'Dual Gravity', Spatial Interdependence of Trade Flows among European Countries

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Abstract

In this paper, we adopt the 'dual gravity' model proposed in Behrens et al. (2012) and apply it to the trade flows data for European countries. We use new methodology and estimation techniques from the spatial econometric literature to study the effect of spatial distribution of countries on trade flows within Europe. Regression result on data from 2006 to 2010 suggests that distance, wage, membership of EU do have different, but expected effects on the normalized trade volumes.

JEL classification:

Keywords: Spatial dependence, Trade, Gravity Model

1. Introduction

Trade is spatially determined by nature. The trade pattern among different regions arises from the distribution of economic activity. In the the literature of international trade, gravity model has been playing a major role in capturing the distance effect on the diffusion of economic activity. Anderson (1979) uses properties of expenditure systems to provide a theoretical explanation for the gravity equation applied to commodities. Anderson and van Wincoop (2003) develop an estimable empirical specification of gravity model which closely follows a CES expenditure system. They suggest that trade flow can be spatial correlated, which implies the multi-lateral trade resistance in the gravity equation. Specifically, the export from region A to region B not only depends on their bilateral distance, but also is influenced by the trade flows from all other regions to region B. Helpman et al. (2008) compute a generalized gravity equation to account for firms' self-selection behavior into export market and its impact on trade volumes. Essentially, the spirit of gravity model is the spatial distribution of economic

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activity, and one of the main aspects is the trade cost resulting from the spatial distribution. Rossi-Hansberg (2005) adopts a trade model with a continuum of regions, transportation cost and agglomeration effects caused by production externalities, so as to study the relationship between trade pattern and spatial distribution of economic activity. Eaton and Kortum (2002) apply a Richardian model of international trade that captures the competing forces of comparative advantage promoting trade and of geographic barriers inhibiting trade.

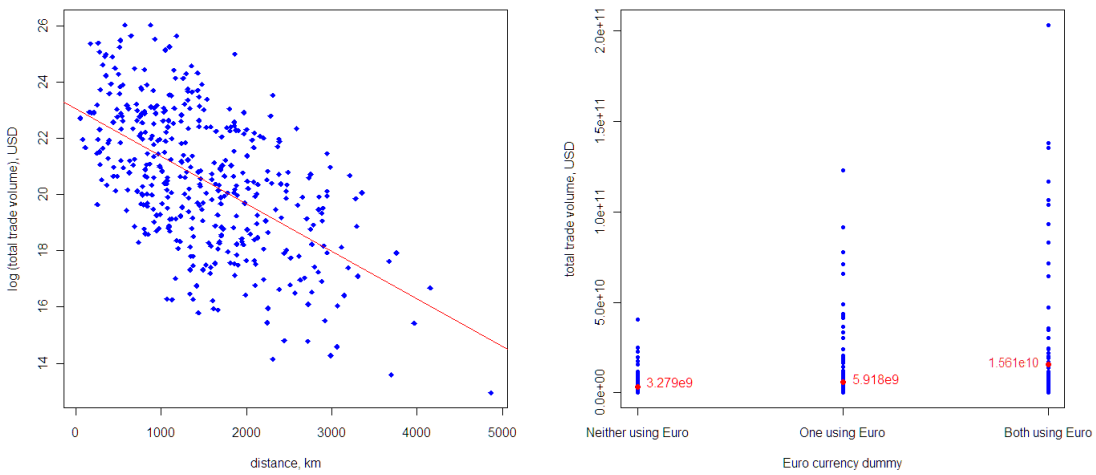
Spatial econometrics has been developed rapidly in the past twenty years. The main idea in the literature is that, instead of treating all the spatial units as isolated, neighbors are interdependent with less dependence farther away.[1] LeSage and Pace (2004) show that the use of least-squares in the presence of spatial dependence in the dependent variable vector leads to biased and inconsistent estimates. Spatial dependence structure is usually captured by a spatial weights matrix, with each entry to be either a contiguity dummy or an inverse measure of the bilateral distance. The weights are smaller for units that are farther away. This approach has been adopted in many applied analysis.[2] Even though standard spatial econometric model only applies to data set for n spatial units, which is generally not suitable for the export-import trade framework, there are some exceptions. Lesage and Pace (2008) model the connectivity among a sample of $N = n^2$ origin-destination (OD) pairs. Behrens et al. (2012) develop a structural 'dual' gravity model and estimate the spatial trade pattern among U.S. states and Canada provinces. They include the international border (i.e. the Canada-U.S. border) as a dummy variable to capture the country specific effect.

There are few papers that directly analyze the spatial trade pattern among European countries. To the best of our knowledge, Porojan (2001) is the only recent paper that uses data of year 1995 and studies this pattern in a preliminary regression framework. A more theory-based regression framework of the gravity model is crucial for better estimating the distance effect. As can be seen from Panel (a) of Figure 1, there is a clear negative relationship between the distance and bilateral trade volume. Moreover, the membership effect among European countries can be captured by Euro currency dummy or European Union (EU) membership dummy. The intuition is that, the trade costs between two countries using the same currency (Euro) or belonging to EU tend to be lower. This is illustrated in Panel (b) of Figure 1, where the data is divided into three groups. The dummy variable is $0, 0.5, 1$ if neither country uses Euro, one uses Euro, and both use Euro, respectively. The average trade volume in the case where both countries use Euro is 4.76 times greater than the case where neither country uses Euro.

In this paper, we adopt the theory-based spatial econometric specification from the 'dual'

[1] See Lesage and Pace (2009)

[2] To name a few, Maddison (2006) uses the spatial econometric techniques to re-estimate the environmental Kuznets curve. Ertur and Koch (2007) develop a spatial version of neoclassical growth model and test the growth convergence hypothesis using a spatial econometric specification. Brueckner (1998) applies the spatial econometric model to investigate the strategic interaction among local governments.



(a) Distance vs. total trade volume

(b) Euro vs. total trade volume

Figure 1: Distance/Euro vs. total trade volume among 29 countries, 2010

gravity model in Behrens et al. (2012), and apply it to the data of European countries to document the spatial trade pattern and distance effects on trade. Our main contribution to the literature is that we are the first paper to use theory-based spatial econometric gravity model to study the recent (2006−2010) spatial trade pattern among European countries.

The remainder of this paper will proceed as follows. In Section 2, we introduce and discuss the 'dual' gravity model proposed in Behrens et al. (2012), with some modifications to adapt to the case of European countries. In Section 3, we discuss the spatial econometric specification derived directly from the theoretical model. In Section 4, we estimate and discuss the proposed spatial econometric model under different specifications of the underlying spatial error structure. Section 5 concludes.

2. The Model

2.1. Preference and Demand

There are n regions in the economy. Each region i has a population of L_i, and each agent consumes and works by supplying one unit of labor inelastically. The aggregate labor supply in region i is L_i. The representative consumer has identical CES utility function over a continuum of horizontally differentiated product varieties, the representative agent in region j solves the following problem:

$$\max \quad U_j = \sum_{i=1}^{n} \int_{\Omega_i} q_{ij}(v)^{\frac{\sigma-1}{\sigma}} dv \quad \text{subject to} \quad \sum_{i=1}^{n} \int_{\Omega_i} q_{ij}(v) p_{ij}(v) dv = y_j \tag{1}$$

where $\sigma > 1$ is the constant elasticity of substitution between any two varieties; y_j is the income of representative agent in region j; $p_{ij}(v)$ is region j's selling price of variety v which is produced in region i; $q_{ij}(v)$ is the amount of consumption of variety v (produced in region i) by a representative agent in region j and Ω_i denotes the set of varieties produced in region i. For simplicity, we assume that varieties produced by the same region are symmetric in the sense that a representative agent consumes the same amount of each variety from a same region, and the price of each variety from the same region is the same. Let m_i be the measure of Ω_i. Standard utility optimization gives the aggregate inverse demand function of region j for the variety produced in region i[3]:

$$p_{ij} = \frac{Q_{ij}^{-\frac{1}{\sigma}}}{\sum_{k=1}^{n} m_k Q_{kj}^{\frac{\sigma-1}{\sigma}}} Y_j \tag{2}$$

where $Q_{ij} = L_j q_{ij}$ and $Y_j = L_j y_j$ are the aggregate demand in region j for a variety produced in region i and the aggregate income in region j, respectively.

2.2. Firms and Technology

Each firm in region i produces a single variety. Labor is the only input of firms, and we assume homogeneous firms in our model. The cost function of a firm is $C(q) = cq + F$, where c is the constant marginal cost, F is fixed cost in units of labor. Shipping varieties from regions to regions is costly in the sense that shipping one unit of variety from region j to region k requires dispatching τ_{jk} units from region j. While one unit arrives at region k eventually, $\tau_{jk} - 1$ becomes the iceberg cost.

A firm located in region j solves the profit maximization problem:

$$\max \ \pi_j \equiv \sum_{k=1}^{n} (p_{jk} - c w_j \tau_{jk}) Q_{jk} - F w_j \ \text{ subject to (2)} \tag{3}$$

The solution to (3) is[4]

$$p_{jk} = \tau_{jk} p_j, \text{ where } p_j = \frac{c w_j \sigma}{\sigma - 1} \tag{4}$$

Imposing free entry such that profits of firms will be zero, impose this and plug (4) into (3), we get

$$\sum_{k=1}^{n} \tau_{jk} Q_{jk} = \frac{F(\sigma - 1)}{c} \equiv \overline{Q} \tag{5}$$

[3]Derivation of the aggregate inverse demand function (2) can be found in the Appendix

[4]Derivation of (4) can be found in the Appendix.

