{\varepsilon}$ which is assumed $\boldsymbol{\varepsilon} \sim N(0, \sigma^2 \mathbf{I})$.

The gravity model was first applied to international trade by Tinbergen (1962) and Poyhonen (1963). Since then, numerous authors have used varying specifications of the gravity model to estimate the trade effects of a wide variety of factors such as the distance effect (Berthelon & Freund, 2008), trading blocs (Frankel, Stein, & Wei, 1998), currency unions (Anderson & Marcouiller, 1999), international borders (McCallum, 1995), intranational borders (Wolf, 2000), and common languages (Nitsch, 2000).

The standard log-normal functional form of the gravity model allows many parameter coefficients to be interpreted as the elasticity of trade with respect to that parameter, for example, if the parameter coefficient for distance in a gravity model were estimated to be -0.72, this can be interpreted as given a 10% increase in the distance

between trading partners, a 7.2% decrease in trade would be expected to occur. This conveniently allows for a viable comparison of parameter estimates found throughout various studies. Many previous gravity model studies have been particularly interested in the elasticity of trade with respect to distance, what has been termed the distance effect. Leamer and Levinsohn (1995) compare previous distance effect estimates and claim an average distance effect of approximately -0.6. Overman, Redding, and Venables (2003) state that the distance effect is normally estimated to be between -0.9 to -1.5, however their estimate only cites three previous studies to support this claim; those of Feenstra, Markusen, and Rose (2001), Frankel (1997), and Soloaga and Winters (2001). In a much more extensive and robust meta-analysis of 1,467 distance effects estimated throughout 103 papers, Disdier and Head (2008) find a mean distance effect of -0.91 and a median of -0.87, with 90% of estimates lying between -0.28 and -1.55. By this analysis, on average, a 10% increase in distance lowers bilateral trade by 9.1%. Of note is the fact that more than half of their samples are below the suggested interval by Overman et al. (2003).

Gravity Model and Spatial Effects

Despite the recognition of the importance of spatial effects when applying the gravity model in other fields such as migration, ecology, or agricultural economics, there has been very little research applying the gravity model to international trade that acknowledges, much less properly accounts for, the presence of spatial dependence in either the observed trade flows or the resulting error terms. While these two problems

have not been completely ignored in the literature they have been given very little due diligence.

Baldwin (1994) discusses generic issues with the empirical implementation of least squares estimated gravity models such as data aggregation, zero trade flows, imports vs. exports, even distance measurement, but makes no mention of spatial effects.

Applying the gravity model to migration flows that exhibit heteroskedasticity, Flowerdew (1982) suggests the use of an iterative weighting method when using least squares estimation, which McCallum (1995) applies in his analysis, but makes no mention of the potential for spatially autocorrelated errors. In differentiating the effects of spatial proximity and regional trading blocs, Poon (1997) addresses the issue of “locational heterogeneity” by proposing a model with variable coefficients by applying Casetti’s (1972) expansion method to a gravity based model of trade. Acknowledging the possible presence of spatially autocorrelated error terms Bougheas, Demetriades, and Morgenroth (1999) use a Seemingly Unrelated Regression (SUR) to allow for correlation between error terms in their estimate of infrastructure impediments to trade in the European Union. However, for identical independent variables, OLS and SUR are identical and there is no gain in efficiency by using the alternative estimate (Greene, 2007). Burger, van Oort, and Linders (2009) point to three issues when using the log-normal least squares estimation of the gravity model; logarithm transformation bias, heteroskedasticity, and zero trade flows, and suggest an extension of the Poisson model specification put forward by Silva and Tenreyro (2006), but do not mention any possible

problems arising from spatial dependence in observed trade flows or the resulting error terms.

