Composite Input-Output Production Functions

An algorithm to linear combination of subsector cost shares

Randall Jackson

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Abstract. This document describes the algorithm used for creating an aggregated linear production function for an industry by weighting subsector production functions. The result can be used as a column in an interindustry (IxI) coefficients table or in a standard Use table (CxI) depending on the units (C or I) of the input data.

Introduction

Each power generating technology \( k \in K \) has a corresponding production function. When the production function is assumed to be linear, each technology’s production function corresponds to a set of cost shares whose sum over all inputs is 1. Define

\[
A_j^k = \begin{bmatrix}
  a_{ij}^k \\
a_{ij}^k \\
\vdots \\
a_{n-1j}^k \\
a_{nj}^k
\end{bmatrix}
\]

be the cost shares for technology subsector \( k \) in power generation industry \( j \). Then let \( Z_j^k \) be the contribution of subsector \( k \) to industry \( j \) output. \( Z \) can be expressed in dollar terms or in proportionate weights. The industry \( j \) composite cost shares can be computed as

\[
A_j = \frac{\sum_{k=1}^{K} A_j^k Z_j^k}{\sum_{k=1}^{K} Z_j^k}
\]

In matrix notation, \( A \) is a normalized cost or cost share matrix with \( N \) industries and \( K \) technologies, \( z \) is a \( K \) dimensional vector of the weights of the respective sectors in the composite sector, \( x \) is the sum of the weights, and \( i \) is an appropriately dimensioned summing vector. Then the compositing function is

\[
\left( \frac{1}{z_i} \right) A^z_i
\]
Supporting Algorithm(s)/Code.

function [t] = techagg(A,z)
% PURPOSE: create an aggregated input–output column from subsectors,
% given subsector coefficient matrix and weights vector
% ---------------------------------------------------------------
% USAGE: t = techagg(A,z)
% where A is an nxk matrix of coefficient cost shares
% and z is a k dimensional vector of weights, either shares or
% levels
% INPUT:
% -> A is an nxk matrix of coefficient cost shares
% -> z is a k dimensional vector of weights, either shares or levels
% OUTPUT:
% -> t is an n dimensional vector of aggregate cost shares
% ---------------------------------------------------------------
% REFERENCES: None
% ---------------------------------------------------------------
% Written by: Randy Jackson, 08/07/2013
% Current e-mail: info@econalyze.com

wtsum=sum(sum(z)');
t=(A*diag(z)/wtsum)*ones(length(z),1);

% ---------------------------------------------------------------