2.3. Equilibrium

Given the price p_{ij}, aggregate demand Q_{ij} and the measure of varieties produced in region i, the value of trade flow from region i to region j is:

$$X_{ij} = m_i p_{ij} Q_{ij} = m_i \frac{Q_{ij}^{\frac{\sigma-1}{\sigma}}}{\sum_{k=1}^{n} m_k Q_{kj}^{\frac{\sigma-1}{\sigma}}} Y_j \tag{6}$$

Since $Y_i = \sum_{k=1}^{n} m_i p_{ik} Q_{ik}$, by (4) and (5) we have $Y_i = m_i p_i \overline{Q}$. Substituting this into (6) to eliminate the unobserved term m_i, we get

$$X_{ij} = \frac{Q_{ij}^{\frac{\sigma-1}{\sigma}} Y_i Y_j}{\sum_{k=1}^{n} \frac{p_i}{p_k} Y_k Q_{kj}^{\frac{\sigma-1}{\sigma}}} \tag{7}$$

By the definition of value of trade flow, (4) and $Y_i = m_i p_i \overline{Q}$, we obtain

$$Q_{ij} = \frac{X_{ij}}{m_i p_{ij}} = \frac{X_{ij} \overline{Q}}{Y_i \tau_{ij}} \tag{8}$$

Plug (8) into (7) and simplify the expression using the fact that $p_i/p_k = w_i/w_k$ and $w_i = \frac{Y_i}{L_i}$. Thus, we have

$$X_{ij} = Y_j \frac{\tau_{ij}^{\frac{1-\sigma}{\sigma}} \left(\frac{X_{ij}}{Y_i}\right)^{\frac{\sigma-1}{\sigma}}}{\sum_{k=1}^{n} \frac{L_k}{L_i} \tau_{kj}^{\frac{1-\sigma}{\sigma}} \left(\frac{X_{kj}}{Y_k}\right)^{\frac{\sigma-1}{\sigma}}} \tag{9}$$

Solving for X_{ij} from (9), we get

$$X_{ij} = Y_j^{\sigma} \left[\sum_{k=1}^{n} \frac{L_k}{L_i} \left(\frac{\tau_{kj} Y_k}{\tau_{ij} Y_i}\right)^{\frac{1-\sigma}{\sigma}} X_{kj}^{\frac{\sigma-1}{\sigma}} \right]^{-\sigma} \tag{10}$$

As we can clearly see from (10), the value of trade flow from region i to region j depends on income, population of every region and the value of trade flow from every other region to region j, and of course, the many iceberg cost. To close the general equilibrium system, we impose the aggregate income constraint,

$$Y_i - \sum_{k=1}^{n} X_{ik} = 0 \quad \forall\ i \tag{11}$$

3. Econometric Specification

Taking the logarithm of (10), we get

$$\ln X_{ij} = \sigma \ln Y_j - \sigma \ln \left[\sum_{k=1}^{n} \frac{L_k}{L_i} \left(\frac{\tau_{kj} Y_k}{\tau_{ij} Y_i X_{kj}} \right)^{\frac{1-\sigma}{\sigma}} \right] \equiv f(\sigma) \tag{12}$$

In order to obtain a specification that is amenable to estimation with spatial econometric techniques, we linearize f around $\sigma = 1$ to get an approximation that $f(\sigma) \approx f(1) + f'(1)(\sigma - 1)$. The linearized approximation is:[5]

$$\ln Z_{ij} = -\sigma \ln L - (\sigma - 1) \left(\ln \tau_{ij} - \sum_{k=1}^{n} \frac{L_k}{L} \ln \tau_{kj} \right) - \sigma \ln w_i - (\sigma - 1) \sum_{k=1}^{n} \frac{L_k}{L} \ln Z_{kj}, \quad \forall\, i, j \tag{13}$$

where $Z_{ij} = \frac{X_{ij}}{Y_i Y_j}$ is a GDP-standardized value of trade flow from region i to region j, and $L = \sum_{k=1}^{n} L_k$ is the total population of all regions. Equation (13) is intuitive. The standardized trade flow from region i to j depends negatively on the deviation of iceberg cost from the weighted mean, higher iceberg cost decreases the trade flow; $\ln Z_{ij}$ is also negatively related to the domestic wage w_i, since we can clearly see from (4) that a higher w_i will increase the price of good sold in all markets(including domestic market), thus reduce the demand. Moreover, $\ln Z_{ij}$ depends negatively on the weighted export from all regions to region j due to the substitution effect, and since a larger value σ indicates that goods from different regions are more substitutable, then a higher σ will strengthen the substitution effect.

Essentially, (13) represents n^2 equations, we can stack the system of equations into matrix form as follows:

$$\boldsymbol{Z} = -(\sigma \ln L)\boldsymbol{\iota} - (\sigma - 1)(\boldsymbol{I} - \boldsymbol{W})\boldsymbol{\tau} - \sigma \boldsymbol{w} - (\sigma - 1)\boldsymbol{W}\boldsymbol{Z} \tag{14}$$

where $\boldsymbol{\iota}$ is a $n^2 \times 1$ vector of ones; $\boldsymbol{I}$ is a $n^2 \times n^2$ identity matrix; $\boldsymbol{\tau}$ is a $n^2 \times 1$ vector with entries $\ln \tau_{ij}$, where j is the faster running index; $\boldsymbol{w}$ is a $n^2 \times 1$ vector with entries $\ln w_i$ s.t. the first n entries are all $\ln w_1$, the second n entries are all $\ln w_2$, and so on; $\boldsymbol{Z}$ is a $n^2 \times 1$ vector with entries $\ln \left(\frac{X_{ij}}{Y_i Y_j} \right)$, where j is also the faster running index; $\boldsymbol{W} = [\boldsymbol{J} \mathrm{diag}(\boldsymbol{L})] \otimes \boldsymbol{I}_n$, where $\boldsymbol{J}$ is a $n \times n$ matrix of ones, $\mathrm{diag}(\boldsymbol{L})$ is a $n \times n$ diagonal matrix with the k-th diagonal term to be $\frac{L_k}{L}$, $\boldsymbol{I}_n$ is a $n \times n$ identity matrix and $\otimes$ is the Kronecker product.

Our data set consists of different countries in Europe; each trade will cross international borders. For the trade cost, unlike Behrens et al. (2012) assuming the trade cost to be a function of distance and border effects across U.S. states and Canada provinces, here we assume that the trade cost is a log linear function of distance and a dummy variable indicating

[5] The derivation can be found in the Appendix.

whether both countries belong to the EU, since trade cost is supposed to be smaller if two trading countries both in EU. Similarly, we use another dummy variable indicating whether both countries use Euro as the major currency. Specifically,

$$\tau_{ij} = d_{ij}^{\gamma} e^{\xi b_{ij}} \tag{15}$$

$b_{ij} = 1$ if the bilateral trade partners both belong to the EU (or both use Euro), and 0 other wise. Taking log and stack it into a matrix, we get

$$\boldsymbol{\tau} = \gamma \boldsymbol{d} + \xi \boldsymbol{b} \tag{16}$$

where $\boldsymbol{d}$ is a $n^2 \times 1$ vector with entries $\ln d_{ij}$, $\boldsymbol{b}$ is a $n^2 \times 1$ vector with entries b_{ij}, j is the faster running index in both vectors. Plug (16) into (14), we get

$$\boldsymbol{Z} = \beta_0 \boldsymbol{\iota} + \beta_1 \tilde{\boldsymbol{d}} + \beta_2 \boldsymbol{w} + \beta_3 \tilde{\boldsymbol{b}} + \rho \boldsymbol{W} \boldsymbol{Z} \tag{17}$$

where $\tilde{\boldsymbol{d}} = (\boldsymbol{I} - \boldsymbol{W})\boldsymbol{d}$, $\tilde{\boldsymbol{b}} = (\boldsymbol{I} - \boldsymbol{W})\boldsymbol{b}$, $\beta_0 = -(\sigma \ln L) < 0$, $\beta_1 = -(\sigma - 1)\gamma < 0$, $\beta_2 = -\sigma < 0$, $\beta_3 = -(\sigma - 1)\xi > 0$, $\rho = -(\sigma - 1)$. Let $\boldsymbol{W}_{\text{diag}} = \text{diag}(\boldsymbol{L}) \otimes \boldsymbol{I}_n$, then it contains only the diagonal elements of $\boldsymbol{W}$, with the i-th block to be $\frac{L_i}{L}\boldsymbol{I}_n$. Subtracting both sides of (17) by $\rho \boldsymbol{W}_{\text{diag}} \boldsymbol{Z}$, we get

$$(\boldsymbol{I} - \rho \boldsymbol{W}_{\text{diag}})\boldsymbol{Z} = \beta_0 \boldsymbol{\iota} + \beta_1 \tilde{\boldsymbol{d}} + \beta_2 \boldsymbol{w} + \beta_3 \tilde{\boldsymbol{b}} + \rho(\boldsymbol{W} - \boldsymbol{W}_{\text{diag}})\boldsymbol{Z} \tag{18}$$