When the gravity model literature has attempted to consider the spatial relationships inherent to international trade, it is most often through the use simple dummy variables such as for adjacency, trading blocs, or international borders (Frankel, 1997; Frankel et al., 1998; McCallum, 1995). In and of themselves these dummy variables are relevant parameters of interest; however, they do nothing to take into account spatial dependence in the estimation procedure thus still treating each observation of trade as independent and leaving the problem of potential bias and inefficiency unaddressed. The adjacency variable used by Frankel et al. (1998) is an example of this practice. In their gravity model analysis Frankel et al. (1998) use a dummy variable for adjacency, equal to 1 if an OD pair share a common land border and equal to 0 if otherwise. In matrix form this dummy variable becomes similar in structure to that of a contiguity based spatial weights matrix. However, in the form used by Frankel et al. (1998), the adjacency variable represents the influence of a particular trading pair sharing a common border, essentially treating each OD pairs' trade as an independent closed system of either having one neighbor or none; ignoring the fact that a particular origin may also have other neighbors that influence its trade to not only the neighbor in question, but to all other neighbors as well.

The theoretical and empirical misalignments described above are potentially assuaged with the methodology introduced by LeSage and Pace (2008). They develop spatial weights structures which allow for the extension of the traditional spatial lag and

spatial error models to account for spatial dependence in gravity model analysis and use this structure to specify a combination of three spatial weights matrices one each for origin, destination, and what they call origin-to-destination dependence. The use of three separate spatial weights matrices allows for each of their influences to be quantified separately, an important feature for modeling origin-destination flows. Their methodology has since been applied to patent citations (Fischer & Griffith, 2008), migration flows (LeSage & Fischer, 2010), and in forecasting interregional commodity flows (LeSage & Llano, 2012).

Intuitively, factors and economic forces such as natural resources, knowledge spillover, or agglomeration economies, that lead to trade flows from any origin to a particular destination may create similar flows from neighbors of that origin to the same destination, and the origin spatial weights matrix of LeSage and Pace (2008) captures this origin-based spatial dependence.

Similar reasoning extends to the destination spatial weights matrix that is used to capture destination-based spatial dependence. For the same reasons why exports from neighboring origins may be similar, certain factors and economic forces that cause a destination to import from a particular origin may cause the neighbors of that destination to import similar flows from that origin. This influence is captured in the destination spatial weights matrix.

The third spatial weights matrix used to capture, what LeSage and Pace (2008) call, origin-to-destination spatial dependence arises from the product of the origin and

destination spatial weights matrices and reflects the average of flows from neighbors of the origin to neighbors of the destination.

LeSage and Pace (2008) describe their methodology in a system where all origins are also destinations resulting in a square n by n matrix of observations. For a system where all origins are not all destinations (m by n), their methodology requires a slight modification in regards to creating the origin-destination spatial weights matrices. This modification is described in **section #**. Because of this change, where in LeSage and Pace (2008) an N by 1 vector denotes a vector with dimensions $(n \times n)$ by 1, an N by 1 vector in the context of this paper denotes a vector with dimensions $(m \times n)$ by 1. For clarity purposes further description of the notation and the ordering of flows is discussed below

Notation and Ordering of Flows

Let \mathbf{Y} be an m by n matrix of flows from each of the n origins to each of the m destinations where each of the n columns represents a different origin and each of the m rows represents a different destination. You can create an $N(=m \times n)$ by 1 vector of these flows in one of two ways, one representing origin-centric ordering and the other representing destination-centric ordering. Starting with \mathbf{Y} , whose columns represent the origins and rows represent destinations, LeSage and Pace (2008) show that you can obtain a vector of origin-centric ordering with $\mathbf{y} = \text{vec}(\mathbf{Y})$ or obtain a vector of destination-centric ordering with $\mathbf{y} = \text{vec}(\mathbf{Y}')$. From here out, the following description of the methods will be with origin-centric ordering. Where the first m elements in the N by 1 vector \mathbf{y} represent flows from origin 1 to all m destinations and with the last m elements representing flows from origin n to all destinations 1 to m .

Model Formulations and Estimations

While often times specific gravity model studies will add other variables of interest in addition to the standard origin “mass”, destination “mass”, and distance; since the purpose of this study is the comparison of models and estimation procedures the variables included in this study are purposefully catholic to insure the broadest applicability of results. Below is a description of the model formulations used in this study.

Non-Spatial Gravity Model

The non-spatial gravity model is equivalent to the standard approach used in the literature to estimate gravity models in international trade, which is to assume independence of flows and use least squares in its estimation. The formulation of the non-spatial gravity model used in this study is

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\mathbf{X}\boldsymbol{\beta} = \alpha\mathbf{1}_N + \beta_o\mathbf{x}_o + \beta_d\mathbf{x}_d + \gamma\mathbf{d}$$

Equation 2

where \mathbf{y} is an N by 1 vector of the natural log of trade flows plus 1, $\mathbf{1}_N$ is an N by 1 vector of ones with α denoting its constant parameter term, \mathbf{x}_o and \mathbf{x}_d are N by 1 vectors of the natural log of GDP for each the origin and destination, respectively, with β_o and β_d being their associated scalar parameters, \mathbf{d} is an N by 1 vector of the natural log of distance, in

miles, between each OD pair with γ being its associated scalar parameter and the error term is represented by the N by 1 vector $\boldsymbol{\varepsilon}$ which is assumed $\boldsymbol{\varepsilon} \sim N(0, \sigma^2 \mathbf{I}_N)$.

Generation of Gravity Model Spatial Weights Matrices

The first step in implementing the spatial lag and spatial error gravity models with the spatial weights matrices that follow the structure developed by LeSage and Pace (2008) and have dimensions of N by N , is to first generate a standard n by n weights matrix of the regions in question. In the framework of LeSage and Pace (2008), where all origins are also destinations, this only requires the generation of one n by n spatial weights matrix, however, in a setting where all origins are not also destinations, this requires the generation of a separate weights matrix for each the origins and the destinations. The n by n origin weights matrix used for this study was a row-standardized 10 by 10 matrix (corresponding to the 10 Canadian provinces), labeled \mathbf{W}_o and the m by m destination matrix was a row-standardized 48 by 48 matrix (corresponding to the 48 contiguous U.S. states), labeled \mathbf{W}_d .

In order for these spatial weights matrices to be operational in an origin-destination application the next step is to expand their dimensions so they are of equal size, allowing each observation to have corresponding origin and destination weights. LeSage and Pace (2008) show that the Kronecker product of the spatial weights matrix and an identity matrix can be used to accomplish this. In their application where each origin was also a destination, allows for the use of a single spatial weights matrix and an identity matrix equal in size to that of the spatial weights matrix, of size n by n , with n

being the number of regions under study. However, in the case where all origins are not also destinations, as in the case of this study, a slight modification is needed. Instead of the Kronecker product of a single spatial weights matrix and an identity of the same dimensions, it can be seen that to generate the correct N by N origin spatial weights matrix, the Kronecker product of the n by n origin weights matrix and an identity matrix with dimensions corresponding to size of the m by m destination weights matrix can be used and to generate the correct N by N destination spatial weights matrix, ones uses the Kronecker product of an identity matrix with dimensions equal in size to the n by n origin weights matrix and the m by m destination matrix. Formally,

$$\mathbf{W}_O = \mathbf{W}_o \otimes \mathbf{I}_d$$

$$\mathbf{W}_D = \mathbf{I}_o \otimes \mathbf{W}_d$$

The resulting \mathbf{W}_O is an N by N row standardized spatial weights matrix used to capture origin-based spatial dependence, operationally, the average of flows from neighbors of the origin to the destination. Similarly, \mathbf{W}_D is an N by N row standardized spatial weights matrix used to capture destination-based spatial dependence, operationally, the average of flows from the origin to neighbors of the destination.

The third type of spatial dependence that was modeled arises from the product of \mathbf{W}_O and \mathbf{W}_D and is defined as

$$\mathbf{W}_W = \mathbf{W}_O \mathbf{W}_D$$

The resulting \mathbf{W}_W is an N by N row standardized spatial weights matrix used to capture what LeSage and Pace (2008) call origin-to-destination spatial dependence and reflects the average of flows from neighbors of the origin to neighbors of the destination.