The matrix $\boldsymbol{I} - \rho \boldsymbol{W}_{\text{diag}}$ is diagonal with its i-th block to be $\left(1 - \rho \frac{L_i}{L}\right) \boldsymbol{I}_n$. Provided that $1 - \rho \frac{L_i}{L} \neq 0 \forall\, i$, $\boldsymbol{I} - \rho \boldsymbol{W}_{\text{diag}}$ is nonsingular with the i-th block of its inverse to be $\left(1 - \rho \frac{L_i}{L}\right)^{-1} \boldsymbol{I}_n$. Pre-multiply (18) by $(\boldsymbol{I} - \rho \boldsymbol{W}_{\text{diag}})^{-1}$, we get

$$\boldsymbol{Z} = \overline{\boldsymbol{\beta}}_0 \boldsymbol{\iota} + \overline{\boldsymbol{\beta}}_1 \tilde{\boldsymbol{d}} + \overline{\boldsymbol{\beta}}_2 \boldsymbol{w} + \overline{\boldsymbol{\beta}}_3 \tilde{\boldsymbol{b}} + \overline{\boldsymbol{\rho}} \boldsymbol{W}_d \boldsymbol{Z} \tag{19}$$

$\boldsymbol{W}_d = \boldsymbol{W} - \boldsymbol{W}_{\text{diag}}$ is the theory based interaction matrix. $\overline{\boldsymbol{\beta}}_0$ is a $n^2 \times 1$ vector, with its i-th subvector to be of size $n \times 1$, and every element of the subvector is the same, being $\overline{\boldsymbol{\beta}}_{0i} = -\sigma \ln L \left(1 - \rho \frac{L_i}{L}\right)^{-1}$. Similarly, $\overline{\boldsymbol{\beta}}_{1i} = \rho\gamma \left(1 - \rho \frac{L_i}{L}\right)^{-1}$, $\overline{\boldsymbol{\beta}}_{2i} = -\sigma \left(1 - \rho \frac{L_i}{L}\right)^{-1}$, $\overline{\boldsymbol{\beta}}_{3i} = \rho\xi \left(1 - \rho \frac{L_i}{L}\right)^{-1}$, $\overline{\boldsymbol{\rho}}_i = \rho \left(1 - \rho \frac{L_i}{L}\right)^{-1}$. As we can see, the parameters varies across regions, due to the fact that L_i varies across regions, which is complicated to estimate. We first estimate a simpler benchmark model assuming the coefficients are the same across regions, which amounts to assuming that $\boldsymbol{W}_{\text{diag}} = \boldsymbol{0}$, i.e. $\boldsymbol{W} = \boldsymbol{W}_d$. Then the model is simplified to

$$\boldsymbol{Z} = \beta_0 \boldsymbol{\iota} + \beta_1 \tilde{\boldsymbol{d}} + \beta_2 \boldsymbol{w} + \beta_3 \tilde{\boldsymbol{b}} + \rho \boldsymbol{W}_d \boldsymbol{Z} \tag{20}$$

Borrowing from spatial econometrics literature, we assume that error structure as follows:

$$\boldsymbol{u} = \boldsymbol{\varepsilon} + \lambda \boldsymbol{W}_d \boldsymbol{\varepsilon} \tag{21}$$

As we can see, the above error structure is the so-called spatial first order moving average error. Alternatively, we can specify the error structure as

$$\boldsymbol{u} = \lambda \boldsymbol{W}_d \boldsymbol{u} + \boldsymbol{\varepsilon} \tag{22}$$

The above expression is the so-called spatial first order autoregressive error structure. In both cases, the error $\boldsymbol{\varepsilon} \sim N(\boldsymbol{0}, \sigma^2 \boldsymbol{I})$. Solving for $\boldsymbol{u}$ from (22), we get

$$\boldsymbol{u} = (\boldsymbol{I} - \lambda \boldsymbol{W}_d)^{-1} \boldsymbol{\varepsilon} = \sum_{j=1}^{\infty} \lambda^j \boldsymbol{W}_d^j \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon} \tag{23}$$

Provided $|\lambda| < \frac{1}{\eta}$, where η is the spectral radius of $\boldsymbol{W}_d$ (see for example, Kelejian and Prucha, 2009). When the successive powers in the sum converge to 0 sufficiently quickly, the autoregressive structure approximates appropriately the first-order moving average, i.e., $\boldsymbol{u} \approx \boldsymbol{\varepsilon} + \lambda \boldsymbol{W}_d \boldsymbol{\varepsilon}$.

4. Data and Estimation

4.1 Data and control variables

We construct five cross-section data sets for five consecutive years (namely 2006-2010). Each data set features aggregate export X_{ij} across 29 European countries, nominal GDP Y_i by country, internal absorption X_{ii} (all measured in US dollars at current prices), great circle distances d_{ij} between countries (measured in kilometers), and the internal distances d_{ii} within one country (measured in kilometers). Each data set also includes the labor force population (ages 15-64) L_i and the average annual manufacturing wages w_i (measured in US dollars) for each country. A detailed description of the data sets and data sources is provided in the Appendix.

There are several issues in dealing with the data. First, in our model, we require a measure of the internal trade costs because we consider the full interdependence structure in the gravity model. Following Behrens et al. (2012) and Redding and Venables (2004), we measure the internal trade distances as $d_{ii} \equiv \frac{2}{3}\sqrt{\text{surface}_i/\pi}$, where surface_i denotes the country's surface area (measured in square kilometers). As a robust check, we also adopt $\frac{1}{3}\sqrt{\text{surface}_i/\pi}$ as an alternative internal distance measure. For the external distances, we use the great circle distances between country capitals as the measure of d_{ij}.

Second, we compute the average manufacturing wages w_i in the following way. We obtain

the total compensation of employees in manufacturing (measured in Euro at current prices) from national accounts - current account, and divide it by the total employment (ages 15+) in manufacturing, so as to get the average annual manufacturing wages (measured in Euro). Then, we convert the wages to US dollar by multiplying the average daily EUR/USD exchange rate of that year.

Third, our estimation model requires the internal absorption X_{ii} for the 29 countries. We assume that a country's export to itself is the difference between its output and its total export to the world. Hence, we construct X_{ii} by deducting the total export volume to the world from the nominal GDP Y_i (measured in US dollars at current prices) as the measure of internal absorption X_{ii}. We also require the bilateral trade flows across all pair of countries to account for their interdependence. Fortunately, there is only 1 observation with zero trade flow among the 4205 observations (29 countries $\times$ 29 countries $\times$ 5 years) that we have. We simply augment the trade flows by adding 1, such that the log of trade flow is equal to zero. Since 1 observation out of 4205 is an extremely small proportion, we do not include a zero-flow dummy variable to control for this adjustment in the regression.

4.2 Empirical Results

We estimate three econometric models for 2010. In order to set a benchmark result for analysis, we first estimate a basic OLS model

$$\boldsymbol{Z} = \beta_0 \boldsymbol{\iota} + \beta_1 \tilde{\boldsymbol{d}} + \beta_2 \boldsymbol{w} + \beta_3 \tilde{\boldsymbol{b}} + \boldsymbol{u}$$

The OLS result is summarized in the first two columns in Table 1. In the first column, we use $\frac{1}{3}\sqrt{\text{surface}_i/\pi}$ as the internal distance measure. The estimated coefficient on distance is -1.465. As expected, this indicates that a larger physical distance tends to set more barriers to trade thus reduce the export volume. The estimated coefficient on wage is -0.103, which indicates that a higher domestic wage tends to reduce export due to the increased production cost. Adopting Euro has a positive effect on the export volume, with the estimated coefficient to be 0.281. In the second column, we use $\frac{2}{3}\sqrt{\text{surface}_i/\pi}$ as the internal distance measure. The result is very similar to that in the first column and all of them are statistically significant.

Our main estimation is the Spatial Autoregressive Moving-average (SARMA) model (i.e. equations (20) and (21)). The estimation results are reported in the third and fourth columns in Table 1. All of the slope coefficients have the expected signs as in the OLS case. However, they differ from the OLS estimates in magnitude. When the internal distance measure $\frac{1}{3}\sqrt{\text{surface}_i/\pi}$ is used, the estimated coefficient on distance is -1.649, which is slightly larger in absolute value compared with the OLS case. This suggests that when we consider the spatial interdependence, distance has a more negative impact on the export volume, and the same holds for the wage effect. While the estimated coefficient for Euro dummy becomes

smaller with the magnitude 0.125, it becomes statistically insignificant. This implies that sharing the same currency might not be an influencing factor in the presence of a mature foreign exchange market. The estimation results of the slope parameters are similar when we use the $\frac{2}{3}\sqrt{\text{surface}_i/\pi}$ internal distance measure. For the estimates of the spatial correlation parameters, λ is 0.059 and -0.165 for the two internal distance measures, and they are not statistically significant. This shows that there is no spatial correlation in the error structure. $\rho = 0.553$ and 0.614 for the two internal distance measures and they are statistically significant at the level of 0.01. This finding is different from the result in Behrens et al. (2012); their estimate of ρ is -0.208. This finding is also not consistent with the theoretical model. In the model we have $\rho = -(\sigma - 1) < 0$, indicating the existence of spatial competition in exporting under the assumption that varieties produced in different countries are substitutes. However, our estimation of $\rho = 0.553$ and 0.614 seems to be contradictory to such substitution effect assumption.