For this study queen contiguity was used to specify the spatial weights matrix for both the U.S. and Canada for use in both spatial lag and spatial error gravity models. Because Prince Edward Island (PEI) does not share a land border with any other provinces, for economic reasons in addition to standard queen contiguity, it was defined as a neighbor of New Brunswick and Nova Scotia.

Spatial Lag Gravity Model

The spatial lag model is used when the dependent variable itself exhibits spatial autocorrelation. If this spatial autocorrelation is not accounted for the resulting variance structure is such that parameter estimates will be both biased and inefficient. To capture this autocorrelation in a regression model an independent variable is added and specified as a spatial lag of the dependent variable. At its most basic formulation the spatial lag model is as follows

$$\mathbf{y} = \rho \mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

Equation 3

where \mathbf{y} is an n by 1 vector of observations of the dependent variable, $\mathbf{W}\mathbf{y}$ is the corresponding spatially lagged dependent variable n by 1 vector for the spatial weights matrix \mathbf{W} , \mathbf{X} is an n by k matrix of explanatory variables, ρ is the spatial autoregressive parameter, $\boldsymbol{\beta}$ is a k by 1 vector of explanatory parameter estimates, and $\boldsymbol{\varepsilon}$ is an n by 1 vector of error terms assumed $\boldsymbol{\varepsilon} \sim N(0, \sigma^2 \mathbf{I})$.

LeSage and Pace (2008) develop an elegant extension of this spatial lag model to allow for spatial dependence to be accounted for in an origin-destination setting. This spatial lag gravity model is formulated as follows

$$\mathbf{y} = \rho_o \mathbf{W}_o \mathbf{y} + \rho_d \mathbf{W}_d \mathbf{y} + \rho_w \mathbf{W}_w \mathbf{y} + \mathbf{X}\beta + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon} \sim N(0, \sigma^2 \mathbf{I}_N)$$

Equation 4

This model is identical to the standard spatial lag model with the important exception that instead of one n by 1 spatially lagged dependent variable on the right hand side of the equation; there are three N by 1 spatially lagged dependent variables. In conjunction with their associated scalar parameters; ρ_o, ρ_d, ρ_w , the three spatially lagged terms respectively indicate the strength of origin-based, destination-based, and origin-to-destination based spatial dependence. The specification used in this study includes the exact same independent variables as the non-spatial gravity model, as represented by $\mathbf{X}\beta$ which was defined above.

As previously mentioned least squares estimation is no longer valid in the presence of spatial dependence and maximum likelihood is the most common form of estimation, following this maximum likelihood was used to estimate both the spatial lag and spatial error models. The log-likelihood function for the spatial lag gravity model is as follows (LeSage & Pace, 2008)

$$\ln L = \ln |\mathbf{I}_N - \rho_o \mathbf{W}_o - \rho_d \mathbf{W}_d - \rho_w \mathbf{W}_w| - \frac{N}{2} \ln(2\pi) - \frac{N}{2} \ln(\sigma^2) - \frac{\boldsymbol{\varepsilon}' \boldsymbol{\varepsilon}}{2\sigma^2}$$

$$\boldsymbol{\varepsilon} = \mathbf{y} - \rho_o \mathbf{W}_o \mathbf{y} - \rho_d \mathbf{W}_d \mathbf{y} - \rho_w \mathbf{W}_w \mathbf{y} - \mathbf{X}\beta$$

Equation 5

The first term in the log-likelihood function is the natural log of the determinant of the Jacobian matrix of the transformation. The calculation of this term is the main challenge of estimating the log-likelihood of spatial models, however, because of the sparse nature of spatial weights matrices, sparse matrix routines can be used to more efficiently find the equivalent value (Pace & LeSage, 2010). The method of computing the log determinant of the Jacobian that was used for this research was originally put forth by Pace and Barry (1997) and is calculated using LU decomposition as the sum of the logs of the pivots of U. The second term is a constant that in practice is normally removed. The third and fourth terms reduce and thus the concentrated log-likelihood function for the spatial lag gravity model is as follows

$$\ln L = \ln |\mathbf{I}_N - \rho_o \mathbf{W}_o - \rho_d \mathbf{W}_d - \rho_w \mathbf{W}_w| - \frac{N}{2} \ln(\boldsymbol{\varepsilon}' \boldsymbol{\varepsilon})$$