Model	OLS		SARMA		SARAR	
	1/3 dist	2/3 dist	1/3 dist	2/3 dist	1/3 dist	2/3 dist
(Intercept)	-16.675^{***}	-16.631^{***}	-5.968^{***}	-4.750^{***}	-5.903^{***}	-4.974^{***}
	(0.500)	(0.520)	(0.957)	(0.997)	(0.895)	(0.911)
Dist	-1.465^{***}	-1.572^{***}	-1.649^{***}	-1.815^{***}	-1.650^{***}	-1.809^{***}
	(0.045)	(0.057)	(0.039)	(0.044)	(0.038)	(0.043)
Wage	-0.103^{**}	-0.104^{**}	-0.194^{***}	-0.202^{***}	-0.194^{***}	-0.201^{***}
	(0.048)	(0.050)	(0.043)	(0.044)	(0.043)	(0.044)
Euro	0.281^{***}	0.340^{***}	0.125	0.161^{*}	0.123	0.168^{*}
	(0.098)	(0.100)	(0.088)	(0.091)	(0.088)	(0.090)
λ	–	–	0.059	-0.165	0.031	-0.101
	–	–	(0.220)	(0.174)	(0.148)	(0.168)
ρ	–	–	0.553^{***}	0.614^{***}	0.556^{***}	0.602^{***}
	–	–	(0.044)	(0.047)	(0.041)	(0.042)
R^2	0.622	0.596	–	–	–	–
Log likelihood	–	–	-1.142×10^3	-1.166×10^3	-1.143×10^3	-1.166×10^3

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 1: Estimation result for 2010 (EURO currency)

For the robustness check, we alternatively estimate the Spatial Autoregressive Autocorrelation (SARAR) model (i.e. equations (20) and (22)). The results are summarized in columns 5 and 6 in Figure 1. As can be seen, the results are almost identical as that in the SARMA model, which confirms our previous results. Another robustness check is performed by using the EU member dummy instead of the Euro dummy. The estimation results are reported in

Table 2, for OLS, SARMA and SARAR models, respectively. The estimated parameters are similar to that using Euro dummy. However, there is one interesting finding. In both SARMA and SARAR models, estimated coefficients for EU membership dummy become significant and have a larger magnitude. It suggests that the membership in EU tends to facilitate the bilateral trade. This is probably because of the trade agreements and treaties among the EU member countries, which promote the economic integration. In comparison the Euro dummy does not have such statistic significance level as the EU dummy, which indicates that sharing the same currency does not have a significant influencing power on bilateral trade.

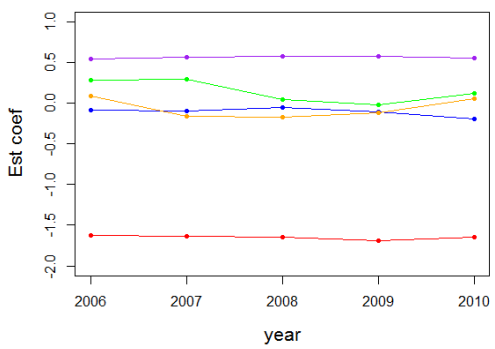
Model	OLS		SARMA		SARAR	
	1/3 dist	2/3 dist	1/3 dist	2/3 dist	1/3 dist	2/3 dist
(Intercept)	-17.332^{***}	-17.387^{***}	-6.841^{***}	-5.579^{***}	-6.684^{***}	-5.751^{***}
	(0.478)	(0.499)	(1.027)	(1.076)	(0.959)	(0.979)
Dist	-1.476^{***}	-1.587^{***}	-1.643^{***}	-1.810^{***}	-1.646^{***}	-1.806^{***}
	(0.044)	(0.056)	(0.038)	(0.044)	(0.038)	(0.043)
Wage	-0.037	-0.029	-0.161^{***}	-0.165^{***}	-0.163^{***}	-0.164^{***}
	(0.046)	(0.048)	(0.042)	(0.044)	(0.042)	(0.043)
EUmem	0.755^{***}	0.805^{***}	0.286^{**}	0.281^{**}	0.278^{**}	0.289^{**}
	(0.153)	(0.155)	(0.116)	(0.119)	(0.114)	(0.117)
λ	–	–	0.135	-0.119	0.069	-0.073
	–	–	(0.236)	(0.184)	(0.145)	(0.166)
ρ	–	–	0.522^{***}	0.589^{***}	0.530^{***}	0.580^{***}
	–	–	(0.046)	(0.049)	(0.043)	(0.044)
R^2	0.635	0.610	–	–	–	–
Log likelihood	–	–	-1.140×10^3	-1.165×10^3	-1.141×10^3	-1.165×10^3

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

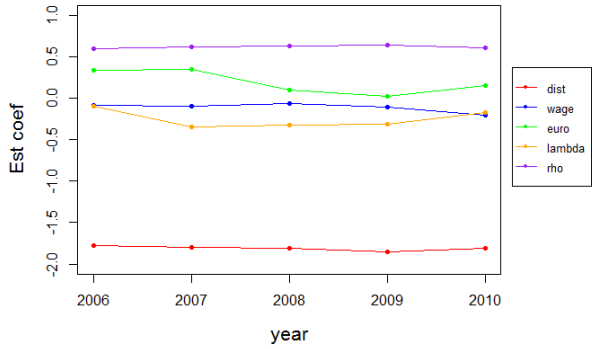
Table 2: Estimation result for 2010 (EU member)

In order to check if our finding is stable across different years, we do the same regression analysis for each year from 2006 to 2009. The results are very stable in terms of sign and magnitude of the estimated parameters. For the sake of compactness, we only plot the results for SARMA model in Figure 2 as an illustration[6]. For instance, in Panel (c) of Figure 2, the distance coefficient stays below -1.5 and is very stable from 2006 to 2010. The wage coefficient remains negative, despite of a small magnitude. There seems to be some slight fluctuation in the estimated EU membership coefficient; nevertheless it stays positive over the five years, which indicates the positive benefit of joining EU.

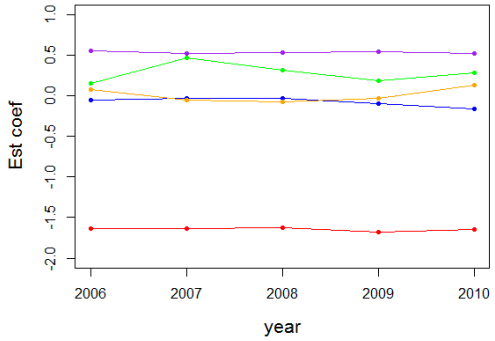
[6]The Matlab code, as well as detailed estimation results for all different regression models across different years are available upon request



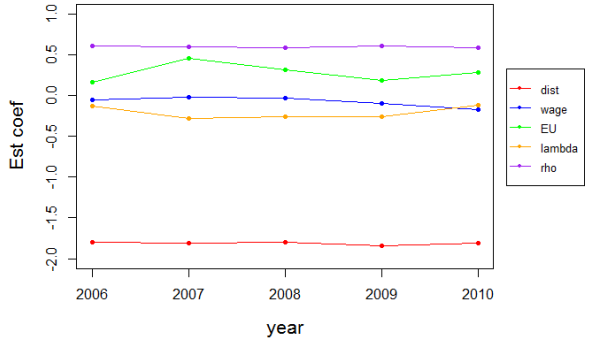
(a) 1/3 distance, Euro currency dummy



(b) 2/3 distance, Euro currency dummy



(c) 1/3 distance, EU membership dummy



(d) 2/3 distance, EU membership dummy

Figure 2: SARMA Estimation for Five Years

5. Conclusion

In this paper, we investigate the spatial trade pattern among European countries in a gravity model. Particularly, we adopt the theoretical framework of 'dual' gravity proposed in Behrens et al. (2012), and apply the structural spatial econometric specification to European countries trade data. The estimation result is basically in line with findings in previous literature and the prediction of gravity model. On one hand, a larger geographical distance between two countries tends to impede trade due to more trade barriers as well as a higher trade cost. On the other hand, a higher domestic wage has a negative impact on the export because of the increased production cost. Furthermore, being an EU member has a positive impact on the export volume while we find a positive but not statistically significant effect of sharing Euro currency. Lastly, the estimated spatial competition coefficient is positive for European

data, which is quite counterintuitive if we assume that varieties from different countries are substitutes.