Equation 6

Expanding on $\boldsymbol{\varepsilon}$ as defined above, it is the N by 1 column vector error term resulting from a least squares regression on a spatially filtered dependent variable. Thus $\boldsymbol{\varepsilon}' \boldsymbol{\varepsilon}$ is simply the sum of the squared errors. The spatial lag gravity model expressed with a spatially filtered dependent variable is as shown below

$$(\mathbf{I}_N - \rho_o \mathbf{W}_o - \rho_d \mathbf{W}_d - \rho_w \mathbf{W}_w) \mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

Equation 7

Spatial Error Gravity Model

A spatial error model is used when spatial autocorrelation is present in an unobserved influence and either manifests itself in the dependent variable or the resulting error terms. Much like the spatial lag model, a spatial error model uses a spatial weights matrix to control of the spatial effects of neighboring observances, but unlike the spatial lag model the spatial weights matrix is applied to the error terms, not the dependent variable. The resulting error variance will be such that if not accounted for estimates will be inefficient, although they will remain unbiased. The most common specification of the spatial error model is

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

$$\mathbf{u} = \lambda\mathbf{W}\mathbf{u} + \boldsymbol{\varepsilon}$$

Equation 8

where \mathbf{y} is an n by 1 vector of observations of the dependent variable, \mathbf{X} is an n by k matrix of explanatory variables, $\boldsymbol{\beta}$ is a k by 1 vector of explanatory parameter estimates, and \mathbf{u} being a composite error term where λ is the spatial autoregressive parameter on $\mathbf{W}\mathbf{u}$, the corresponding spatially lagged error vector (λ is used to distinguish the notation from the spatial autoregressive term ρ in the spatial lag model, but their role is identical), and $\boldsymbol{\varepsilon}$ is an n by 1 vector of disturbances assumed $\boldsymbol{\varepsilon} \sim N(0, \sigma^2\mathbf{I})$.

In similar fashion to the spatial lag model, LeSage and Pace (2008) extend the spatial error model such that the spatial error gravity model is formulated as follows

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

$$\mathbf{u} = \lambda_o\mathbf{W}_o\mathbf{u} + \lambda_d\mathbf{W}_d\mathbf{u} + \lambda_w\mathbf{W}_w\mathbf{u} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon} \sim N(0, \sigma^2 \mathbf{I}_N)$$

Equation 9

The spatial error gravity model is similar to the spatial lag gravity model, with again the inclusion of three spatially lagged terms instead of only one, they differ however in that the spatial weights matrices are moved to the error term to create the composite error term \mathbf{u} . As mentioned early, the spatial autoregressive terms that in the spatial lag model were notated as ρ , are notated as λ in the spatial error context to distinguish their formulations, however their role is identical.

For this study the same $\mathbf{X}\boldsymbol{\beta}$ as defined for the non-spatial and spatial lag gravity model was used in the formation of the spatial error gravity model.

As Anselin and Bera (1998) describe, maximum likelihood estimation for the spatial error model can be approached by considering it as a special case of general parameterized nonspherical error terms for which $var(\mathbf{u}) = \sigma^2 \boldsymbol{\Omega}(\lambda)$. Where for the spatial error gravity model

$$\boldsymbol{\Omega}(\lambda) = [(\mathbf{I}_N - \lambda_o \mathbf{W}_o - \lambda_d \mathbf{W}_d - \lambda_w \mathbf{W}_w)'(\mathbf{I}_N - \lambda_o \mathbf{W}_o - \lambda_d \mathbf{W}_d - \lambda_w \mathbf{W}_w)]^{-1}$$