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Appendix I. Some Derivations

Derivation of Aggregate Inverse Demand

$$\max U_j \equiv \sum_i \int_{\Omega_i} q_{ij}(v)^{\frac{\sigma-1}{\sigma}} dv \quad \text{s.t.} \quad \sum_i \int_{\Omega_i} q_{ij}(v) p_{ij}(v) dv = y_j$$

Since varieties produced in the same region are assumed to be symmetric, thus we alleviate notation by dropping v. Let m_k stand for the measure of Ω_k. Using Lagrangian method, we have

$$\mathscr{L} = \sum_i \int_{\Omega_i} q_{ij}(v)^{\frac{\sigma-1}{\sigma}} dv - \lambda \left[\sum_i \int_{\Omega_i} q_{ij}(v) p_{ij}(v) dv - y_j \right]$$

$$\mathscr{L} = \sum_i (q_{ij}^{\frac{\sigma-1}{\sigma}} - \lambda q_{ij} p_{ij}) m_i + \lambda y_j$$

$$\frac{\partial \mathscr{L}}{\partial q_{ij}} = \frac{\sigma-1}{\sigma} q_{ij}^{-\frac{1}{\sigma}} - \lambda p_{ij} = 0 \tag{1}$$

$$\frac{\partial \mathscr{L}}{\partial \lambda} = \sum_i q_{ij} p_{ij} m_i - y_j = 0 \tag{2}$$

From equation (1) we get

$$\lambda = \frac{\frac{\sigma-1}{\sigma} q_{ij}^{-\frac{1}{\sigma}}}{p_{ij}} = \frac{\frac{\sigma-1}{\sigma} \sum_i q_{ij}^{-\frac{1}{\sigma}} q_{ij} m_i}{\sum_i p_{ij} q_{ij} m_i} = \frac{\frac{\sigma-1}{\sigma} \sum_i q_{ij}^{1-\frac{1}{\sigma}} m_i}{y_j}$$

Plug λ back into equation (1), we get

$$q_{ij}^{-\frac{1}{\sigma}} = \frac{\sum_k q_{kj}^{1-\frac{1}{\sigma}} m_k}{y_j} p_{ij}$$

Multiply both sides by $L_j^{-\frac{1}{\sigma}}$, then we get the aggregate inverse demand function

$$p_{ij} = \frac{Q_{ij}^{-\frac{1}{\sigma}}}{\sum_k Q_{kj}^{1-\frac{1}{\sigma}} m_k} Y_j$$

where $Q_{ij} \equiv L_j q_{ij}$ and $Y_j \equiv L_j y_j$.

Derivation of Equation (4)

A firm located in region j maximizes its profit, given by

$$\pi_j = \sum_k (p_{jk} - cw_j\tau_{jk})Q_{jk} - Fw_j \quad \text{s.t.} \quad p_{jk} = \frac{Q_{jk}^{-\frac{1}{\sigma}}}{\sum_i Q_{ik}^{1-\frac{1}{\sigma}} m_i} Y_k$$

Plug in p_{jk}, we get

$$\pi_j = \sum_k \left(\frac{Q_{jk}^{1-\frac{1}{\sigma}}}{\sum_i Q_{ik}^{1-\frac{1}{\sigma}} m_i} Y_k - cw_j\tau_{jk}Q_{jk}\right) - Fw_j$$

$$\frac{\partial \pi_j}{\partial Q_{jk}} = \frac{(1-\frac{1}{\sigma})Q_{jk}^{-\frac{1}{\sigma}}}{\sum_i Q_{ik}^{1-\frac{1}{\sigma}} m_i} Y_k - cw_j\tau_{jk} + \frac{Q_{jk}^{1-\frac{1}{\sigma}}(-1)}{(\sum_i Q_{ik}^{1-\frac{1}{\sigma}} m_i)^2} Y_k m_j Q_{jk}^{-\frac{1}{\sigma}} (1-\frac{1}{\sigma}) = 0$$

Then we have

$$(1-\frac{1}{\sigma})p_{jk} = (1-\frac{1}{\sigma})\frac{Q_{jk}^{-\frac{1}{\sigma}}}{\sum_i Q_{ik}^{1-\frac{1}{\sigma}} m_i} Y_k = cw_j\tau_{jk}$$

$$p_{jk} = cw_j\tau_{jk}(\frac{\sigma}{\sigma-1}) \equiv \tau_{jk}p_j$$

Derivation of Equation (13)

Take the logarithm of the equation X_{ij} we get previously, we obtain

$$\ln X_{ij} = \sigma \ln Y_j - \sigma \ln \left[\sum_{k=1}^{n} \frac{L_k}{L_i} \left(\frac{\tau_{kj}Y_k}{\tau_{ij}Y_i}\right)^{\frac{1}{\sigma}-1} X_{kj}^{1-\frac{1}{\sigma}}\right] \equiv f(\sigma)$$

The linear approximation of f at $\sigma = 1$ is given by $f(\sigma) \approx f(1) + (\sigma - 1) f'(1)$. Let $L \equiv \sum_k L_k$, we get

$$f(1) = \ln Y_j - \ln\left(\sum_{k=1}^{n} \frac{L_k}{L_i}\right) = \ln Y_j - \ln L + \ln L_i \tag{3}$$

$$f'(\sigma) = \ln Y_j - \ln\left[\sum_k \frac{L_k}{L_i}\left(\frac{\tau_{kj} Y_k}{\tau_{ij} Y_i X_{kj}}\right)^{\frac{1}{\sigma}-1}\right] - \sigma \frac{\sum_k \frac{L_k}{L_i}\left(\ln \frac{\tau_{kj} Y_k}{\tau_{ij} Y_i X_{kj}}\right)\left(\frac{\tau_{kj} Y_k}{\tau_{ij} Y_i X_{kj}}\right)^{\frac{1}{\sigma}-1}\left(\frac{-1}{\sigma^2}\right)}{\sum_k \frac{L_k}{L_i}\left(\frac{\tau_{kj} Y_k}{\tau_{ij} Y_i X_{kj}}\right)^{\frac{1}{\sigma}-1}}$$

$$f'(1) = \ln Y_j - \ln L + \ln L_i + \sum_k \frac{L_k}{L} \ln \frac{\tau_{kj}}{\tau_{ij}} + \sum_k \frac{L_k}{L} \ln \frac{Y_k}{Y_i} - \sum_k \frac{L_k}{L} \ln X_{kj} \tag{4}$$

Using equations (3) and (4), the linear approximation of f is

$$\begin{aligned}\ln X_{ij} =& \sigma \ln L_i - \sigma \ln L + \sigma \ln Y_j - (\sigma - 1) \ln Y_i - (\sigma - 1) \ln \tau_{ij} \\ &+ (\sigma - 1) \sum_k \frac{L_k}{L} \ln \tau_{kj} + (\sigma - 1) \sum_k \frac{L_k}{L} \ln Y_k - (\sigma - 1) \sum_k \frac{L_k}{L} \ln X_{kj}\end{aligned}$$

Given $Y_i = w_i L_i$ (the aggregate income constraint) and $\sum_k \frac{L_k}{L} = 1$, we have

$$\begin{aligned}\ln\left(\frac{X_{ij}}{Y_i Y_j}\right) =& \sigma \sum_k \frac{L_k}{L} \ln \frac{L_k}{L} + (\sigma - 1) \ln Y_j - (\sigma - 1)\left(\ln \tau_{ij} - \sum_k \frac{L_k}{L} \ln \tau_{kj}\right) \\ &- \sigma\left(\ln w_i - \sum_k \frac{L_k}{L} \ln w_k\right) - \sum_k \frac{L_k}{L} \ln Y_k - (\sigma - 1) \sum_k \frac{L_k}{L} \ln X_{kj} \\ =& -\sigma \ln L - (\sigma - 1)\left(\ln \tau_{ij} - \sum_k \frac{L_k}{L} \ln \tau_{kj}\right) - \sigma \ln w_i - (\sigma - 1) \sum_k \frac{L_k}{L} \ln\left(\frac{X_{kj}}{Y_k Y_j}\right)\end{aligned}$$

We construct the dependent variable as $Z_{ij} \equiv \frac{X_{ij}}{Y_i Y_j}$, thus we obtain

$$\ln Z_{ij} = -\sigma \ln L - (\sigma - 1)\left(\ln \tau_{ij} - \sum_k \frac{L_k}{L} \ln \tau_{kj}\right) - \sigma \ln w_i - (\sigma - 1) \sum_k \frac{L_k}{L} \ln Z_{kj}$$

Derivation of Equation (14)

Let $\boldsymbol{Z} = (\boldsymbol{Z}_1', \boldsymbol{Z}_2', ..., \boldsymbol{Z}_n')'$, where $\boldsymbol{Z}_i = (\ln Z_{i1}, ..., \ln Z_{in})'$.

$$\boldsymbol{J}\text{diag}(\boldsymbol{L}) = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & & & \\ 1 & 1 & \cdots & 1 \end{pmatrix} \begin{pmatrix} \frac{L_1}{L} & 0 & \cdots & 0 \\ 0 & \frac{L_2}{L} & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & \frac{L_n}{L} \end{pmatrix} = \begin{pmatrix} \frac{L_1}{L} & \frac{L_2}{L} & \cdots & \frac{L_n}{L} \\ \frac{L_1}{L} & \frac{L_2}{L} & \cdots & \frac{L_n}{L} \\ \vdots & & & \\ \frac{L_1}{L} & \frac{L_2}{L} & \cdots & \frac{L_n}{L} \end{pmatrix}$$