Equation 10

Once $\boldsymbol{\Omega}(\lambda)$ is obtained it can be plugged into the standard log-likelihood function for the spatial error model as follows

$$\ln L = -\frac{1}{2} \ln |\boldsymbol{\Omega}(\lambda)| - \frac{N}{2} \ln(2\pi) - \frac{N}{2} \ln(\sigma^2) - \frac{(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})' \boldsymbol{\Omega}(\lambda)^{-1} (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})}{2\sigma^2}$$

Equation 11

Or in concentrated form

$$\ln L = \ln |\mathbf{\Omega}(\lambda)| - \frac{N}{2} \ln \left(\frac{\boldsymbol{\varepsilon}' \boldsymbol{\varepsilon}}{N} \right)$$

Equation 12

where $\boldsymbol{\varepsilon}$ is the column vector of error terms from a least squares estimation of the spatially filtered spatial error model. The formulation for the spatially filtered spatial error model differs from the spatially filtered spatial lag model in that the filter is applied on both sides of the equation. Functionally,

$$(\mathbf{I}_N - \lambda_o \mathbf{W}_o - \lambda_d \mathbf{W}_d - \lambda_w \mathbf{W}_w) \mathbf{y} = (\mathbf{I}_N - \lambda_o \mathbf{W}_o - \lambda_d \mathbf{W}_d - \lambda_w \mathbf{W}_w) \mathbf{X} \boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

Equation 13

Results

The results of the gravity model estimations can be seen in table 1. The least squares non-spatial gravity model resulted in the worst overall performance as measured by sigma-squared with a sigma-squared of 1.10 compared to 0.90 for the spatial lag gravity model and 0.81 for the spatial error model. The residuals of the non-spatial gravity model were tested for spatial autocorrelation using Moran's I and significant amounts of positive spatial autocorrelation remained when using the weights structure of both the destination-based and origin-based spatial weights matrix.

As expected when modeling spatial dependence in the error terms, the coefficient estimates for the intercept, both GDPs, and the distance parameter in the spatial error gravity model are asymptotically equivalent to those obtained from the non-spatial gravity model. The coefficient standard errors increased when using the spatial error

gravity model compared to the non-spatial, reflecting the loss of global information available that the presence of spatial autocorrelation produces. Least squares estimation essentially believes there is more information available than there actually is because it ignores the redundancy that the spatial autocorrelation creates. This results in standard errors for the coefficients in the non-spatial gravity model that are misleadingly small.

Out of the three spatial weights matrices in the composite error term, destination-based spatial dependence, \mathbf{W}_d proved to be the most influential in the spatial error gravity model. Origin based spatial dependence, \mathbf{W}_o had relatively low estimates but remained significant, while origin-to-destination dependence captured by \mathbf{W}_w showed no statistical significance.

When the residuals of the spatial error gravity model were tested for any remaining spatial autocorrelation, no significant levels remained when using any of the three spatial weights structures.

Compared to the non-spatial gravity model, the spatial lag gravity model saw a lessening of the influence by roughly one-third for both the GDP coefficients as well as the distance coefficient as influence previously attributed to these parameters in the non-spatial gravity model were captured by the spatial autoregressive parameters in the spatial lag gravity model. Destination-based spatial dependence was the most influential, however less so than its estimated influence in the spatial error gravity model. The influence of origin-based spatial dependence in the spatial lag gravity model proved almost twice as strong as it did in the spatial error gravity model. Also a departure from the spatial error gravity model, origin-to-destination based spatial dependence was

significant and had influence as strong as origin-based dependence except in the opposite direction.

When the residuals of the spatial lag gravity model were tested for remaining spatial autocorrelation, there were no significant levels of spatial autocorrelation remaining for any of the three spatial weights structures.

Distance measurement robustness check

When transportation networks were used to create a network weighted distance for each OD pair in an effort to see if the standard approach of using great circle distance introduced bias in the parameter estimates, it resulted in no significant differences in estimates for any parameter in any of the non-spatial, spatial error, or spatial lag gravity models. Although great circle distance on average underestimated the distance between OD pairs by 20%, it did so consistently, in such a way to essentially shift all distance measurements up and thus not changing the variation of the distances between OD pairs.