Then

$$\boldsymbol{W}\boldsymbol{Z} = \{[\boldsymbol{J}\mathrm{diag}(\boldsymbol{L})] \otimes \boldsymbol{I}_n\}\,\boldsymbol{Z} = \begin{pmatrix} \frac{L_1}{L}\boldsymbol{I}_n & \frac{L_2}{L}\boldsymbol{I}_n & \cdots & \frac{L_n}{L}\boldsymbol{I}_n \\ \frac{L_1}{L}\boldsymbol{I}_n & \frac{L_2}{L}\boldsymbol{I}_n & \cdots & \frac{L_n}{L}\boldsymbol{I}_n \\ \vdots & & & \\ \frac{L_1}{L}\boldsymbol{I}_n & \frac{L_2}{L}\boldsymbol{I}_n & \cdots & \frac{L_n}{L}\boldsymbol{I}_n \end{pmatrix} \begin{pmatrix} \boldsymbol{Z}_1 \\ \boldsymbol{Z}_2 \\ \vdots \\ \boldsymbol{Z}_n \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^n \frac{L_k}{L}\boldsymbol{Z}_k \\ \sum_{k=1}^n \frac{L_k}{L}\boldsymbol{Z}_k \\ \vdots \\ \sum_{k=1}^n \frac{L_k}{L}\boldsymbol{Z}_k \end{pmatrix}$$

where

$$\sum_{k=1}^n \frac{L_k}{L}\boldsymbol{Z}_k = \frac{L_1}{L}\boldsymbol{Z}_1 + \frac{L_2}{L}\boldsymbol{Z}_2 + \cdots + \frac{L_n}{L}\boldsymbol{Z}_n$$

$$= \frac{L_1}{L}\begin{pmatrix} \ln Z_{11} \\ \ln Z_{12} \\ \vdots \\ \ln Z_{1n} \end{pmatrix} + \frac{L_2}{L}\begin{pmatrix} \ln Z_{21} \\ \ln Z_{22} \\ \vdots \\ \ln Z_{2n} \end{pmatrix} + \cdots + \frac{L_n}{L}\begin{pmatrix} \ln Z_{n1} \\ \ln Z_{n2} \\ \vdots \\ \ln Z_{nn} \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^n \frac{L_k}{L}\ln Z_{k1} \\ \sum_{k=1}^n \frac{L_k}{L}\ln Z_{k2} \\ \vdots \\ \sum_{k=1}^n \frac{L_k}{L}\ln Z_{kn} \end{pmatrix}$$

where the last vector is exactly the typical block vector when we stack the term $\sum_{k=1}^n \frac{L_k}{L}\ln Z_{kj}$ of (13). Similarly, let $\boldsymbol{\tau} = (\boldsymbol{\tau}_1', \boldsymbol{\tau}_2', ..., \boldsymbol{\tau}_n')'$, where $\boldsymbol{\tau}_i = (\ln \tau_{i1}, ..., \ln \tau_{in})'$. Then

$$\boldsymbol{W}\boldsymbol{\tau} = \begin{pmatrix} \frac{L_1}{L}\boldsymbol{I}_n & \frac{L_2}{L}\boldsymbol{I}_n & \cdots & \frac{L_n}{L}\boldsymbol{I}_n \\ \frac{L_1}{L}\boldsymbol{I}_n & \frac{L_2}{L}\boldsymbol{I}_n & \cdots & \frac{L_n}{L}\boldsymbol{I}_n \\ \vdots & & & \\ \frac{L_1}{L}\boldsymbol{I}_n & \frac{L_2}{L}\boldsymbol{I}_n & \cdots & \frac{L_n}{L}\boldsymbol{I}_n \end{pmatrix} \begin{pmatrix} \boldsymbol{\tau}_1 \\ \boldsymbol{\tau}_2 \\ \vdots \\ \boldsymbol{\tau}_n \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^n \frac{L_k}{L}\boldsymbol{Z}_k \\ \sum_{k=1}^n \frac{L_k}{L}\boldsymbol{Z}_k \\ \vdots \\ \sum_{k=1}^n \frac{L_k}{L}\boldsymbol{Z}_k \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^n \frac{L_k}{L}\boldsymbol{\tau}_k \\ \sum_{k=1}^n \frac{L_k}{L}\boldsymbol{\tau}_k \\ \vdots \\ \sum_{k=1}^n \frac{L_k}{L}\boldsymbol{\tau}_k \end{pmatrix}$$

where

$$\sum_{k=1}^n \frac{L_k}{L}\boldsymbol{\tau}_k = \frac{L_1}{L}\boldsymbol{\tau}_1 + \frac{L_2}{L}\boldsymbol{\tau}_2 + \cdots + \frac{L_n}{L}\boldsymbol{\tau}_n$$

$$= \frac{L_1}{L}\begin{pmatrix} \ln \tau_{11} \\ \ln \tau_{12} \\ \vdots \\ \ln \tau_{1n} \end{pmatrix} + \frac{L_2}{L}\begin{pmatrix} \ln \tau_{21} \\ \ln \tau_{22} \\ \vdots \\ \ln \tau_{2n} \end{pmatrix} + \cdots + \frac{L_n}{L}\begin{pmatrix} \ln \tau_{n1} \\ \ln \tau_{n2} \\ \vdots \\ \ln \tau_{nn} \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^n \frac{L_k}{L}\ln \tau_{k1} \\ \sum_{k=1}^n \frac{L_k}{L}\ln \tau_{k2} \\ \vdots \\ \sum_{k=1}^n \frac{L_k}{L}\ln \tau_{kn} \end{pmatrix}$$

where the last vector is exactly the typical block vector when we stack the term $\sum_{k=1}^n \frac{L_k}{L}\ln \tau_{kj}$ of (13).

Appendix II. Models and Log-likelihood functions

SARMA Model

Log-likelihood function

Assuming homogeneous coefficients model and Moving Average error structure, we have

$$\boldsymbol{Z} = \beta_0 \boldsymbol{\iota} + \beta_1 \tilde{\boldsymbol{d}} + \beta_2 \boldsymbol{w} + \beta_3 \tilde{\boldsymbol{b}} + \rho \boldsymbol{W}_d \boldsymbol{Z} + \boldsymbol{u}$$

$$\boldsymbol{u} = \boldsymbol{\varepsilon}_1 + \lambda \boldsymbol{W}_d \boldsymbol{\varepsilon}_1, \quad \boldsymbol{\varepsilon}_1 \sim N(\boldsymbol{0}, \sigma^2 \boldsymbol{I})$$

Let $\boldsymbol{S}_1(\lambda) = \boldsymbol{I} + \lambda \boldsymbol{W}_d$, $\boldsymbol{S}_2(\rho) = \boldsymbol{I} - \rho \boldsymbol{W}_d$, $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{\iota} & \tilde{\boldsymbol{d}} & \boldsymbol{w} & \tilde{\boldsymbol{b}} \end{bmatrix}$, $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2, \beta_3)'$, $\boldsymbol{\theta} = (\beta_0, \beta_1, \beta_2, \beta_3, \rho, \lambda)'$. Simple algebra shows that $\boldsymbol{\varepsilon}_1(\boldsymbol{\theta}) = \boldsymbol{S}_1(\lambda)^{-1}[\boldsymbol{S}_2(\rho)\boldsymbol{Z} - \boldsymbol{X}\boldsymbol{\beta}]$. Then the log-likelihood function is

$$\ell_1(\boldsymbol{\theta}, \sigma^2) = -\frac{n^2}{2} \ln 2\pi - \frac{n^2}{2} \ln \sigma^2 + \ln |\boldsymbol{S}_2(\rho)| - \ln |\boldsymbol{S}_1(\lambda)| - \frac{1}{2\sigma^2} \boldsymbol{\varepsilon}_1'(\boldsymbol{\theta}) \boldsymbol{\varepsilon}_1(\boldsymbol{\theta})$$

Score function and Concentrated log-likelihood function

Denote $\boldsymbol{s}$ as the full score vector, then

$$\begin{aligned}
\boldsymbol{s}_{\boldsymbol{\beta}} &= \frac{1}{\sigma^2} \boldsymbol{X}' \boldsymbol{S}_1'(\lambda)^{-1} \boldsymbol{\varepsilon}_1(\boldsymbol{\theta}) \\
\boldsymbol{s}_{\lambda} &= -\mathrm{tr}\left[\boldsymbol{S}_1(\lambda)^{-1} \boldsymbol{W}_d\right] + \frac{1}{\sigma^2} \boldsymbol{\varepsilon}_1'(\boldsymbol{\theta}) \boldsymbol{S}_1(\lambda)^{-1} \boldsymbol{W}_d \boldsymbol{\varepsilon}_1(\boldsymbol{\theta}) \\
\boldsymbol{s}_{\rho} &= -\mathrm{tr}\left[\boldsymbol{S}_2(\rho)^{-1} \boldsymbol{W}_d\right] + \frac{1}{\sigma^2} \boldsymbol{\varepsilon}_1'(\boldsymbol{\theta}) \boldsymbol{S}_1(\lambda)^{-1} \boldsymbol{W}_d \boldsymbol{Z} \\
\boldsymbol{s}_{\sigma^2} &= -\frac{n^2}{2\sigma^2} + \frac{1}{2(\sigma^2)^2} \boldsymbol{\varepsilon}_1'(\boldsymbol{\theta}) \boldsymbol{\varepsilon}_1(\boldsymbol{\theta})
\end{aligned}$$

Set $\boldsymbol{s}_{\boldsymbol{\beta}} = \boldsymbol{0}$, $\boldsymbol{s}_{\sigma^2} = 0$, let $\widehat{\boldsymbol{S}}_1 = \boldsymbol{S}_1(\widehat{\lambda})$, $\widehat{\boldsymbol{S}}_2 = \boldsymbol{S}_2(\widehat{\rho})$. We get

$$\widehat{\boldsymbol{\beta}} = \left[\boldsymbol{X}' \widehat{\boldsymbol{S}}_1'^{-1} \widehat{\boldsymbol{S}}_1^{-1} \boldsymbol{X}\right]^{-1} \left[\boldsymbol{X}' \widehat{\boldsymbol{S}}_1'^{-1} \widehat{\boldsymbol{S}}_1^{-1} \widehat{\boldsymbol{S}}_2 \boldsymbol{Z}\right]$$

$$\widehat{\sigma}^2 = \frac{1}{n^2} \boldsymbol{\varepsilon}_1'(\widehat{\boldsymbol{\theta}}) \boldsymbol{\varepsilon}_1(\widehat{\boldsymbol{\theta}}) = \frac{1}{n^2} \boldsymbol{Z}' \widehat{\boldsymbol{S}}_2' \widehat{\boldsymbol{S}}_1'^{-1} \widehat{\boldsymbol{M}}_1 \widehat{\boldsymbol{S}}_1^{-1} \widehat{\boldsymbol{S}}_2 \boldsymbol{Z}$$

where $\widehat{\boldsymbol{M}}_1 = \boldsymbol{I} - \widehat{\boldsymbol{S}}_1^{-1} \boldsymbol{X} \left(\boldsymbol{X}' \widehat{\boldsymbol{S}}_1'^{-1} \widehat{\boldsymbol{S}}_1^{-1} \boldsymbol{X}\right)^{-1} \boldsymbol{X}' \widehat{\boldsymbol{S}}_1'^{-1}$ is the $n^2 \times n^2$ annihilator matrix. Thus the concentrated log-likelihood function is

$$\ell_1^c(\lambda, \rho) = -\frac{n^2}{2}[\ln(2\pi) + 1] + \ln |\boldsymbol{S}_2(\rho)| - \ln |\boldsymbol{S}_1(\lambda)| - \frac{n^2}{2} \ln \widehat{\sigma}^2$$

Second order derivatives

Denote $\boldsymbol{H}$ as the hessian matrix, we get

$$\boldsymbol{h}_{\boldsymbol{\beta}\boldsymbol{\beta}'} = -\frac{1}{\sigma^2}\boldsymbol{X}'\boldsymbol{S}_1'(\lambda)^{-1}\boldsymbol{S}_1(\lambda)^{-1}\boldsymbol{X},\ \boldsymbol{h}_{\boldsymbol{\beta}\lambda} = -\frac{1}{\sigma^2}\boldsymbol{X}'\boldsymbol{S}_1'(\lambda)^{-1}\left[\boldsymbol{W}_d'\boldsymbol{S}_1'(\lambda)^{-1} + \boldsymbol{S}_1(\lambda)^{-1}\boldsymbol{W}_d\right]\boldsymbol{\varepsilon}_1(\boldsymbol{\theta})$$

$$\boldsymbol{h}_{\boldsymbol{\beta}\boldsymbol{\rho}} = -\frac{1}{\sigma^2}\boldsymbol{X}'\boldsymbol{S}_1'(\lambda)^{-1}\boldsymbol{S}_1(\lambda)^{-1}\boldsymbol{W}_d\boldsymbol{Z},\ \boldsymbol{h}_{\boldsymbol{\beta}\sigma^2} = -\frac{1}{(\sigma^2)^2}\boldsymbol{X}'\boldsymbol{S}_1'(\lambda)^{-1}\boldsymbol{\varepsilon}_1(\boldsymbol{\theta})$$

$$\boldsymbol{h}_{\lambda\lambda} = \operatorname{tr}\left[\boldsymbol{S}_1(\lambda)^{-1}\boldsymbol{W}_d\boldsymbol{S}_1(\lambda)^{-1}\boldsymbol{W}_d\right] - \frac{1}{\sigma^2}\boldsymbol{\varepsilon}_1'(\boldsymbol{\theta})\left[\boldsymbol{W}_d'\boldsymbol{S}_1'(\lambda)^{-1} + 2\boldsymbol{S}_1(\lambda)^{-1}\boldsymbol{W}_d\right]\boldsymbol{S}_1(\lambda)^{-1}\boldsymbol{W}_d\boldsymbol{\varepsilon}_1(\boldsymbol{\theta})$$

$$\boldsymbol{h}_{\lambda\rho} = -\frac{1}{\sigma^2}\boldsymbol{\varepsilon}_1'(\boldsymbol{\theta})\left[\boldsymbol{W}_d'\boldsymbol{S}_1'(\lambda)^{-1} + \boldsymbol{S}_1(\lambda)^{-1}\boldsymbol{W}_d\right]\boldsymbol{S}_1(\lambda)^{-1}\boldsymbol{W}_d\boldsymbol{Z}$$

$$\boldsymbol{h}_{\lambda\sigma^2} = -\frac{1}{(\sigma^2)^2}\boldsymbol{\varepsilon}_1'(\boldsymbol{\theta})\boldsymbol{S}_1(\lambda)^{-1}\boldsymbol{W}_d\boldsymbol{\varepsilon}_1(\boldsymbol{\theta})$$

$$\boldsymbol{h}_{\rho\rho} = -\operatorname{tr}\left[\boldsymbol{S}_2(\rho)^{-1}\boldsymbol{W}_d\boldsymbol{S}_2(\rho)^{-1}\boldsymbol{W}_d\right] - \frac{1}{\sigma^2}\boldsymbol{Z}'\boldsymbol{W}_d'\boldsymbol{S}_1'(\lambda)^{-1}\boldsymbol{S}_1(\lambda)^{-1}\boldsymbol{W}_d\boldsymbol{Z}$$

$$\boldsymbol{h}_{\rho\sigma^2} = -\frac{1}{(\sigma^2)^2}\boldsymbol{\varepsilon}_1'(\boldsymbol{\theta})\boldsymbol{S}_1(\lambda)^{-1}\boldsymbol{W}_d\boldsymbol{Z},\ \boldsymbol{h}_{\sigma^2\sigma^2} = \frac{n^2}{2(\sigma^2)^2} - \frac{1}{(\sigma^2)^3}\boldsymbol{\varepsilon}_1'(\boldsymbol{\theta})\boldsymbol{\varepsilon}_1(\boldsymbol{\theta})$$

SARAR Model

Log-likelihood function

Alternatively, we can specify the error structure as

$$\boldsymbol{u} = \lambda\boldsymbol{W}_d\boldsymbol{u} + \boldsymbol{\varepsilon}_2, \quad \boldsymbol{\varepsilon}_2 \sim N(\boldsymbol{0}, \sigma^2\boldsymbol{I})$$

Let $\boldsymbol{S}_3(\lambda) = \boldsymbol{I} - \lambda\boldsymbol{W}_d$, we get $\boldsymbol{\varepsilon}_2(\boldsymbol{\theta}) = \boldsymbol{S}_3(\lambda)[\boldsymbol{S}_2(\rho)\boldsymbol{Z} - \boldsymbol{X}\boldsymbol{\beta}]$, then the log-likelihood function is

$$\ell_2(\boldsymbol{\theta}, \sigma^2) = -\frac{n^2}{2}\ln 2\pi - \frac{n^2}{2}\ln\sigma^2 + \ln|\boldsymbol{S}_2(\rho)| + \ln|\boldsymbol{S}_3(\lambda)| - \frac{1}{2\sigma^2}\boldsymbol{\varepsilon}_2'(\boldsymbol{\theta})\boldsymbol{\varepsilon}_2(\boldsymbol{\theta})$$

Score function and Concentrated log-likelihood function

Denote $\boldsymbol{s}$ as the full score vector, then

$$\begin{aligned}
\boldsymbol{s}_{\boldsymbol{\beta}} &= \frac{1}{\sigma^2}\boldsymbol{X}'\boldsymbol{S}_3'(\lambda)\boldsymbol{\varepsilon}_2(\boldsymbol{\theta}) \\
\boldsymbol{s}_{\lambda} &= -\operatorname{tr}\left[\boldsymbol{S}_3(\lambda)^{-1}\boldsymbol{W}_d\right] + \frac{1}{\sigma^2}\boldsymbol{\varepsilon}_2'(\boldsymbol{\theta})\boldsymbol{W}_d[\boldsymbol{S}_2(\rho)\boldsymbol{Z} - \boldsymbol{X}\boldsymbol{\beta}] \\
\boldsymbol{s}_{\rho} &= -\operatorname{tr}\left[\boldsymbol{S}_2(\rho)^{-1}\boldsymbol{W}_d\right] + \frac{1}{\sigma^2}\boldsymbol{\varepsilon}_2'(\boldsymbol{\theta})\boldsymbol{S}_3(\lambda)\boldsymbol{W}_d\boldsymbol{Z} \\
\boldsymbol{s}_{\sigma^2} &= -\frac{n^2}{2\sigma^2} + \frac{1}{2(\sigma^2)^2}\boldsymbol{\varepsilon}_2'(\boldsymbol{\theta})\boldsymbol{\varepsilon}_2(\boldsymbol{\theta})
\end{aligned}$$