This supports the observation of Disdier and Head (2008) who find that distance coefficient estimates from papers that used sea transportation routes in distance measurement showed no significant differences from the estimates obtained in studies that use great circle distance.

Conclusion

This work shows empirically that spatial dependence is an important factor when modeling international trade flows. The traditional approach of assuming independence

of flows and using least squares to estimate gravity model in international trade, proved to produce unreliable estimates when compared to the spatial gravity model approaches.

Empirical rationale for using the spatial econometric gravity models can be seen in the poor performance of the non-spatial least squares gravity model in this study, however theoretical rationale would provide even stronger reasoning for their implementation. Studies such as Curry (1972) and Griffith and Jones (1980) support spatial econometric gravity modeling in general, however, they do not speak to theoretical motivations in respect to international trade. In this respect there has been little theoretical work that explicitly includes spatial dependence in deriving the occurrence of international trade. One exception is the work of Behrens, Ertur, and Koch (2012) who extend the work of Anderson and Wincoop (2004) . They use monopolistic competition coupled with a CES utility function to derive a gravity model for trade flows that contain spatial lags of the dependent variable.

As to which spatial gravity model is theoretically suited to model this interdependence of trade flows, depends on how one interprets the proper use of the spatial lag and spatial error models.

It is not disputed that a spatial lag model is the proper specification when the values of the dependent variable at one location cause a change in the value of the dependent variable at a neighboring location. The discrepancy arises when one infers the definition of cause. In a strict sense, it is not intuitive to assume that because Ontario sends a large amount of exports to Michigan that these exports themselves cause more exports to be sent to Ohio. It might make more sense that similar industries, natural

resource needs, or similar distance might be the actual underlying cause for similar levels of exports from Ontario to these two states. When interpreted in this manner, the spatial error model becomes the theoretically proper model for estimation. However, the majority of spatial econometric literature, including the relevant works of Behrens et al. (2012), LeSage and Pace (2008), and LeSage and Fischer (2010), either interpret cause in a more lenient fashion or use purely econometric reasons to conclude that the spatial lag specification is appropriate.

The role of spatial dependence in international trade flows is relatively new to explicit investigation and accordingly there many areas which remain largely unexplored. Future research should focus on extending spatial econometric modeling in empirical international trade analysis, while also developing stronger theoretical justifications for including spatial dependence in international trade modeling.

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Estimates of Non-Spatial, Spatial Error, and Spatial Lag Gravity Models

Variable	Non-Spatial			Spatial Error			Spatial Lag		
	Coefficient	Standard Error	p-value	Coefficient	Standard Error	p-value	Coefficient	Standard Error	p-value
Intercept	-7.3204	0.9100	0.0000	-7.7452	1.4996	0.0000	-5.7879	1.1889	0.0000
Log(Origin GDP)	1.0144	0.0354	0.0000	0.9885	0.0641	0.0000	0.6514	0.0491	0.0000
Log(Destination GDP)	0.9697	0.0471	0.0000	1.0639	0.0492	0.0000	0.7350	0.0566	0.0000
Log(Distance)	-1.5747	0.0818	0.0000	-1.6341	0.1507	0.0000	-1.0454	0.1100	0.0000
W_o	-	-	-	0.1298	0.0436	0.0031	0.2282	0.0381	0.0000
W_d	-	-	-	0.4894	0.0453	0.0000	0.3514	0.0443	0.0000
W_w	-	-	-	0.0216	0.0679	0.7500	-0.2248	0.0442	0.0000
Residual Moran's I W_o	0.1887			-0.0024			-0.0222		
Residual Moran's I W_d	0.3543			-0.0240			0.0942		
Residual Moran's I W_w	0.0865			0.0030			0.0976		
Sigma-Squared	1.1027			0.8075			0.9008		