Set $\boldsymbol{s}_{\boldsymbol{\beta}} = \boldsymbol{0}$, $\boldsymbol{s}_{\sigma^2} = 0$, and let $\widehat{\boldsymbol{S}}_3 = \boldsymbol{S}_3(\widehat{\lambda})$, $\widehat{\boldsymbol{S}}_2 = \boldsymbol{S}_2(\widehat{\rho})$. We get

$$\widehat{\boldsymbol{\beta}} = \left[\boldsymbol{X}'\widehat{\boldsymbol{S}}_3'\widehat{\boldsymbol{S}}_3\boldsymbol{X}\right]^{-1}\left[\boldsymbol{X}'\widehat{\boldsymbol{S}}_3'\widehat{\boldsymbol{S}}_3\widehat{\boldsymbol{S}}_2\boldsymbol{Z}\right]$$

$$\widehat{\sigma}^2 = \frac{1}{n^2}\boldsymbol{\varepsilon}_2'(\widehat{\boldsymbol{\theta}})\boldsymbol{\varepsilon}_2(\widehat{\boldsymbol{\theta}}) = \frac{1}{n^2}\boldsymbol{Z}'\widehat{\boldsymbol{S}}_2'\widehat{\boldsymbol{S}}_3'\widehat{\boldsymbol{M}}_2\widehat{\boldsymbol{S}}_3\widehat{\boldsymbol{S}}_2\boldsymbol{Z}$$

where $\widehat{\boldsymbol{M}}_2 = \boldsymbol{I} - \widehat{\boldsymbol{S}}_3\boldsymbol{X}\left(\boldsymbol{X}'\widehat{\boldsymbol{S}}_3'\widehat{\boldsymbol{S}}_3\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\widehat{\boldsymbol{S}}_3'$ is the $n^2 \times n^2$ annihilator matrix. Thus the concentrated log-likelihood function is

$$\ell_2^c(\lambda,\rho) = -\frac{n^2}{2}[\ln(2\pi)+1] + \ln|\boldsymbol{S}_2(\rho)| + \ln|\boldsymbol{S}_3(\lambda)| - \frac{n^2}{2}\ln\widehat{\sigma}^2$$

Second order derivatives

Let $\boldsymbol{H}$ be the hessian matrix, then

$$\begin{aligned}
h_{\boldsymbol{\beta}\boldsymbol{\beta}'} &= -\frac{1}{\sigma^2}\boldsymbol{X}'\boldsymbol{S}_3'(\lambda)\boldsymbol{S}_3(\lambda)\boldsymbol{X} \\
h_{\boldsymbol{\beta}\lambda} &= -\frac{1}{\sigma^2}\boldsymbol{X}'[\boldsymbol{W}_d'\boldsymbol{S}_3(\lambda) + \boldsymbol{S}_3'(\lambda)\boldsymbol{W}_d][\boldsymbol{S}_2(\rho) - \boldsymbol{X}\boldsymbol{\beta}] \\
h_{\boldsymbol{\beta}\rho} &= -\frac{1}{\sigma^2}\boldsymbol{X}'\boldsymbol{S}_3'(\lambda)\boldsymbol{S}_3(\lambda)\boldsymbol{W}_d\boldsymbol{Z} \\
h_{\boldsymbol{\beta}\sigma^2} &= -\frac{1}{(\sigma^2)^2}\boldsymbol{X}'\boldsymbol{S}_3'(\lambda)\boldsymbol{\varepsilon}_2(\boldsymbol{\theta}) \\
h_{\lambda\lambda} &= -\mathrm{tr}\left[\boldsymbol{S}_3(\lambda)^{-1}\boldsymbol{W}_d\boldsymbol{S}_3(\lambda)^{-1}\boldsymbol{W}_d\right] - \frac{1}{\sigma^2}[\boldsymbol{S}_2(\rho)\boldsymbol{Z} - \boldsymbol{X}\boldsymbol{\beta}]'\boldsymbol{W}_d'\boldsymbol{W}_d[\boldsymbol{S}_2(\rho)\boldsymbol{Z} - \boldsymbol{X}\boldsymbol{\beta}] \\
h_{\lambda\rho} &= -\frac{1}{\sigma^2}[\boldsymbol{S}_2(\rho)\boldsymbol{Z} - \boldsymbol{X}\boldsymbol{\beta}]'[\boldsymbol{W}_d'\boldsymbol{S}_3(\lambda) + \boldsymbol{S}_3'(\lambda)\boldsymbol{W}_d]\boldsymbol{W}_d\boldsymbol{Z} \\
h_{\lambda\sigma^2} &= -\frac{1}{(\sigma^2)^2}\boldsymbol{\varepsilon}_2'(\boldsymbol{\theta})\boldsymbol{W}_d[\boldsymbol{S}_2(\rho)\boldsymbol{Z} - \boldsymbol{X}\boldsymbol{\beta}] \\
h_{\rho\rho} &= -\mathrm{tr}\left[\boldsymbol{S}_2(\rho)^{-1}\boldsymbol{W}_d\boldsymbol{S}_2(\rho)^{-1}\boldsymbol{W}_d\right] - \frac{1}{\sigma^2}[\boldsymbol{S}_3(\lambda)\boldsymbol{W}_d\boldsymbol{Z}]'\boldsymbol{S}_3(\lambda)\boldsymbol{W}_d\boldsymbol{Z} \\
h_{\rho\sigma^2} &= -\frac{1}{(\sigma^2)^2}\boldsymbol{\varepsilon}_2'(\boldsymbol{\theta})\boldsymbol{S}_3(\lambda)\boldsymbol{W}_d\boldsymbol{Z} \\
h_{\sigma^2\sigma^2} &= \frac{n^2}{2(\sigma^2)^2} - \frac{1}{(\sigma^2)^3}\boldsymbol{\varepsilon}_2'(\boldsymbol{\theta})\boldsymbol{\varepsilon}_2(\boldsymbol{\theta})
\end{aligned}$$

Appendix III. Some Supplemental Information

Variables	Definition	Data source
X_{ij}	Export from country i to country j HS2002, all commodities	United Nations Commodity Trade Statistics Database
Y_i	GDP at current US dollar	World Bank Databank
X_{ii}	X_{ii} = GDP at current US dollar − total export to world HS2002, all commodities	United Nations Commodity Trade Statistics Database
L_i	Labor force population ages 15-64 = Labor force participation rate (% of population ages 15-64) * total population ages 15-64	World Bank Databank
d_{ij}	Great circle distance between the geographic coordinates of countries' capitals	coordinates from worldatlas.com distances calculated by R script
d_{ii}	$d_{ii} \equiv \frac{2}{3}\sqrt{\text{surface}_i/\pi}$ alternatively $d'_{ii} \equiv \frac{1}{3}\sqrt{\text{surface}_i/\pi}$ surface_i = country's surface area (km_2)	World Bank Databank
w_i	$w_i = \frac{\text{compensation of employees in manufacturing}}{\text{total employment ages 15+ in manufacturing}}$ compensation from national accounts (current account) NACE.R2 - manufacturing	Eurostat

Table 3: Data description, definition and sources

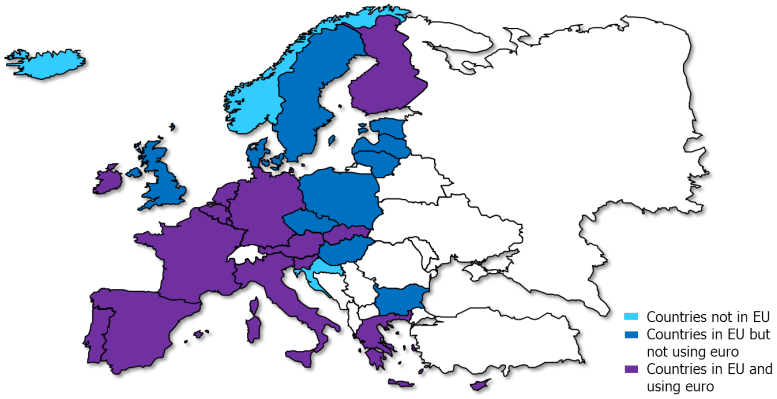

Figure 3: Countries of interest (Euro and EU member as of Jan-01-2010)

Countries	**EU member since**	**Euro currency since**
Austria	1995-01-01	1999-01-01
Belgium	Founder	1999-01-01
Bulgaria	2007-01-01	×
Croatia	×	×
Cyprus	2004-05-01	2008-01-01
Czech Republic	2004-05-01	×
Denmark	1973-01-01	×
Estonia	2004-05-01	2011-01-01
Finland	1995-01-01	1999-01-01
France	Founder	1999-01-01
Germany	Founder	1999-01-01
Greece	1981-01-01	2001-01-01
Hungary	2004-05-01	×
Iceland	×	×
Ireland	1973-01-01	1999-01-01
Italy	Founder	1999-01-01
Latvia	2004-05-01	×
Lithuania	2004-05-01	×
Luxembourg	Founder	1999-01-01
Malta	2004-05-01	2008-01-01
Netherlands	Founder	1999-01-01
Norway	×	×
Poland	2004-05-01	×
Portugal	1986-01-01	1999-01-01
Slovakia	2004-05-01	2009-01-01
Slovenia	2004-05-01	2007-01-01
Spain	1986-01-01	1999-01-01
Sweden	1995-01-01	×
United Kingdom	1973-01-01	×

Table 4: List of countries, EU membership and Euro currency